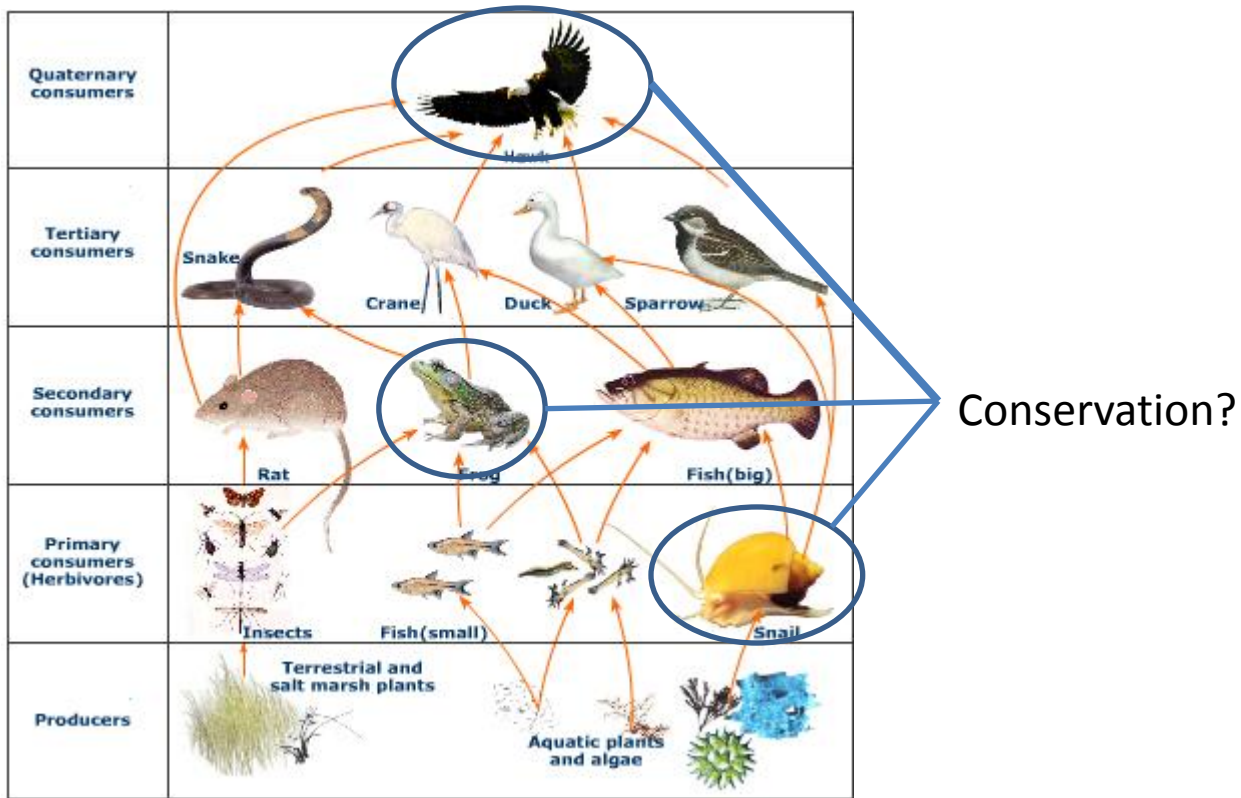


# The Mathematics of Ecological Networks Management

Nathalie Peyrard and Régis Sabbadin  
Applied Mathematics and Computer Science  
Laboratory, INRA-Toulouse  
Unité de Mathématiques et Informatique appliqués,  
Toulouse (MIAT)

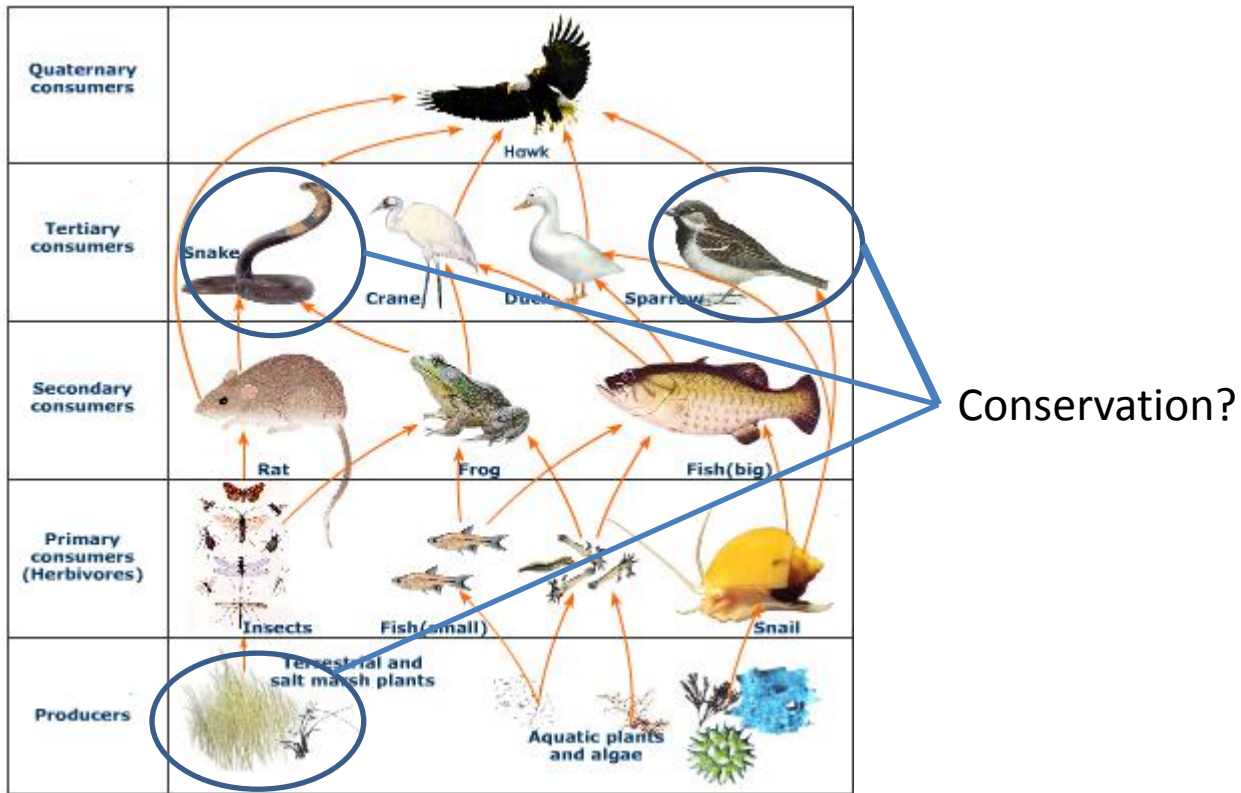
*Workshop on Mathematics for Ecological Management  
Toulouse, March 11, 2013*

# Conservation of multiple species in food webs



On which species should we spend our money if our goal is to preserve biodiversity?

# Conservation of multiple species in food webs



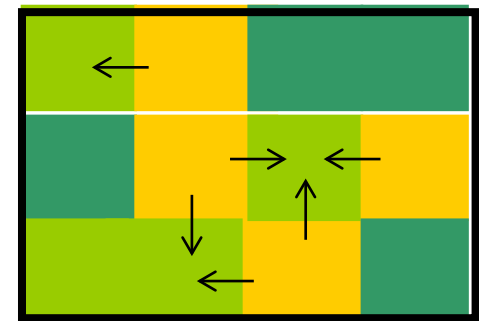
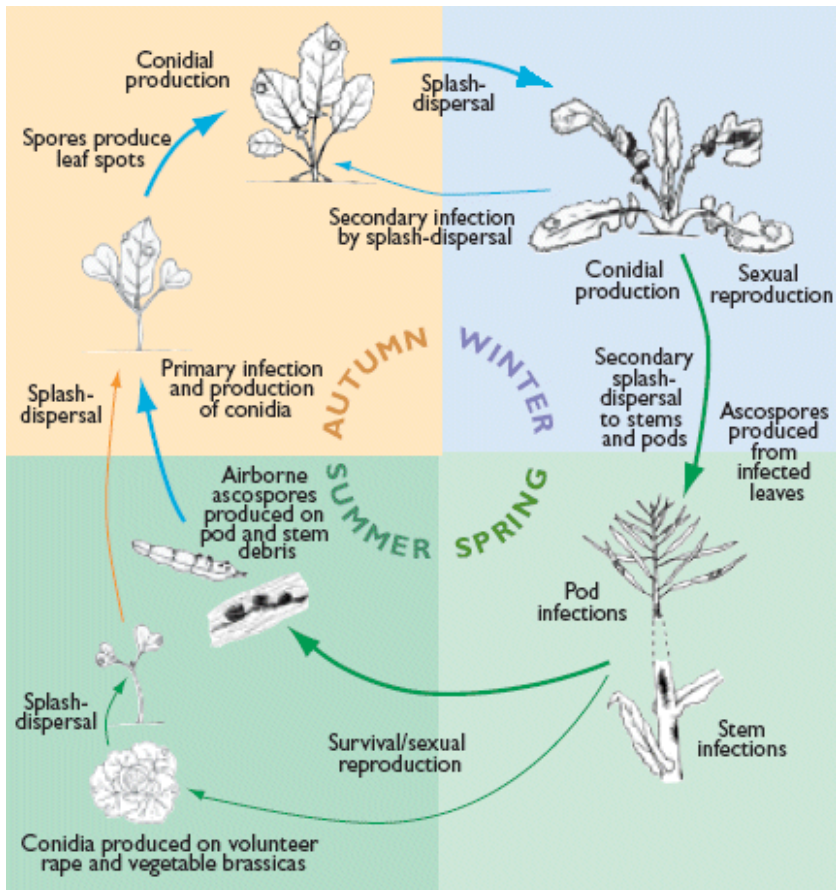
On which species should we spend our money if our goal is to preserve biodiversity?

# Spatial sampling of weeds for map reconstruction



How can we choose adaptively the locations to sample in order to reconstruct a “reliable” weed map?

# Collective management of crop resistance to pathogens

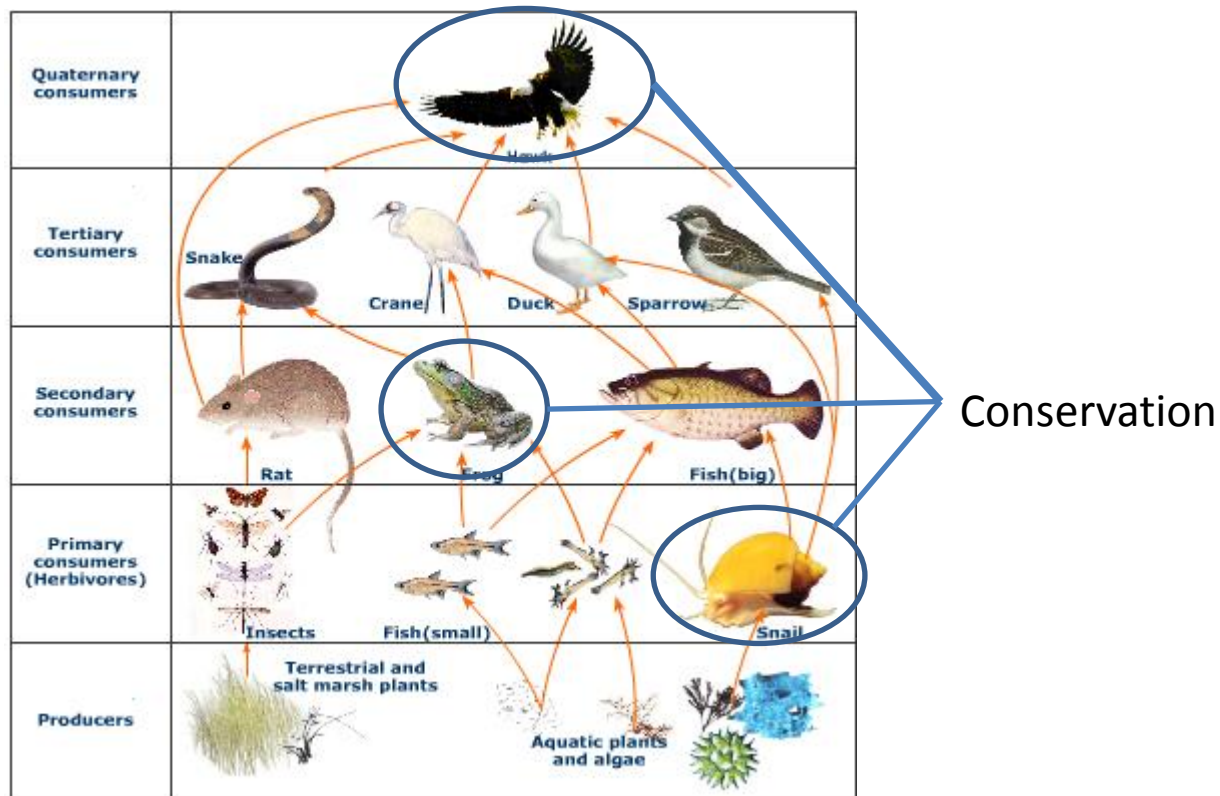


Where and when should we allocate Resistant crops/protection actions?



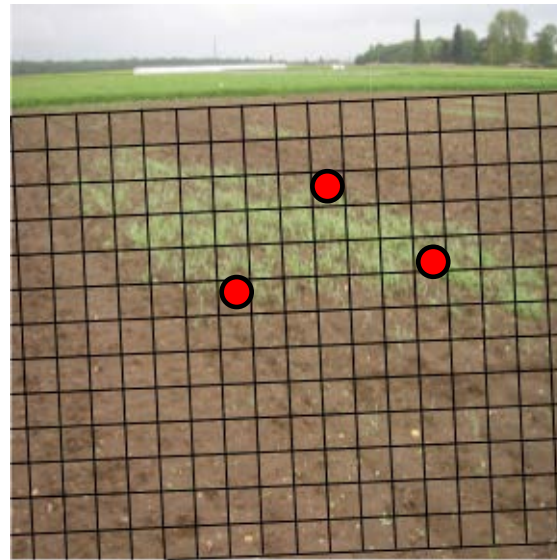
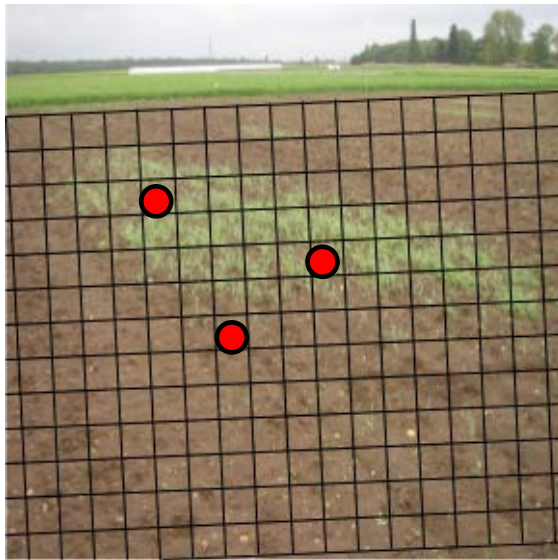
# These problems are ecological **management** problems

- **Choosing** and **applying** conservation actions!



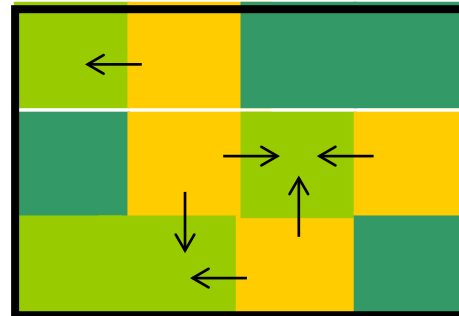
# These problems are ecological **management** problems

- Choosing and applying conservation actions!
- **Choosing** (adaptively) weed **sample locations**



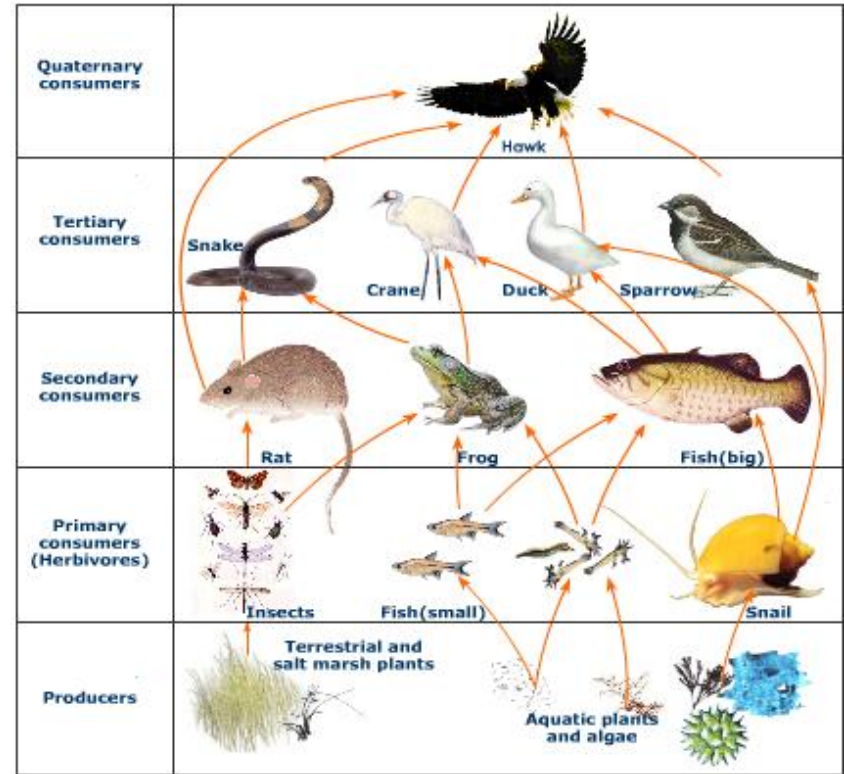
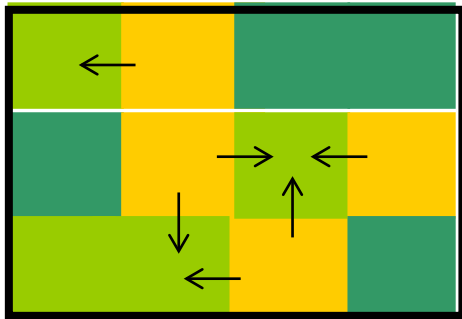
# These problems are ecological **management** problems

- Choosing and applying conservation actions!
- Choosing (adaptively) weed sample locations
- **Allocating** crop systems in space/time





# These problems involve **networks**



# These problems involve **uncertainty**

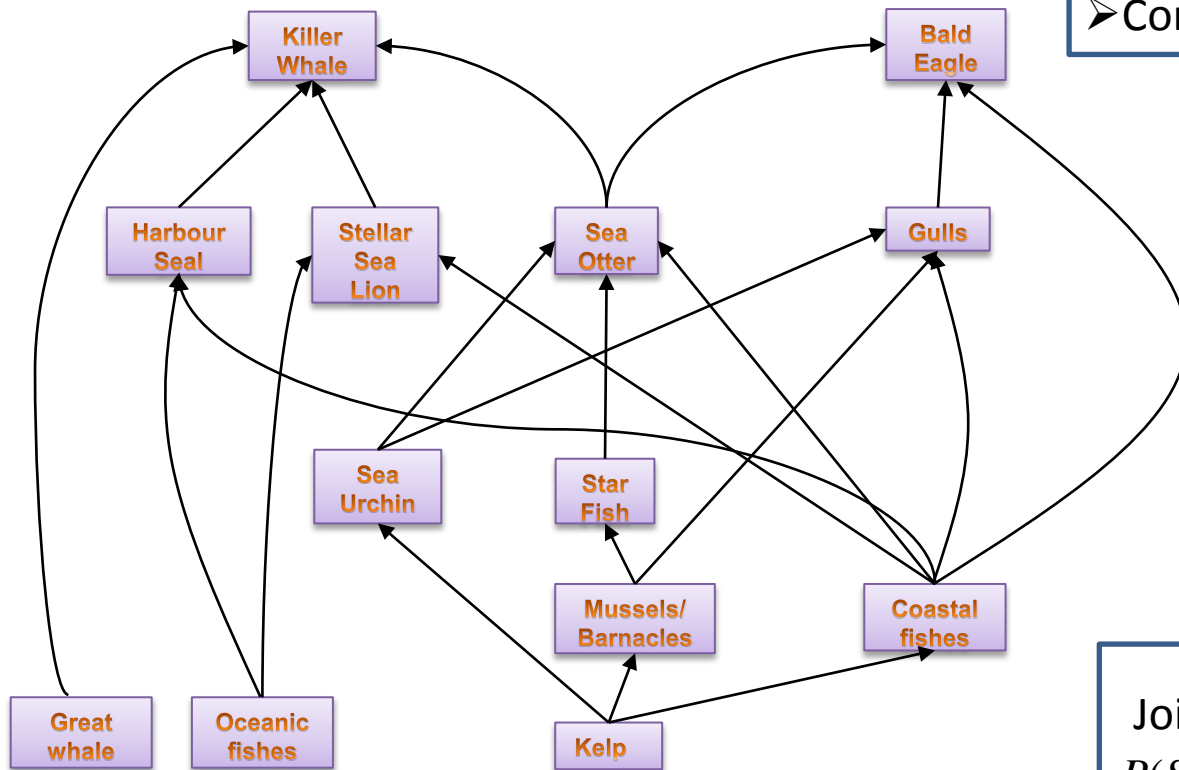
- Threatened species persistence is **uncertain!**
- Weeds (especially the seed bank) are barely **detectable!**
- Pathogens dynamics/spread are **uncertain!**

# Mathematics of ecological networks management

- Mathematical tools for the *Management of stochastic processes on networks*
- These mathematical tools are based on **Stochastic graphical models**
  - Bayesian networks
  - Markov Random Fields
  - Factored Markov Decision Processes
- Plus the use of **optimization/approximation methods**
  - Dynamic programming
  - Reinforcement learning
  - Heuristics

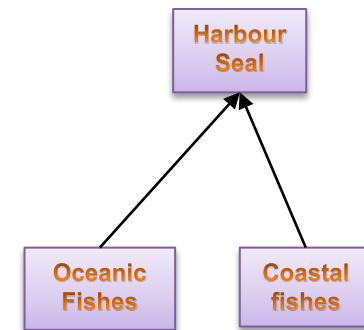
# Bayesian networks

Concisely express joint probability distributions over sets of variables



- A directed acyclic graph over variables
- Conditional probability tables

Conditional probabilities  
 $P(HS|OF,CF)$



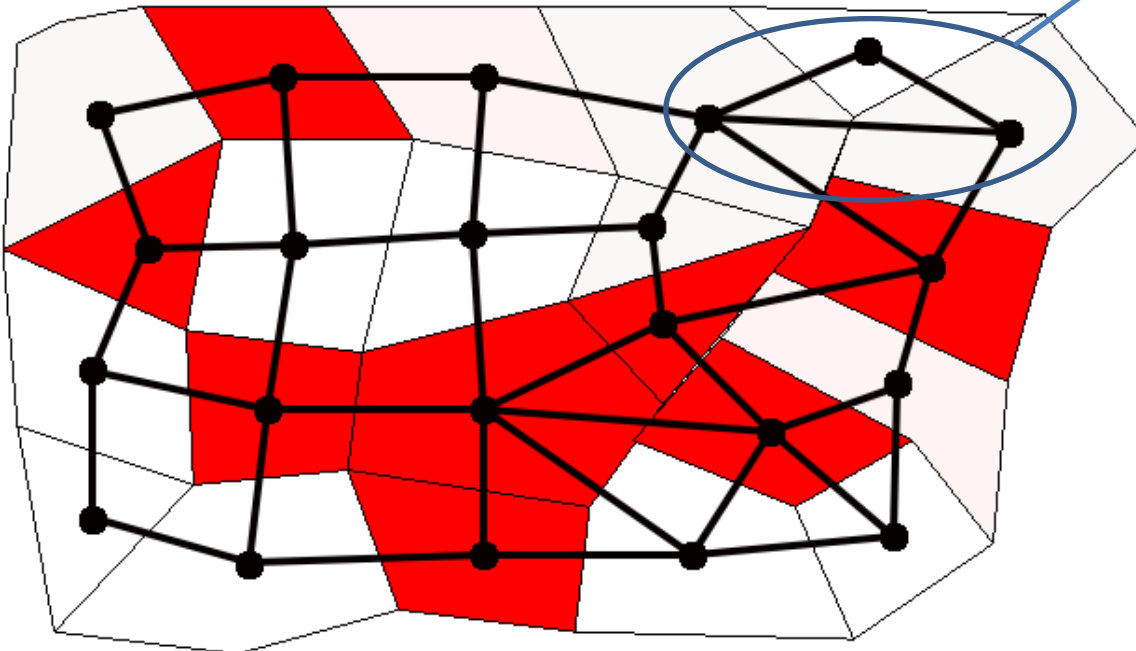
Joint probability model:

$$P(S_1, \dots, S_n) = \prod_{i=1..n} P_i(S_i | \text{Preys}(S_i))$$

# Markov random fields

A framework for representing uncertain knowledge about spatial processes

Network representation of a spatial process



$\Psi_c(x_c)$

- Undirected graph with cycles
- A set of potential functions over cliques:  $\Psi_c(x_c) > 0, \forall x_c$

MRF probability distribution:

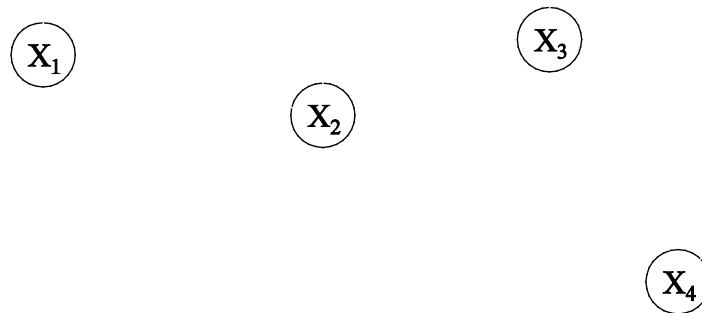
$$P(x) \propto \prod_{c \in \mathcal{C}} \Psi_c(x_c)$$



# Graph-based Markov Decision Processes

Structured problems of sequential decision under uncertainty

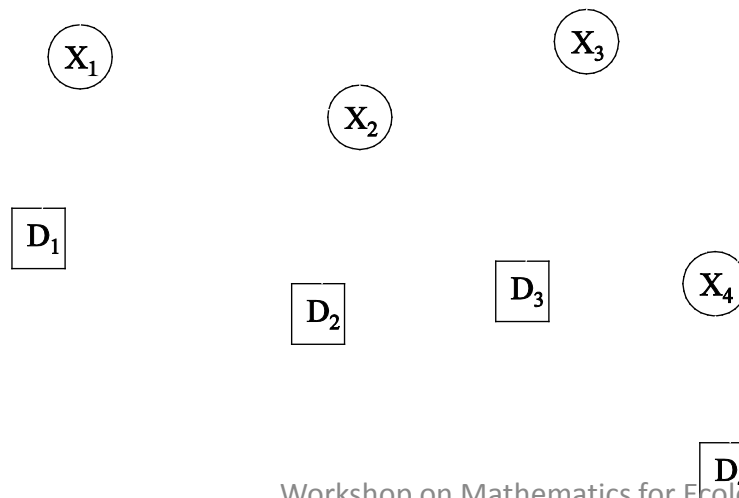
- Several **state variables**  $\{X_i\}$



# Graph-based Markov Decision Processes

Structured problems of sequential decision under uncertainty

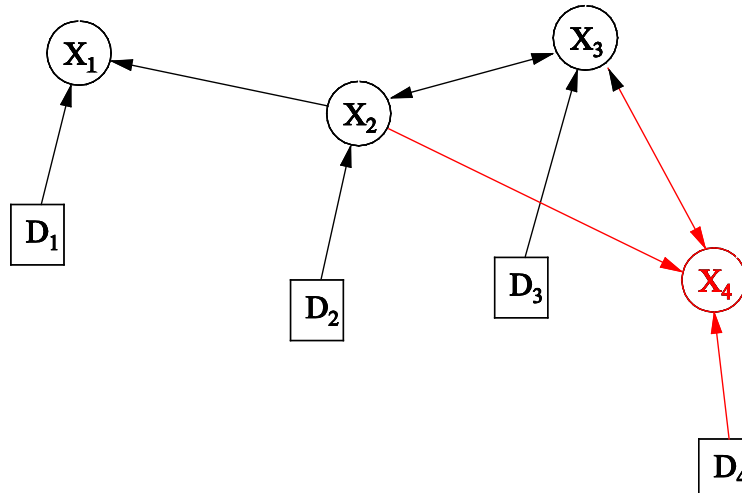
- Several state variables  $\{X_i\}_{i \in V}$  and decision variables  $\{A_i\}_{i \in V}$



# Graph-based Markov Decision Processes

## Structured problems of sequential decision under uncertainty

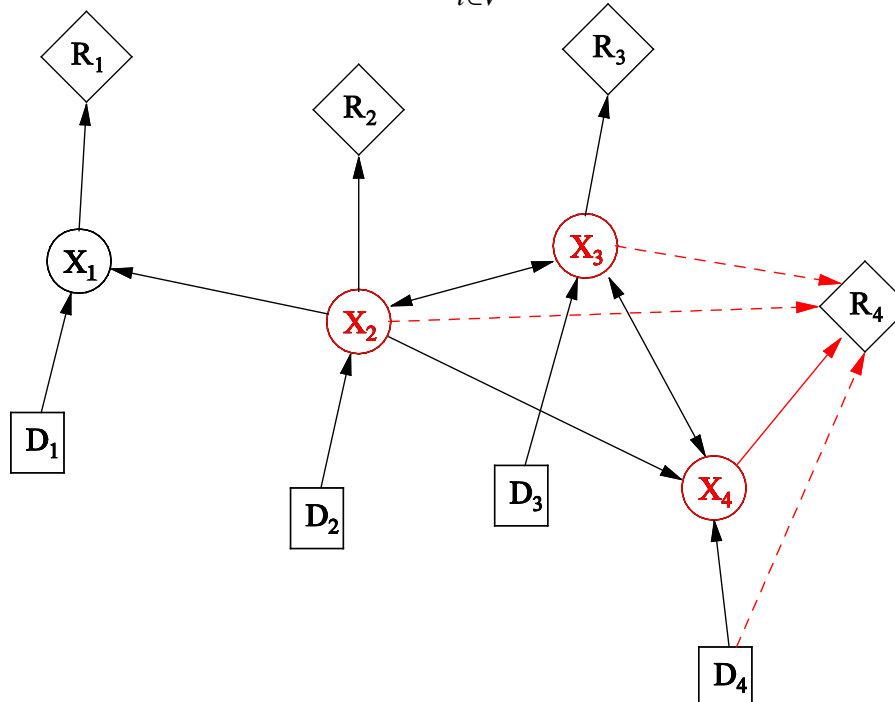
- Several state variables  $\{X_i\}_{i \in V}$  and decision variables  $\{A_i\}_{i \in V}$
- A **factored stochastic transitions model**: 
$$p(x^{t+1} | x^t, a^t) = \prod_{i \in V} p_i(x_i^{t+1} | x_{N(i)}^t, a_i^t)$$



# Graph-based Markov Decision Processes

## Structured problems of sequential decision under uncertainty

- Several state variables  $\{X_i\}_{i \in V}$  and decision variables  $\{A_i\}_{i \in V}$
- A factored stochastic transitions model
- A **local reward model**  $r(x^t, a^t, x^{t+1}) = \sum_{i \in V} r_i(x_i^t, a_i^t, x_i^{t+1})$



# Graph-based Markov Decision Processes

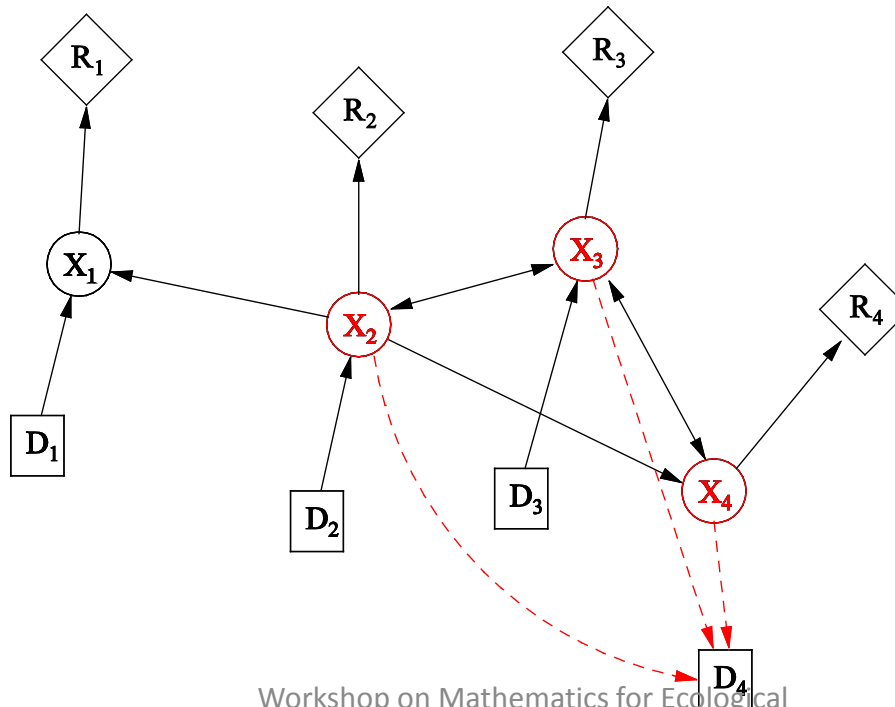
Structured problems of sequential decision under uncertainty

➤ Optimization problem:

*Find a policy  $\delta : X \rightarrow A$  assigning an action  $\delta(x)$  to every possible states of the system, maximizing the expected discounted sum of future rewards*

➤ Local policies (approximate):

$$\left\{ \delta_i \left( x_{N(i)} \right) \right\}_{i \in V}$$





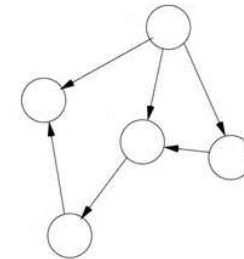
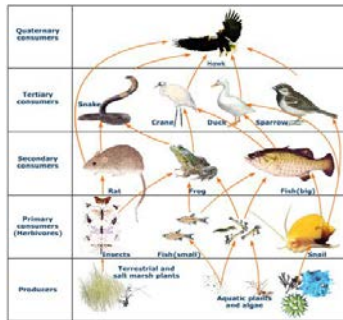
# Conservation of multiple species in food webs

*Eve MacDonald-Madden & Iadine Chadès (CSIRO and University of Queensland)*

*Peter Baxter, William Probert & Hugh Possingham (University of Queensland)*

*Edward Game (The Nature Conservancy)*

*Nathalie Peyrard & Régis Sabbadin (INRA-MIAT)*



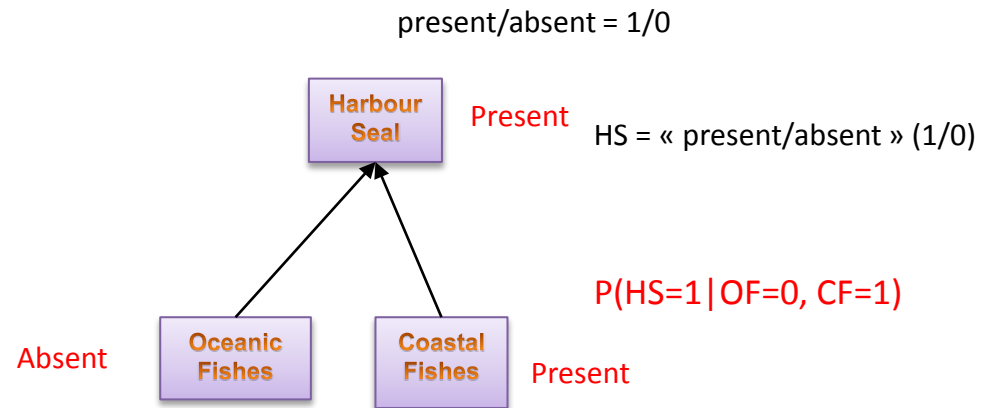
- **Problem** : Multiple species protection, within a network of trophic relations
- **Management decision** : which species should we protect (and when) , in order to make the network the most resilient?

# Food webs and bayesian networks

- Classically, trophic relations in food webs are:
  - **deterministic, qualitative** or quantified by **mass flows** (dynamical systems)
- The Bayesian network approach is:
  - **stochastic** and quantified by conditional **probabilities of presence**

Joint probability distribution over species occurrences:

$$P(S_1, \dots, S_n) = \prod_{i=1..n} P_i(S_i | \text{Preys}(S_i))$$

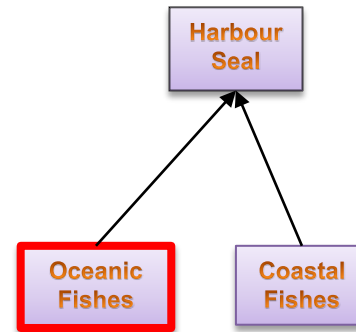


$$P(HS, OF, CF) = P(HS | OF, CF) P(OF) P(CF)$$

# Food webs, bayesian networks and « optimal » conservation

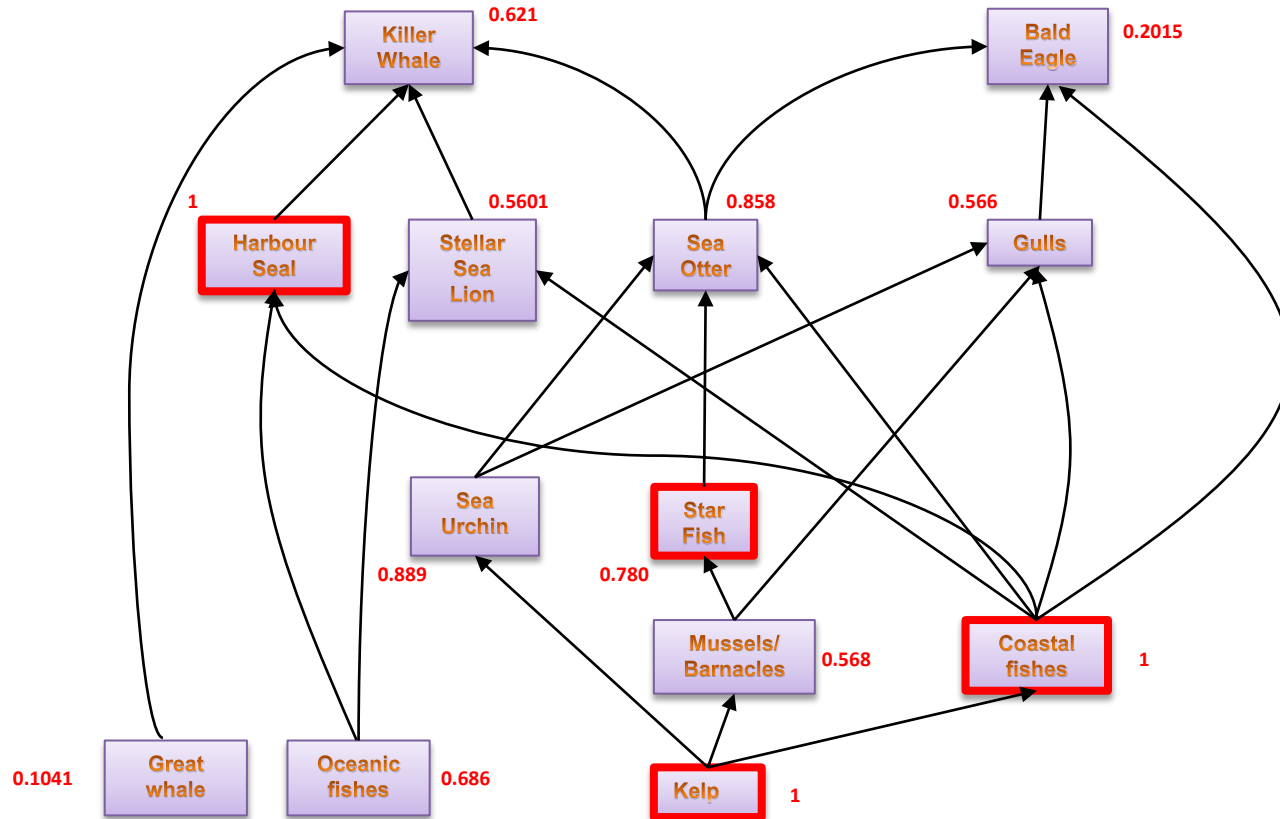
- « Conserving » a species increases its survival probability
- And the survival probability of its predators

- And thus, the species richness of the food web!



- Find the « optimal » feasible set of species to conserve, given
  - A conservation budget  $B$
  - Species conservation costs  $C_i$
  - A global criterion (expectation of the number of surviving species)

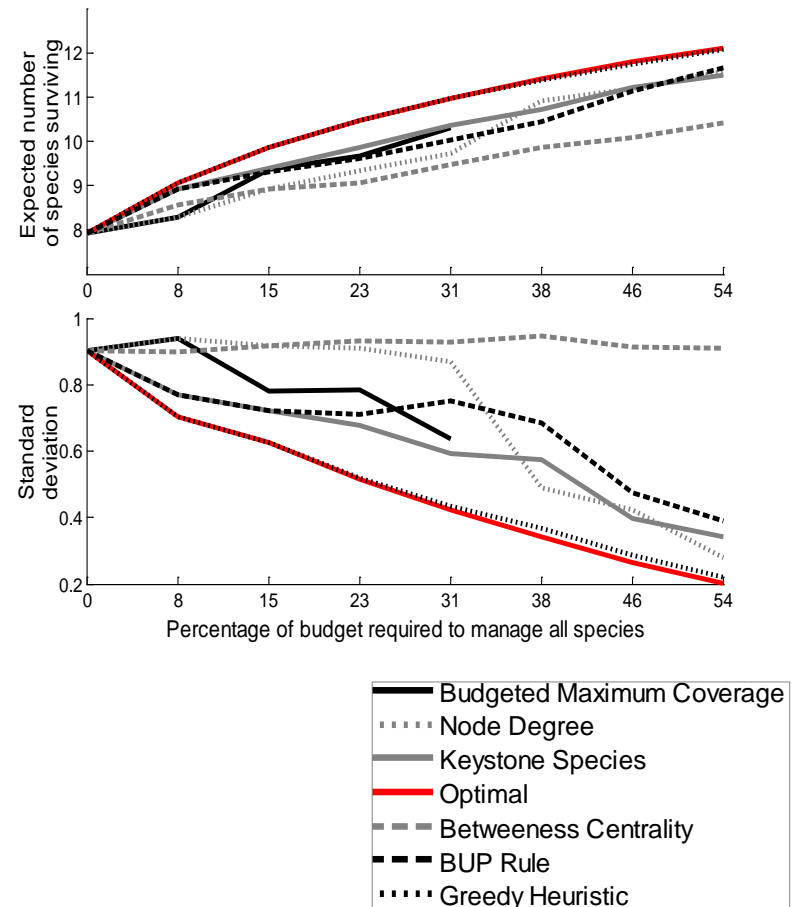
# Food webs, bayesian networks and « optimal » conservation



Problem: Optimal conservation is too difficult in general!

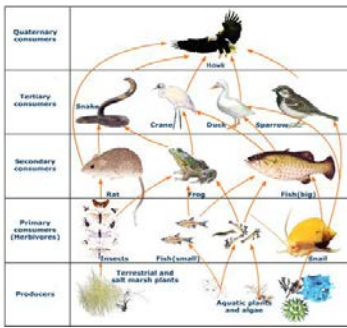
# Food webs, bayesian networks and conservation heuristics

- Structure : Alaskan Network
- 20 sets of randomly generated probability tables
- Comparison of the expectation and variance of the number of surviving species





# Conclusions



- A Bayesian network and combinatorial optimization model for species conservation within food webs
- Efficient heuristics for approximating optimal conservation
- No theoretical guarantee about the heuristics' performance
- Even more difficult in the “dynamic” case (work in progress)

# Spatial weeds sampling for map reconstruction

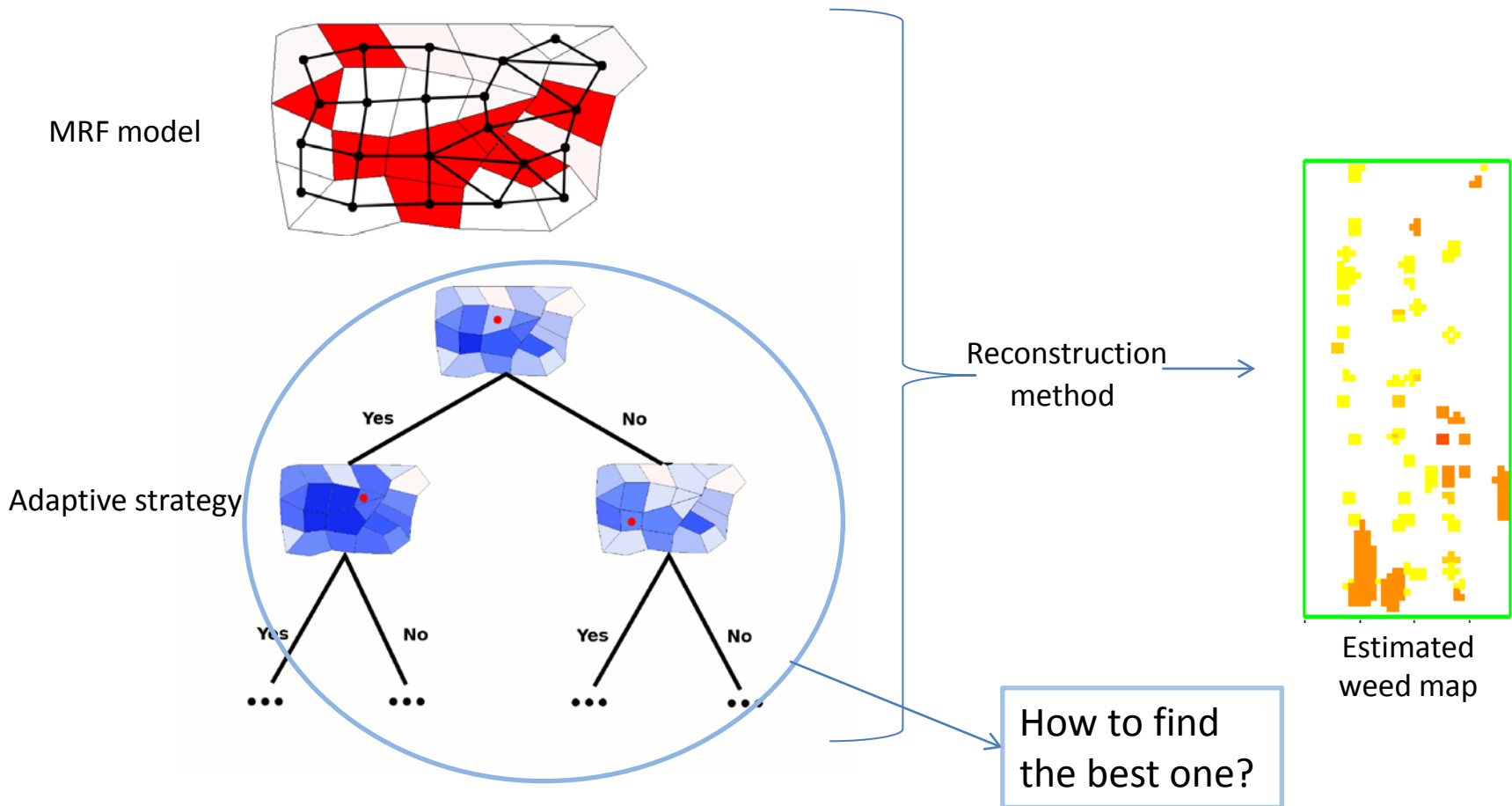
*Sabrina Gaba (INRA- UMR Agroécologie)*  
*Mathieu Bonneau, Nathalie Peyrard & Régis Sabbadin (INRA-MIAT)*



- **Problem:** an accurate map of weed repartition in the crop field
  - is a useful tool for studying weeds populations
  - but observations are costly
- « **Management** » **decision** : where to get sample observations in order to achieve a good compromise between map accuracy and sampling cost?

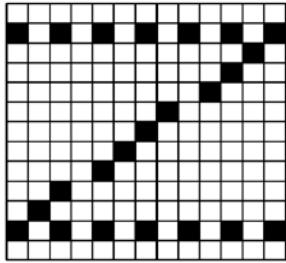
# Optimal adaptive sampling strategy

We combine **MRF and MDP** to model the problem of designing an adaptive strategy by optimization and we propose **two heuristic solutions**

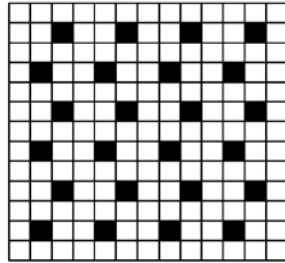


# Performance of adaptive sampling heuristics

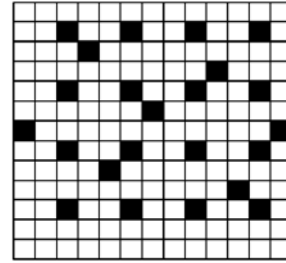
Comparison with 8 static sampling strategies



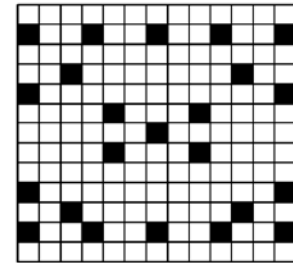
**Z**



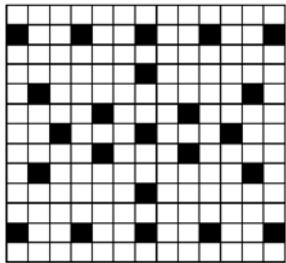
**Reg1**



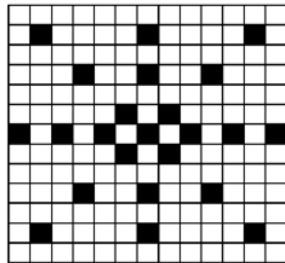
**Reg2**



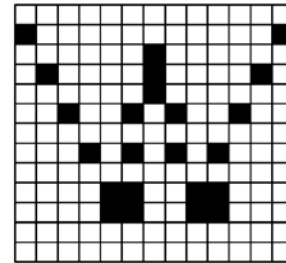
**Reg3**



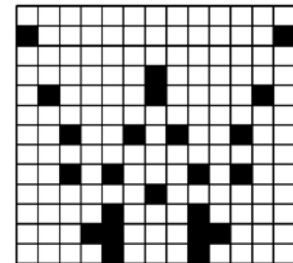
**Z**



**Star**



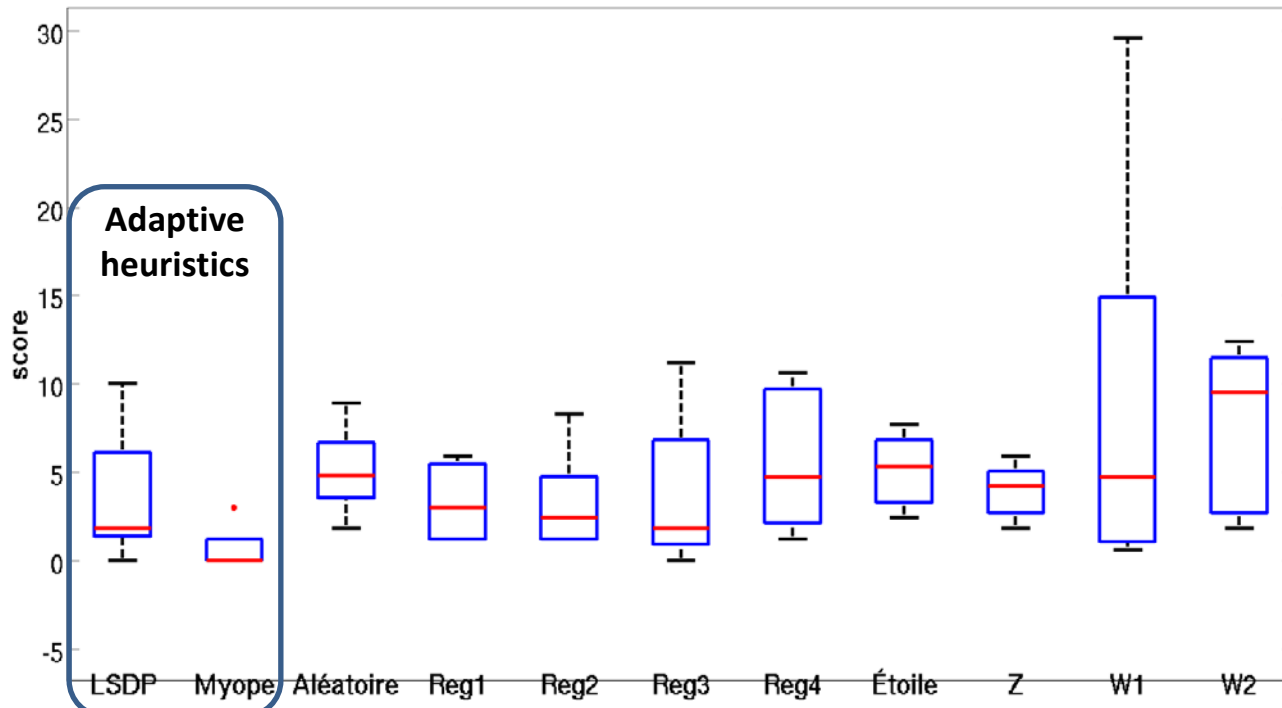
**W1**



**W2**

# Performances of adaptive sampling heuristics

- Benchmark of 6 real weed maps
- Field of 13 by 13 quadrats, sample size = 13,5 % of total
- **NWC** = number of well classified quadrats
- **Score** = NWC(best strat) – NWC (strat)



➔ Adaptive sampling is more efficient



# Conclusions

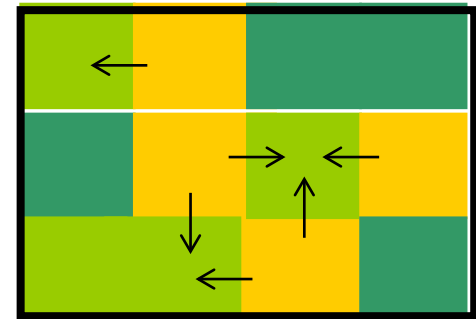
- A framework and two heuristics for adaptive sampling under cost constraints
- Adaptive sampling gives more accurate maps for the same cost
- Method also applied on a problem of fire ants sampling
- The sampled system is assumed static

# Collective management of crop resistance to pathogens

*Benjamin Borgy (ex INRA-AGIR- MIAT)*

*Jean-Noël Aubertot (INRA-AGIR)*

*Nathalie Peyrard & Régis Sabbadin (INRA-MIAT)*



- **Problem** : Cultivar resistance enables to avoid fungicides but can be broken down
- **Management decision** : Where and when should we allocate resistant crops to maintain both resistance and yield?

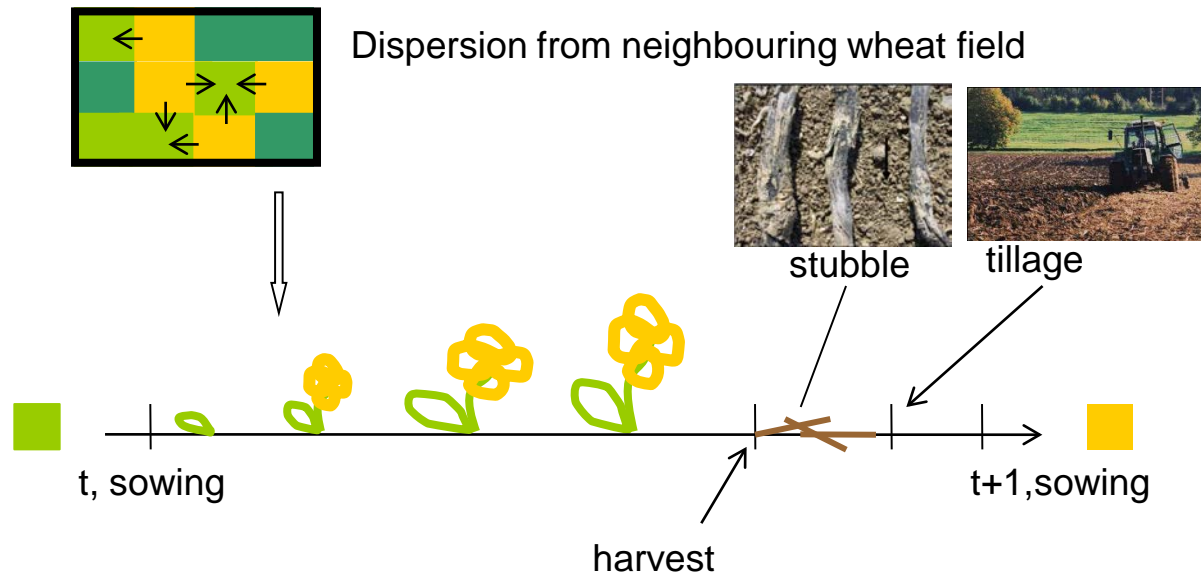
# Case study: blackleg on canola

## Crop rotation

- canola
- wheat
- barley

## Host-pathogen interaction

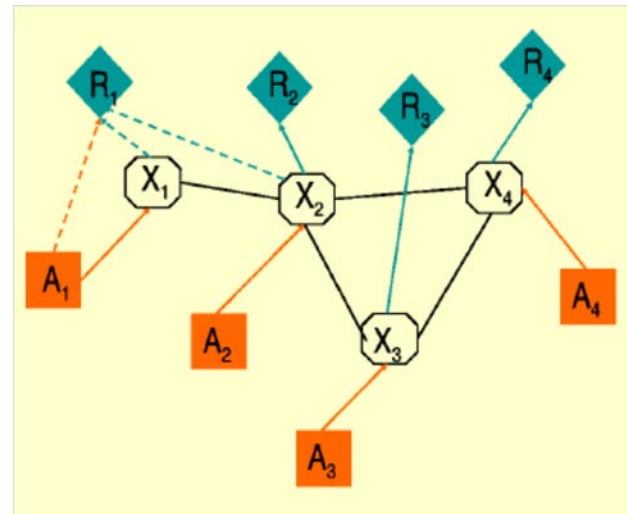
	resistant	susceptible
virulent	+	+
avirulent	-	+



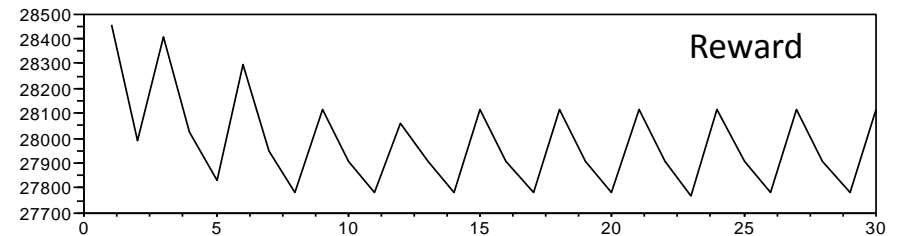
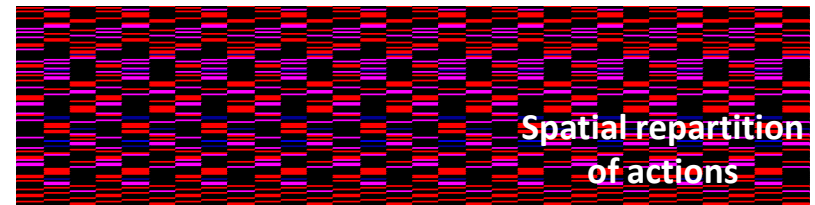
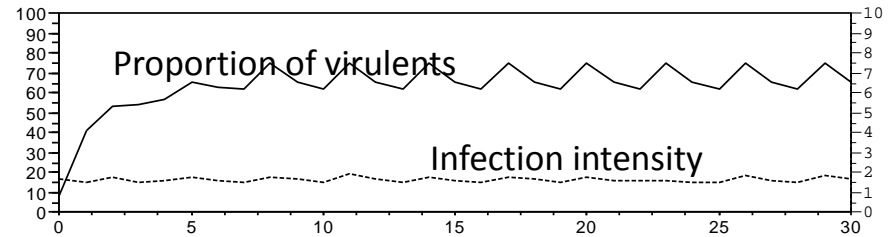
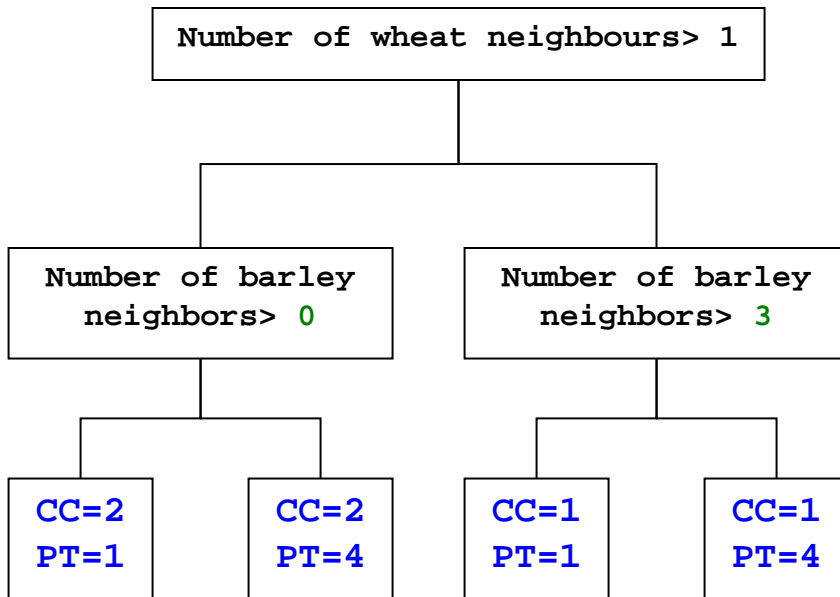


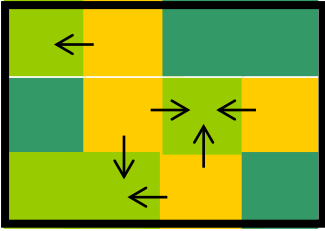
# A GMDP model for the control of blackleg on canola

- **State variables in each field**
  - crop
  - infection intensity
  - composition of pathogen population
- **Actions (on canola fields)**
  - cultivar choice (CC)
  - ploughing threshold (PT)
- **Transition functions:**
  - learned from simulations (SIPPOM )
- **Reward:**
  - sum of local gross margins



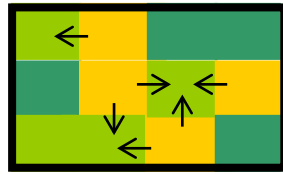
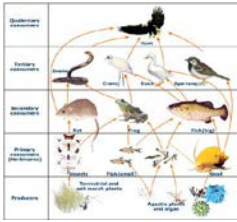
# A tool for strategies simulation





# Conclusions

- A framework for designing collective strategies by optimization
- A model for comparing and simulating management strategies
- BUT no satisfying solution to the problem of design by optimization



# General conclusions

## ➤ **Managing ecological networks**

- **Networks can be:** spatial, causal, ...
- **Management can be:** control, conservation, sampling ..
- **Ecosystems and agricultural systems :** management share similarities

## ➤ **Common tools for all these problems**

- graphical models, simulation, optimization
- **Computing exactly** the optimal strategy is out of reach

**Current research focuses on approximate resolution**

# Still some challenges!

## Design by optimization of strategies for managing ecological networks

- How to improve solutions of problems with large factored state and action spaces?
  - We have heuristic solutions but there is still room for improvement
- How to manage a network when observations are costly?
  - Combine sampling and control/conservation actions
- Dynamics
  - How to control a system where the network changes through time?
- ...

# References

## **Food webs management**

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## **Adaptive spatial sampling**

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## **GMDP, collective management of crop fields, forests or reserves**

- R Sabbadin, N Peyrard and N Forsell. A framework and a mean-field algorithm for the local control of spatial processes. *International Journal of Approximate Reasoning*, 2011.
- N Peyrard, R Sabbadin, E Lo-Pelzer and J.N. Aubertot. A graph-based Markov decision process applied to the optimization of strategies for integrated management of diseases. American Phytopathological Society and Society of Nematologist joint meeting, San Diego, California, 2007.
- N Forsell, P Wikström, F Garcia, R Sabbadin, K Blennow and L.O. Eriksson. Management of the risk of wind damage in forestry: a graph-based Markov decision process approach. *Annals of Operations Research*, 2009.
- R Sabbadin, D Spring and C.E. Rabier. Dynamic reserve site selection under contagion risk of deforestation. *Ecological Modelling*, vol. 201, 2007, pp. 75-81.

11/03/2013

Workshop on Mathematics for Ecological  
Management

# Summary

- Ecological networks management:
  - Ecology: From agricultural systems, in interaction with communities of pathogens to ecological management (food webs)
  - Network: Spatial correlations (fields, sites) and species correlations (food web, weeds communities...)
  - Management: Control (eradication), conservation or sampling for map construction
- Methodological tools:
  - Stochastic models of interactions: Bayesian Networks, Markov Random Fields, Dynamic Bayesian Networks
  - Control : MDP, Combinatorial optimization...



# Management should be

- **Spatially explicit** : long-distance pathogen dispersion
- **Collective**: decisions are taken in each field but are interdependent
- **Long term**: we want to minimize yield losses now and in the future
  
- *Collective strategy design is difficult because of spatial and temporal dependences*

# Message

- Justifier l'utilisation de MGS et de l'opti pour ecological management
  - Why networks: Importance of interactions (spatial, species...) in Ecology.
  - Why stochastic models: Obviously processes are uncertain
  - Why optimization: management implies policy conception and optimization is useful for this. One step beyond comparison or simulation of management strategies.

# Gestion de la santé des cultures : Importance de la composante « spatiale »

Spatialisation



- Parcelles gérées « indépendamment » (hormis rotations)
- Prise en compte de relations « globales » (travail, biodiversité, « stocks » de pathogènes/adventices...)
- Prise en compte des dispersions (adventices, pathogènes...)

*Une gestion « explicitement spatiale » des cultures permet la prise en compte de :*

- La dispersion de pathogènes/adventices
- Les interactions culture/adventices/pathogènes

*Dans la conception de modes de gestion des cultures*

# Management should be

## Contrôle des bioagresseurs des cultures :

### ➤ **Global**

*Stratégies collectives plus efficaces que des stratégies individuelles*

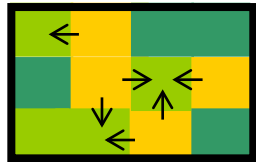
- **Échelle pluri-parcellaire + Échelle pluri-annuelle + Multiples interactions**  
*Conception de stratégies collectives efficaces difficile*

### ➤ **Durable**

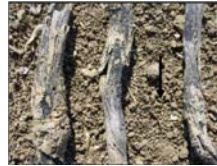
- Cultivars possédant des résistances spécifiques et/ou quantitatives
- Contournement de la résistance si cultivar surexploité

*Comment concevoir des stratégies collectives exploitant durablement les résistances variétales ?*

# Le Modèle



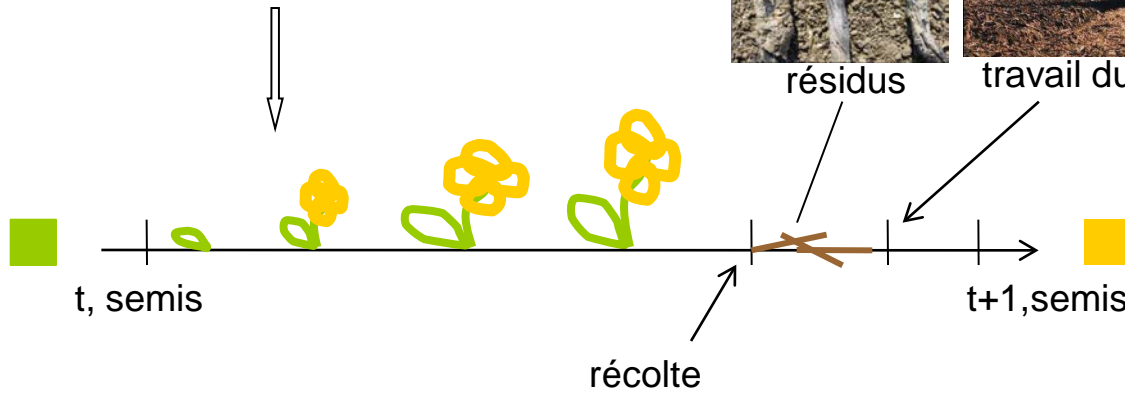
propagation depuis les parcelles de blé voisines



résidus



travail du sol



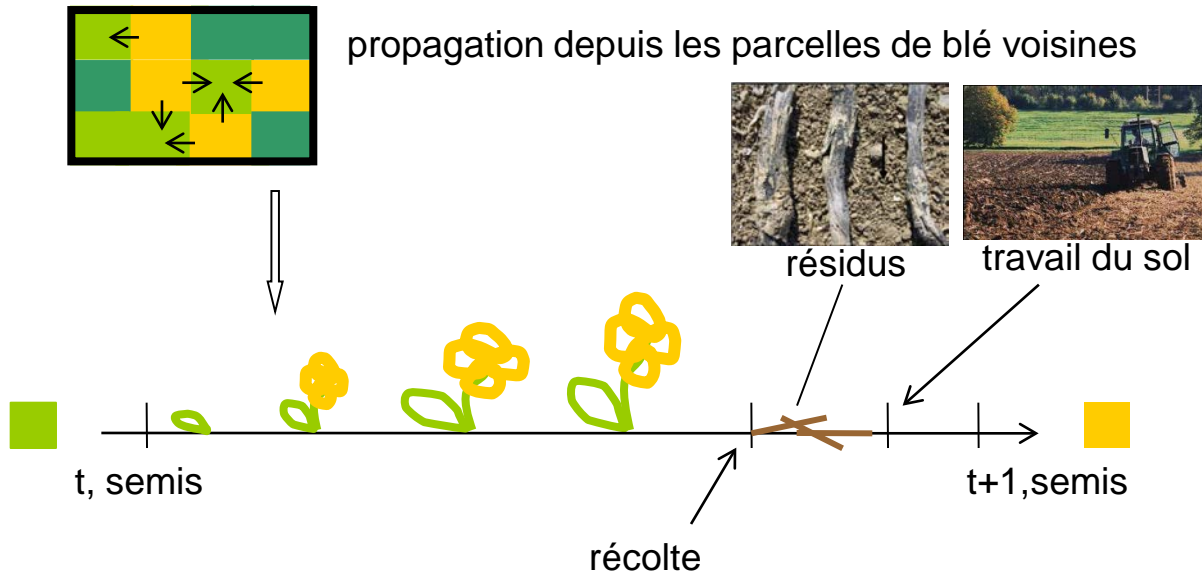
## Rotation des cultures

- colza
- blé
- orge

## Interactions hôte-patho

	résistant	sensible
virulent	+	+
avirulent	-	+

# Le Modèle



propagation depuis les parcelles de blé voisines

Rotation des cultures

- colza
- blé
- orge

Interactions hôte-patho

	résistant	sensible
virulent	+	+
avirulent	-	+

## Variables d'état d'une parcelle

- Culture en cours (C  $\rightarrow$  B  $\rightarrow$  O)
- Sévérité d'infection des résidus (G2)
- % Pathotypes virulents (SG)

## Actions

- Choix variétal (S ou R)
- Seuil de travail du sol avec labour

(labour si  $G2 > \tau$ )

# Graph-based Markov Decision Processes

## Structured problems of sequential decision under uncertainty

- Several state variables  $\{X_i\}_{i \in V}$  and decision variables  $\{A_i\}_{i \in V}$
- A factored stochastic transitions model
- A local reward model
- **Local policies**

$$\left\{ \delta_i(x_{N(i)}) \right\}_{i \in V}$$

