Simplified root architectural models using continuous deformable domains

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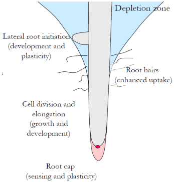
- *Meristems* rule plant architecture.



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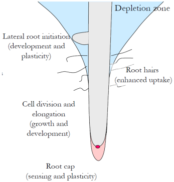

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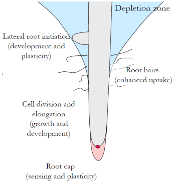

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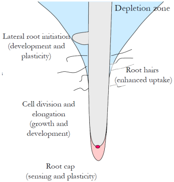

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- To overcome computational limitations, we developed a continuous model for meristem distribution and solved it in a semi-Lagrangian framework.
- Application to a simple case of density dependent growth in a coordinated population of plants.

Models for meristem development in soil

	Features	Limitations
Root depth/distribution models (Hackett and Rose, *Aust. J. biol. Sci.* 1972)	Number of root tips is a function of branching rate, root length is a function of number of root tips and link to increase in root depth	Spatial resolution
Density models of root systems dynamics (Gerwitz and Page, *J. appl. Ecol.* 1974)	Root systems as density distribution, conservation law, simulation algorithm (root fluxes)	Biological interpretation of parameters
Structural functional plant models (Lyndenmayer, *J. of Theoretical Biology* 1968)	Independent virtual meristems, empirical developmental processes and source-sink relationships regulate growth	Difficult to parameterize
Developmental models (Korn, *J. Theor. Biol.* 1969)	Mechanics of growth, gene regulation, transport and signalling	Even more difficult to set parameters

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Model-based analysis of root meristems dynamics

- Assumption: densities to describe root system: ρ_a (meristem), ρ_n (length) and ρ_b (branching); "phase space" to account for root morphology.

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Hyperbolic PDE → Propagation of travelling waves.

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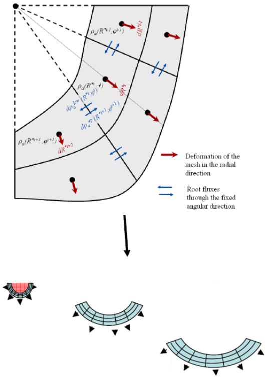
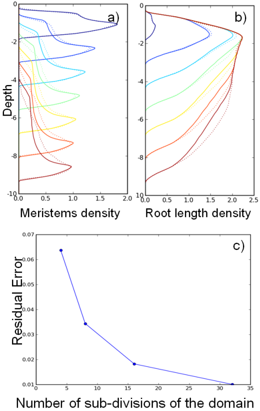
Deformable domains for plant modelling

- We propose an alternative to classical Eulerian framework (densities defined on nodes of a fixed grid).
- Semi-deformable mesh in radial direction (fluxes in azymuth): densities are computed for a fixed proportion of material (meristems).
- Each meristem distorts its neighbourhood within a domain because of growth. Close meristems have close trajectories. → Sounds adapted to plant roots. Another advantage: few elements to consider.



Numerical analysis



(a-b) Numerical semi-Lagrangian simulations (N=16, solid line) compared with 1D explicit solution (dotted line) at different times.

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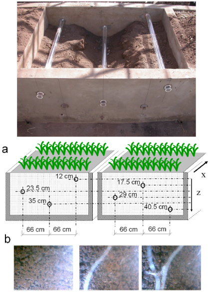
Applying the model to plant systems biology

Experiment

Imaging in plastic tubes going through concrete bins with sown Barley in rows of at different depths → Plots of root length distribution → Characterization of meristem activity.



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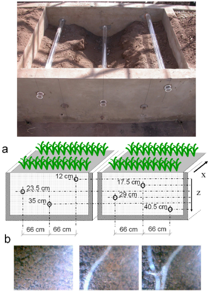
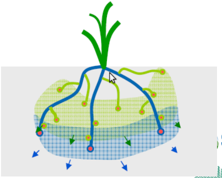


- Superposition of waves for two different root orders (coupled PDEs).
- Heterogeneity can be modelled *via* non-fixed coefficients.
- Architectural features encoded in source term (*e.g.* $b = b_0\rho_a(u \pm \pi/2)/2$).

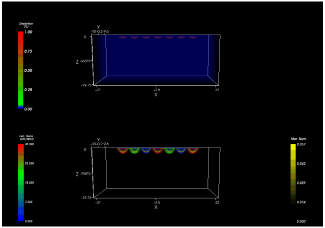


Simulating biology ?

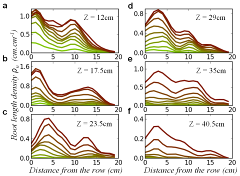
From biology to models and back

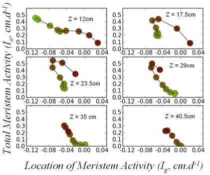
Is meristem location/activity (and more generally developmental mechanisms) obtained from experiments somehow related to the equations shown before ??

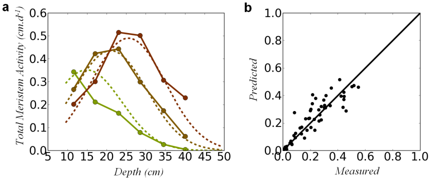
Simulations...

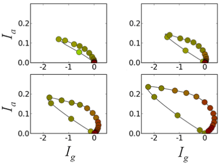


Analysis of meristem trajectories



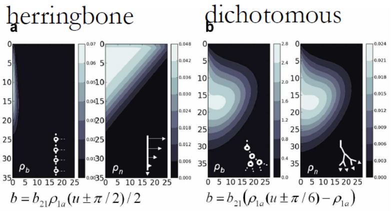


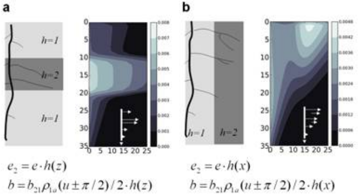


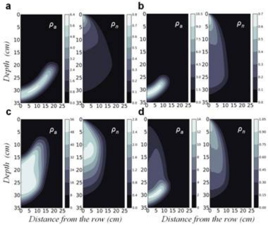


Observations vs. predictions from the model.

Biological interpretation of the model







Modelling [*I*.] branching (source term) [*II*.] heterogeneity in the soil and [*III*.] different root behaviour depending on model parameters.

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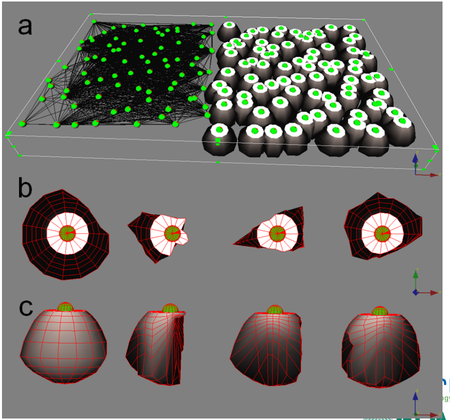
Deformable domains allow us to simulate competition in a population of plants

1. Start from each meristem that has a domain distorting trajectory.
2. Several independent (similar self-avoiding) domains to model a plant.
3. Allocated resources depend on relative densities. Dynamics models simplistic competition.



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Wrap up

- Development of the plant can be viewed as (overlapping) waves of meristems; root architecture = footprint of these waves.
- Simple models with quick computation allows us to obtain interpretation in terms of root developmental mechanisms.
- Deformable domains (Lagrangian solver) for the simulation of ensemble of plants complementary to static mesh approach (Eulerian solver). Application in ecology ?

Future Work

- Models in early stage of development so produce useable package (+ stochasticity, plant/environment feedbacks . . .).
- More experiments to characterize wave morphology, influence of genotype, developmental processes, *etc.*
- Link this kind of study to genome. In a population with different genotypes, a mapping (stat. link) is not enough. Dynamic (not only accouting for final skeleton) model where traits (QTL context) would be developmental precesses.

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谢谢

Any questions?