

Efficient Inference Algorithms for Scene Understanding Problems

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What is scene understanding?





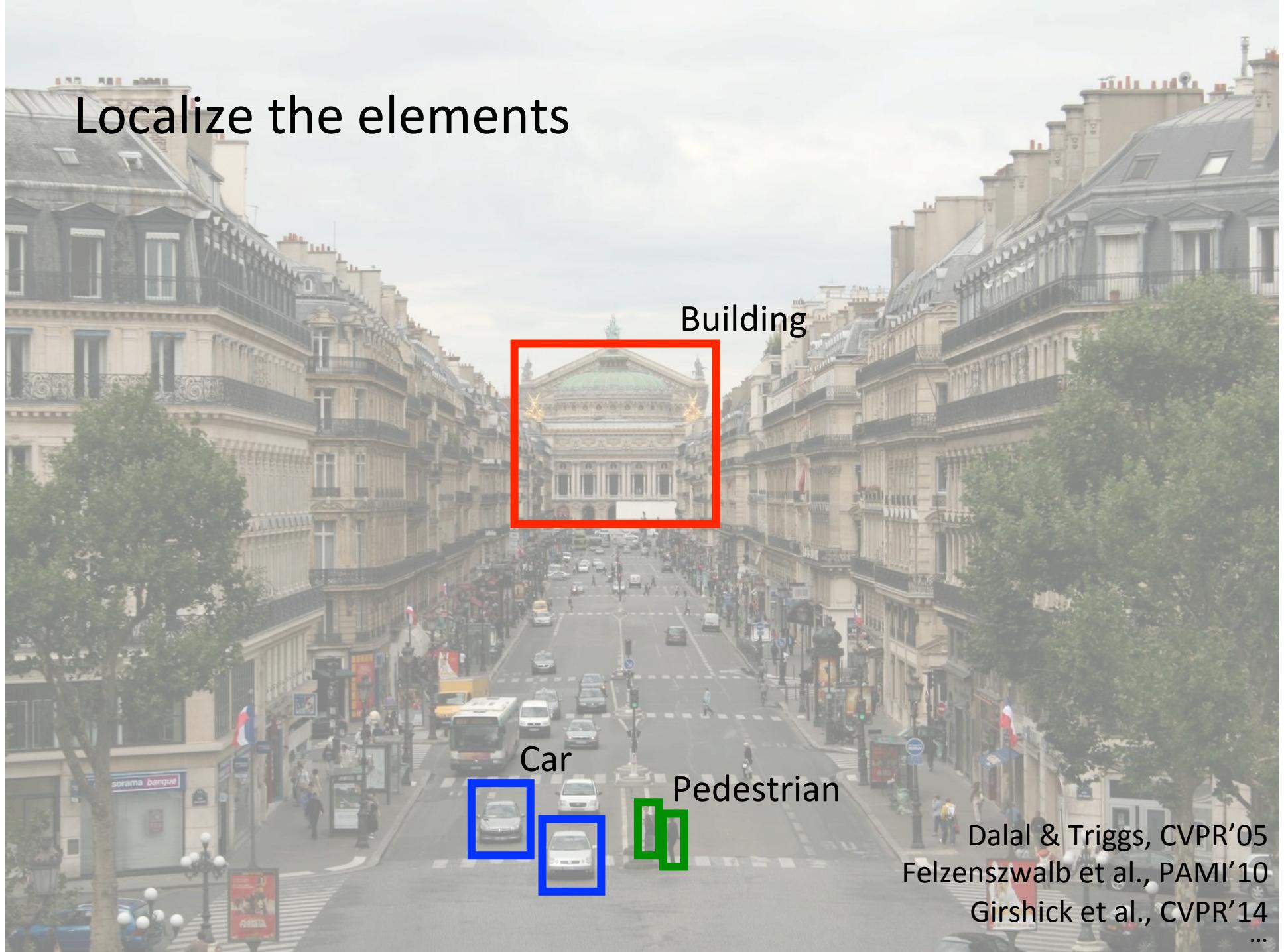
Identify scene type and the elements in the scene

Road
Car
Pedestrian
Building
Sky

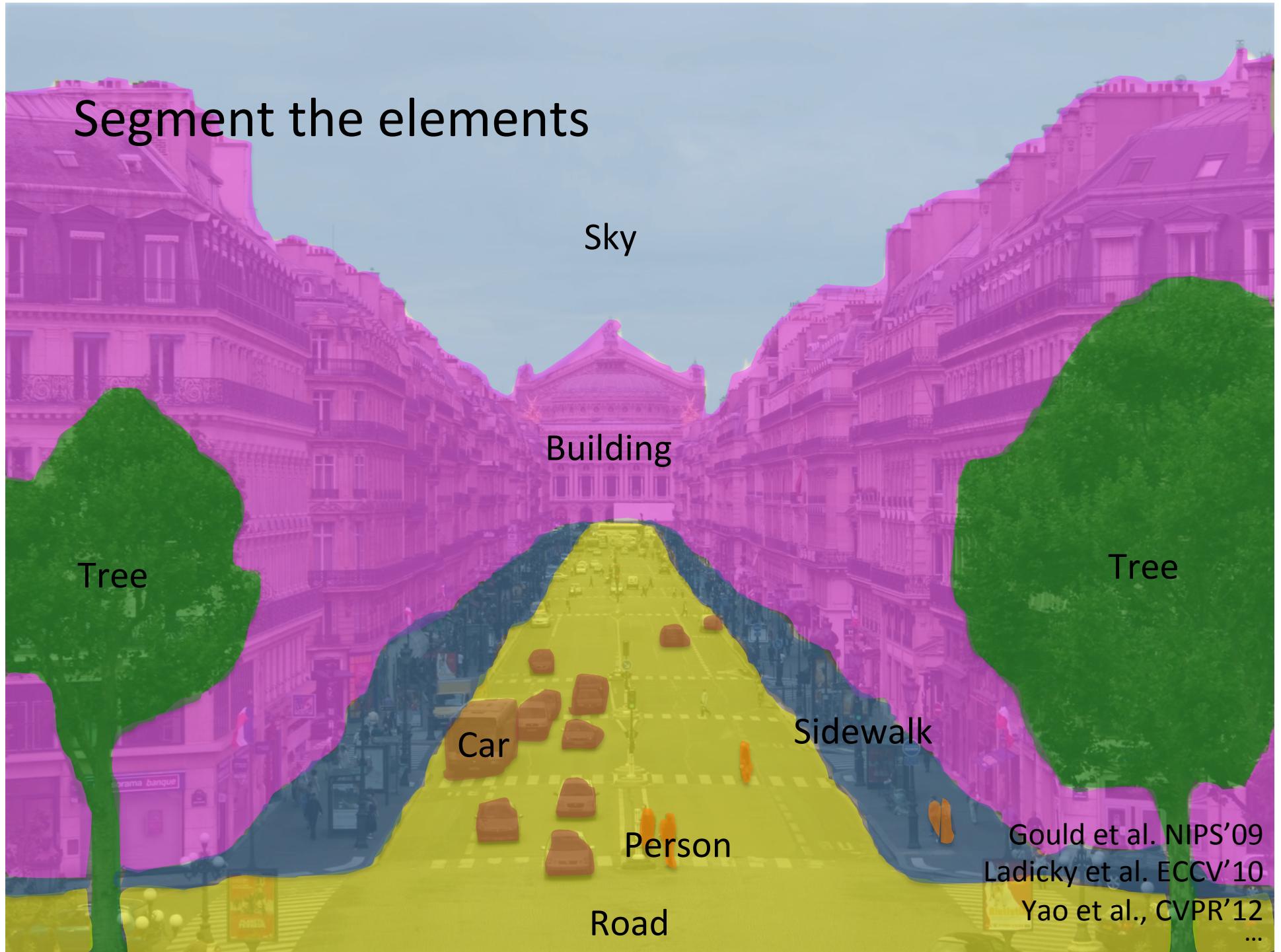
...

Bosch et al., ECCV'06
Deselaers et al., PR'10, DAGM'06
...

Localize the elements



Segment the elements



Not all images are the same!



Image courtesy: L'Obs

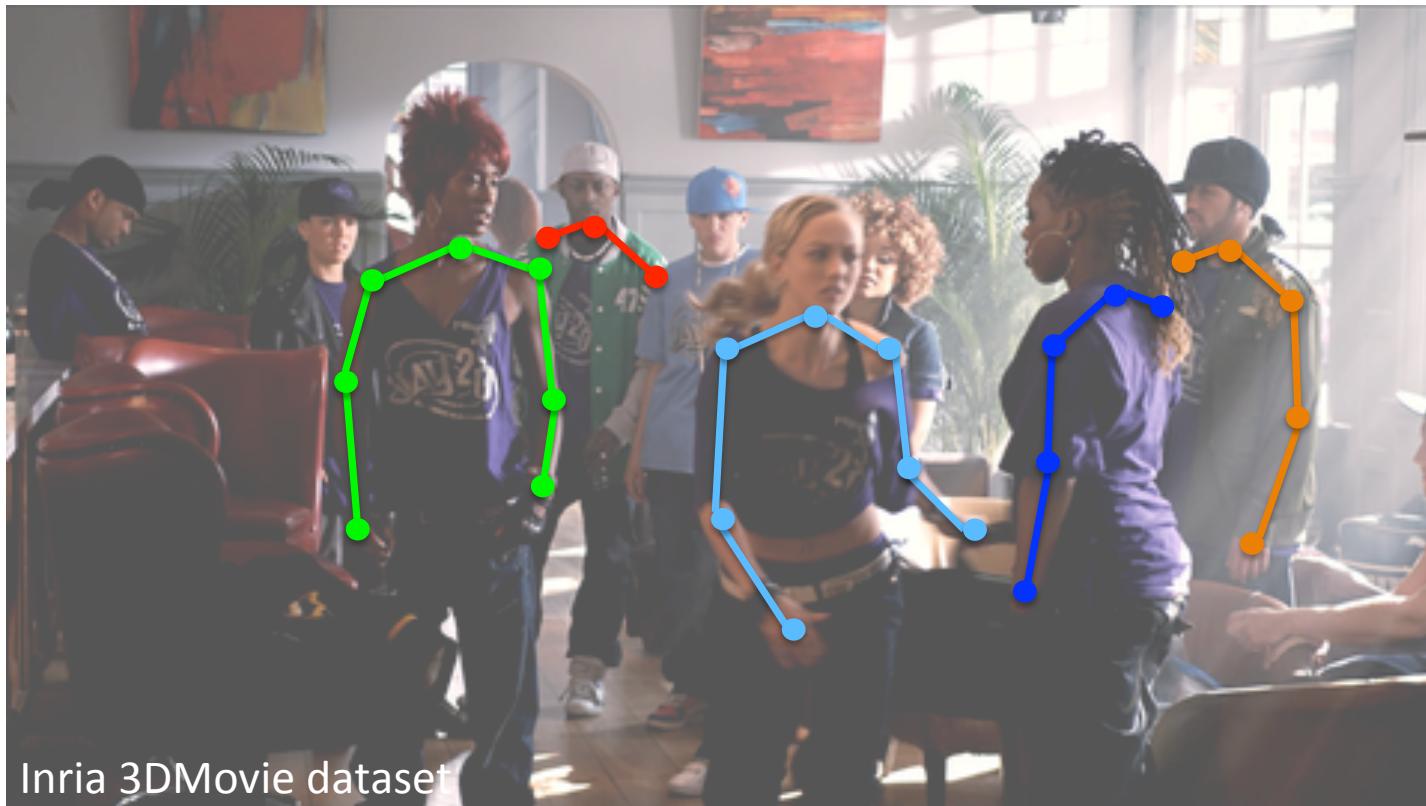
Analyze people in a scene



Inria 3DMovie dataset

Analyze people in a scene

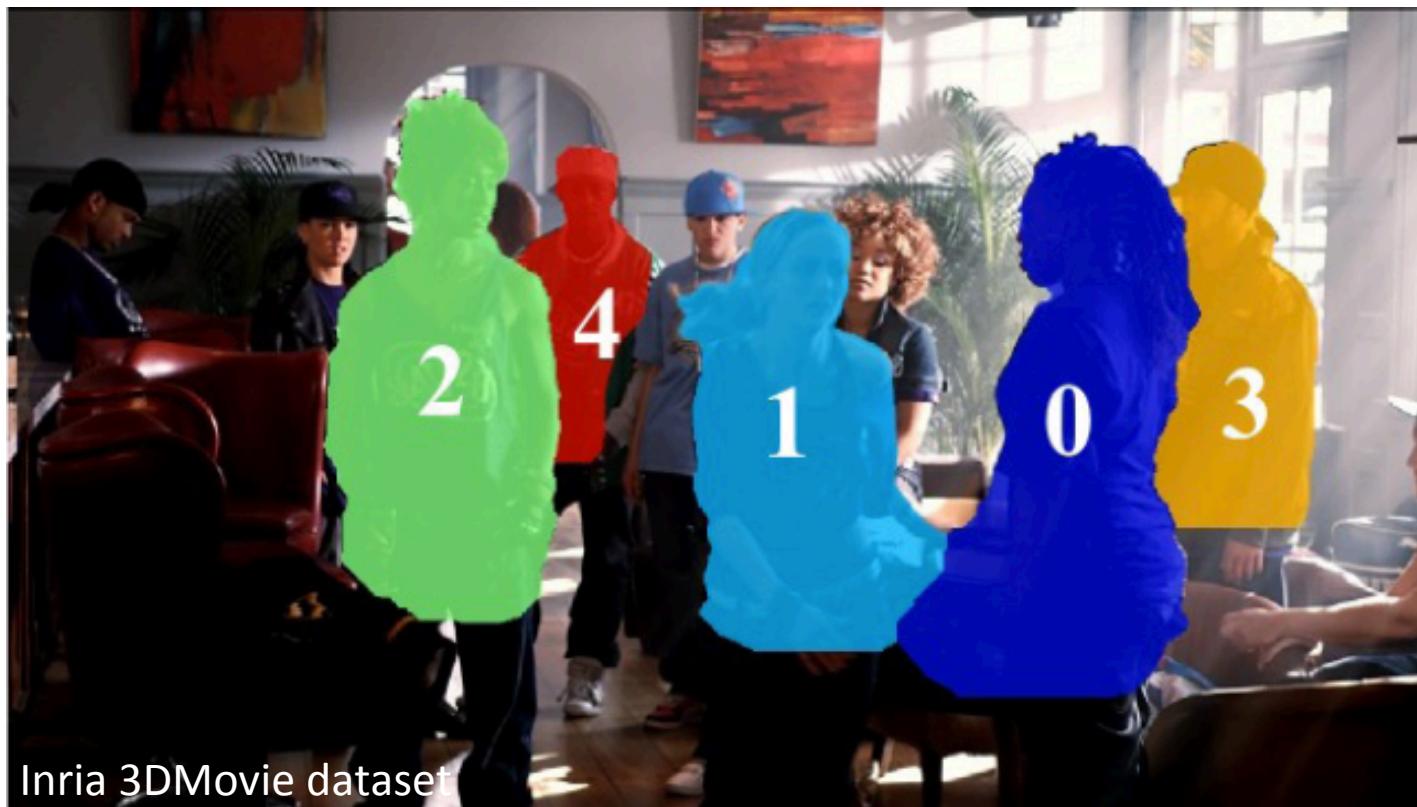
Estimate poses (body-joint locations)



Yang & Ramanan, PAMI'12
Tompson et al., CVPR'14
...

Analyze people in a scene

Segment the individuals



Inria 3D Movie dataset

Yang et al., PAMI'11
Shotton et al., CVPR'11

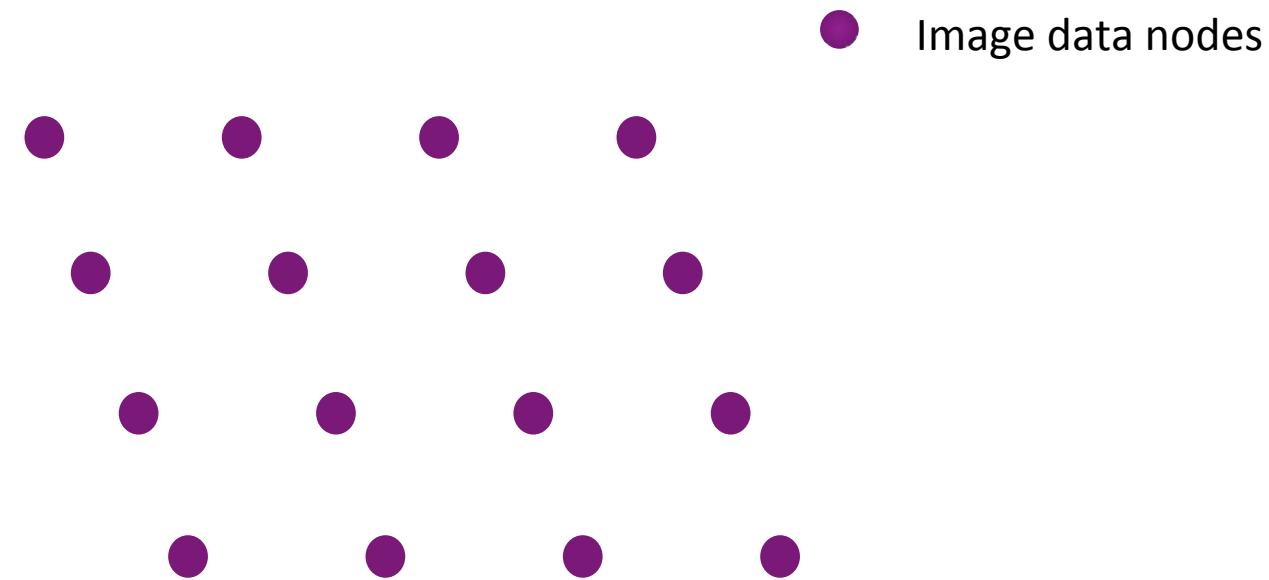
...

Scene Understanding

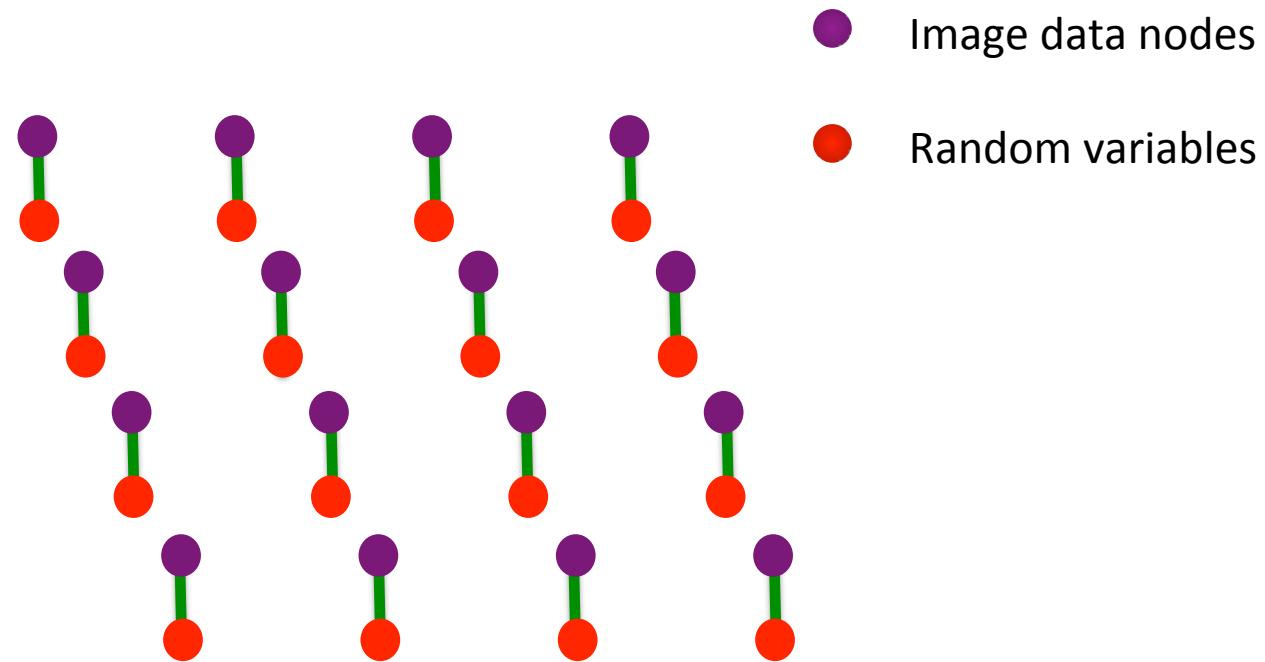


Markov Random Fields [Preston '74]

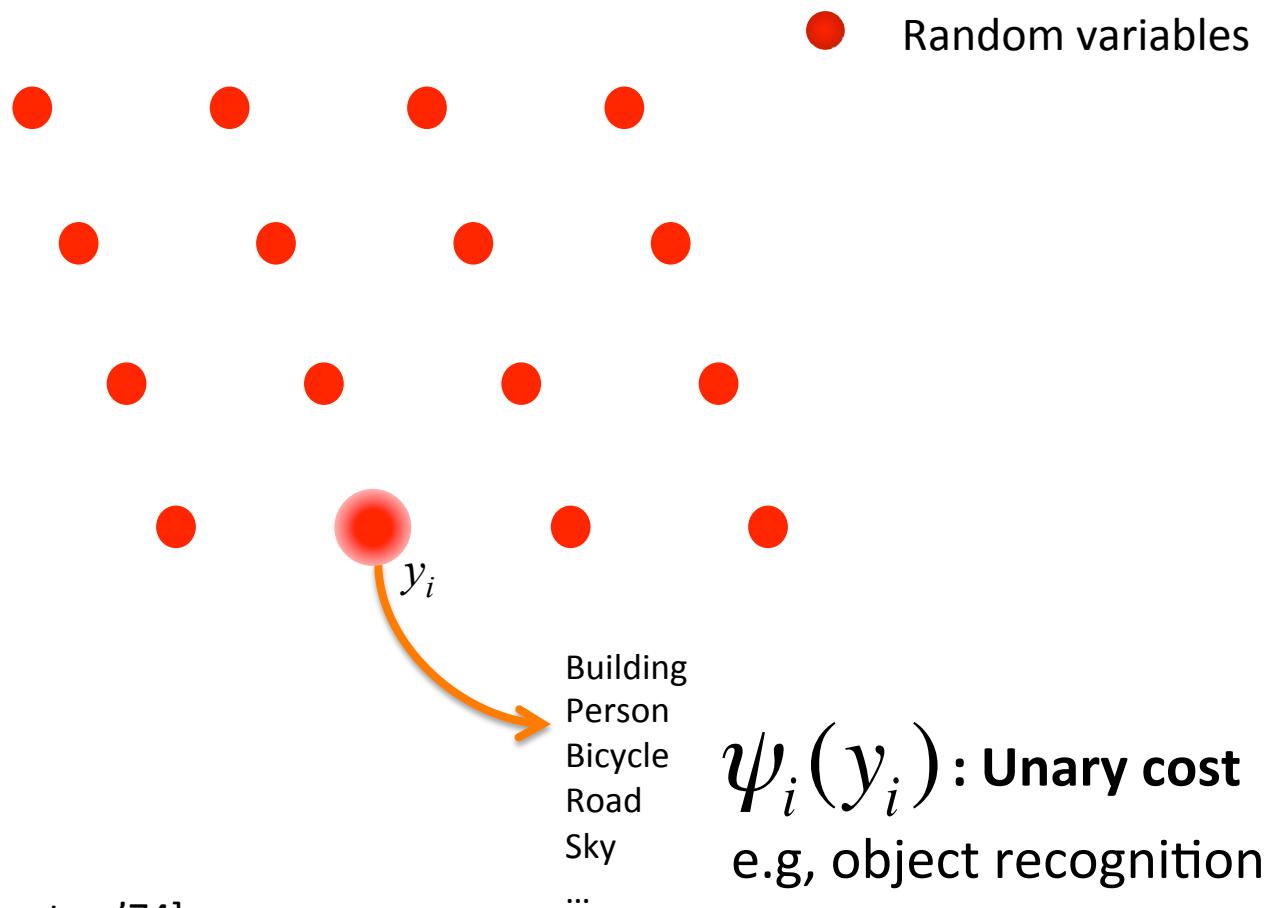
Scene Understanding



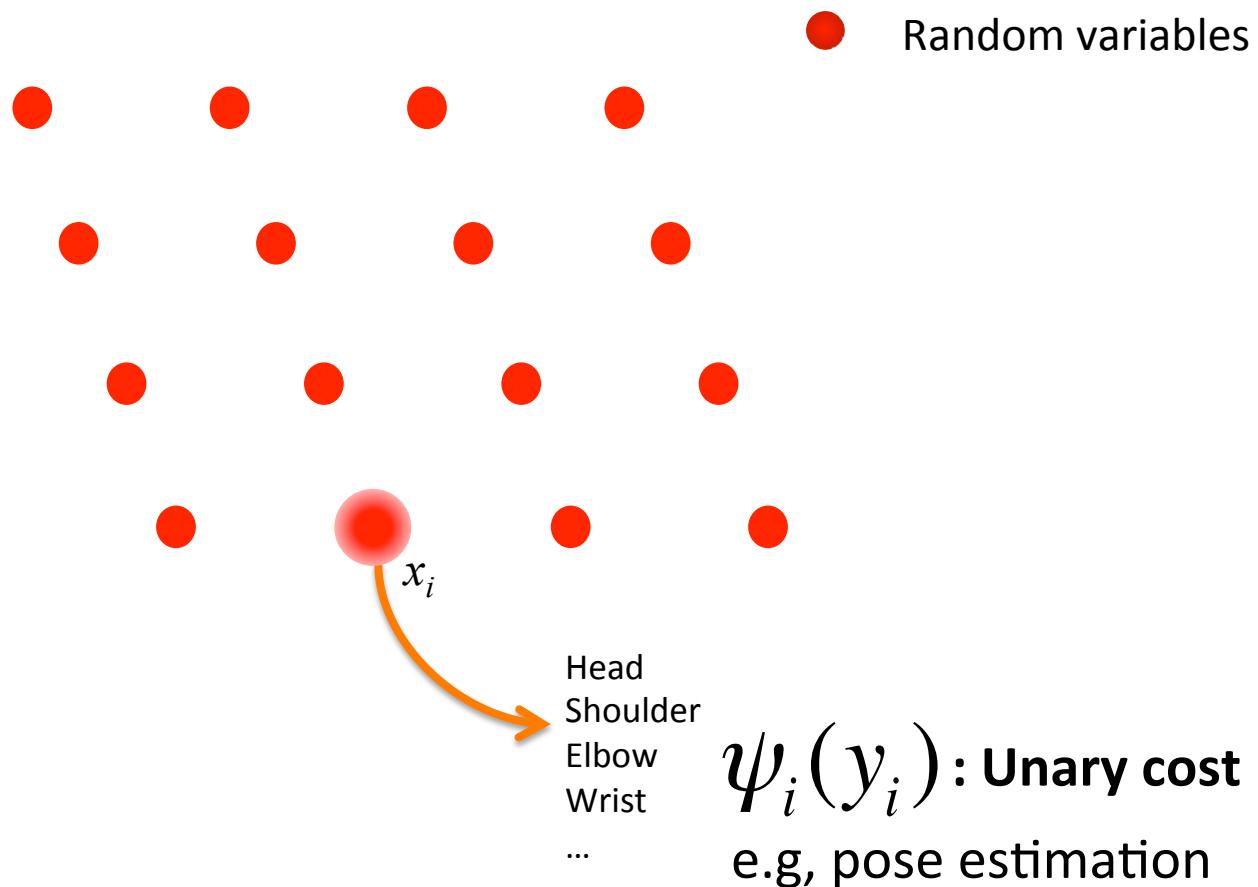
Scene Understanding



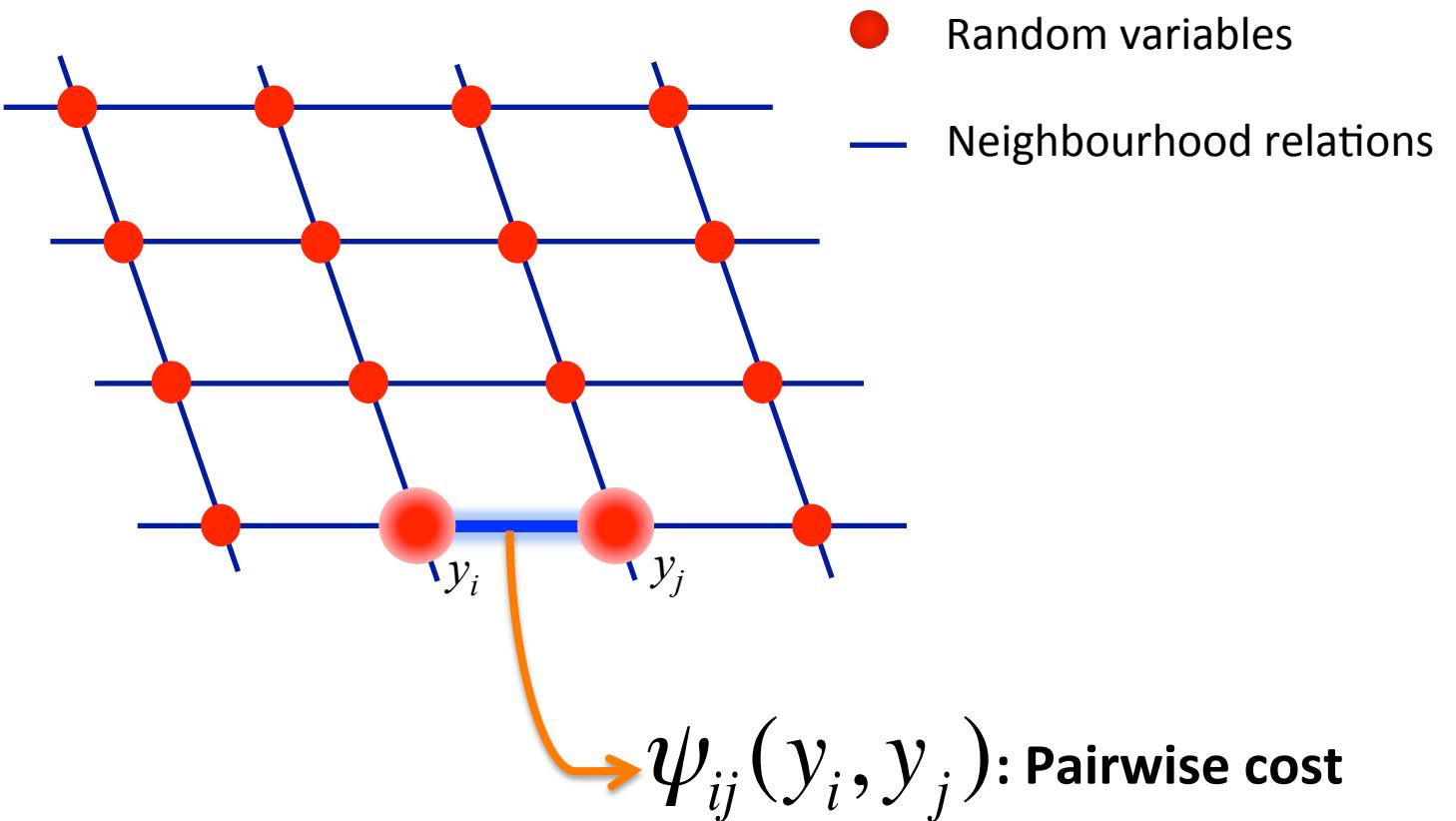
Scene Understanding



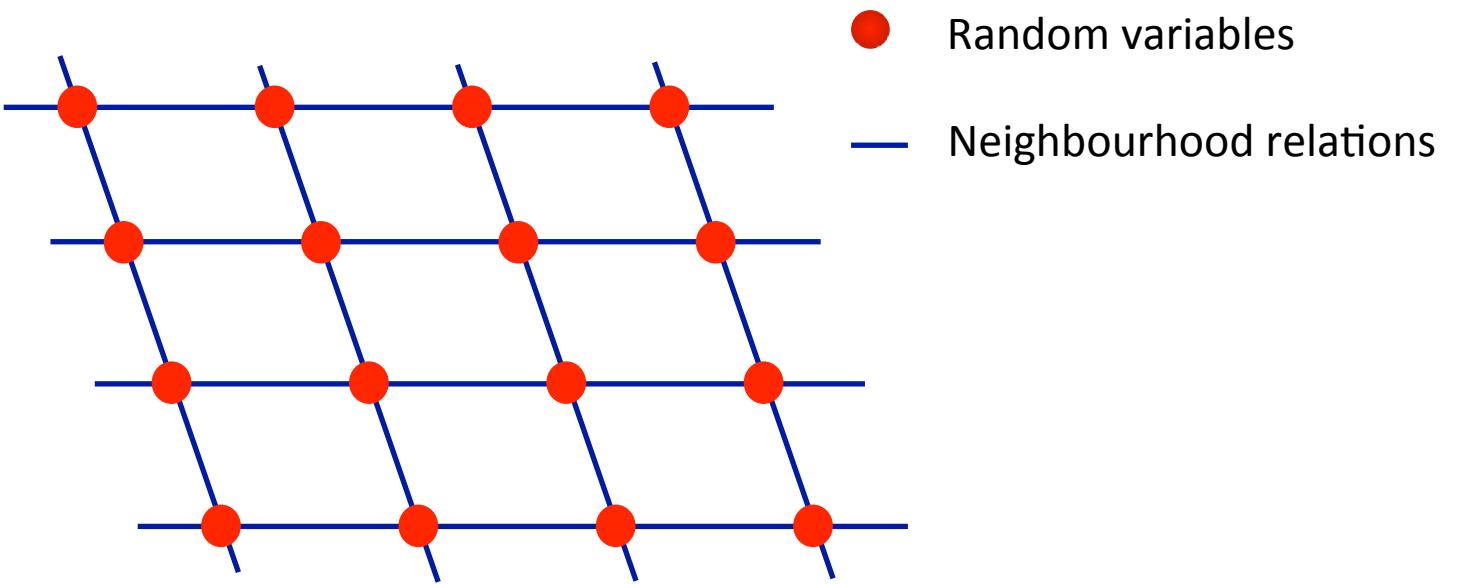
Scene Understanding



Scene Understanding



Scene Understanding



$$E(\mathbf{y}) = \sum_{i \in \mathcal{V}} \psi_i(y_i) + \sum_{(i,j) \in \mathcal{E}} \psi_{ij}(y_i, y_j)$$

: Cost of labelling

Scene Understanding

- In the context of such labelling problems
 - Learning parameters of the function
 - Modelling image priors & temporal constraints
 - Performing inference

Scene Understanding

- In the context of such labelling problems
 - Learning parameters of the function
 - Modelling image priors & temporal constraints
 - Performing inference

Parameter learning

- Consider a 2-label problem (for now)
- e.g., binary image segmentation
- Also, consider a pairwise random field

Parameter learning

- Conditional probability of a labelling \mathbf{y} :

$$\Pr(\mathbf{y}|\mathbf{x}, \boldsymbol{\theta}) = \frac{1}{Z(\boldsymbol{\theta})} \prod_{i \in \mathcal{V}} \exp(y_i \boldsymbol{\theta}_u^\top h_i(\mathbf{x})) \prod_{(i,j) \in \mathcal{E}} \exp(y_i y_j \boldsymbol{\theta}_p^\top \nu_{ij}(\mathbf{x}))$$

Parameter learning

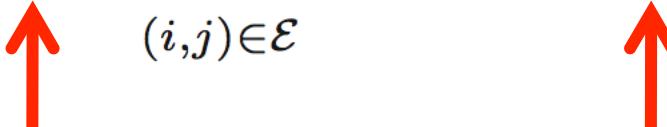
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The label taken by pixel $x_i = -1$ or 1

Parameter learning

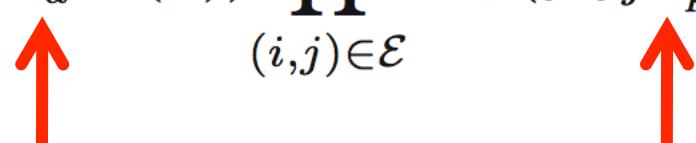
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Unary features Pairwise features

Parameter learning

- Conditional probability of a labelling \mathbf{y} :

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Unary parameters Pairwise parameters

Learning Approaches

- Maximum likelihood-based
- Large margin based
- Other iterative methods

Maximum likelihood-based

- Compute $\hat{\theta}$ which maximizes likelihood

$$\hat{\theta} = \arg \max_{\theta} \Pr(\mathbf{y}|\mathbf{x}, \theta)$$

- Hard to solve for vision problems

Maximum likelihood-based

- Why is it hard?

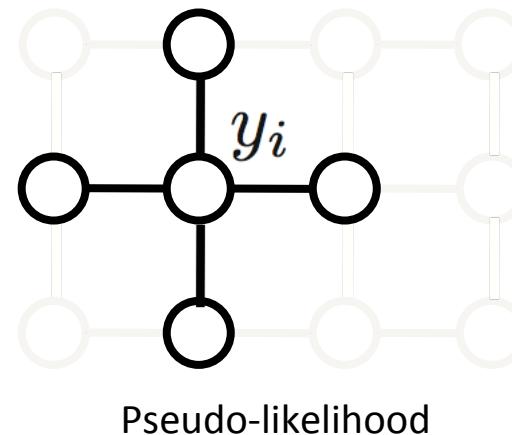
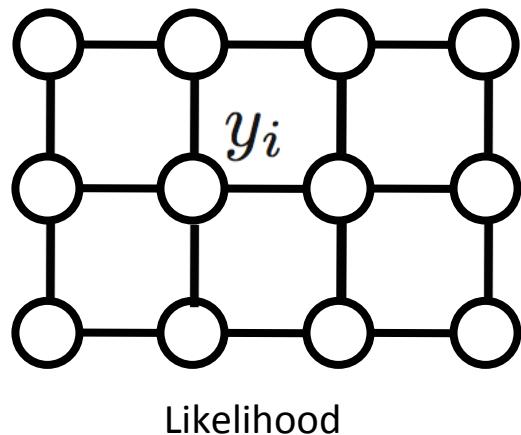
$$\Pr(\mathbf{y}|\mathbf{x}, \boldsymbol{\theta}) = \frac{1}{Z(\boldsymbol{\theta})} \prod_{i \in \mathcal{V}} \exp(y_i \boldsymbol{\theta}_u^\top h_i(\mathbf{x})) \prod_{(i,j) \in \mathcal{E}} \exp(y_i y_j \boldsymbol{\theta}_p^\top \nu_{ij}(\mathbf{x}))$$


Partition function – Infeasible to compute
for vision problems (2^N values)

- Approximate methods: Sampling, mode of the distribution, pseudo-likelihood

Pseudo-likelihood

$$\Pr(\mathbf{y}|\mathbf{x}, \theta) = \prod_{\substack{i \in \mathcal{V} \\ (i,j) \in \mathcal{E}}} \Pr(y_i|\mathbf{x}, y_j, \theta)$$



Besag, The Statistician, 1975

Pseudo-likelihood

$$\Pr(\mathbf{y}|\mathbf{x}, \boldsymbol{\theta}) = \prod_{\substack{i \in \mathcal{V} \\ (i,j) \in \mathcal{E}}} \Pr(y_i|\mathbf{x}, y_j, \boldsymbol{\theta})$$

\uparrow

$$\frac{1}{z_i(\boldsymbol{\theta})} \exp(y_i \boldsymbol{\theta}_u^\top h_i(\mathbf{x})) \dots$$

- ✓ Easy to compute
- ✗ But, can lead to poor accuracy

Max-Margin Learning

- Consider

$$\log \Pr(\mathbf{y}|\mathbf{x}, \boldsymbol{\theta}) = \boldsymbol{\theta}^T \mathbf{X} \mathbf{y} - \log Z(\boldsymbol{\theta})$$

- Maximize confidence margin in true label assignment
- In other words, maximize

$$\log \Pr(\hat{\mathbf{y}}|\mathbf{x}, \boldsymbol{\theta}) - \log \Pr(\mathbf{y}|\mathbf{x}, \boldsymbol{\theta}) = \boldsymbol{\theta}^T \mathbf{X} (\hat{\mathbf{y}} - \mathbf{y})$$

↑ ↑
True label All other label assignments

Max-Margin Learning

- Formulated as SVM learning problem

$$\min \quad \frac{1}{2} \|\boldsymbol{\theta}\|^2 + C\xi$$

$$\text{subject to} \quad \boldsymbol{\theta}^T \mathbf{F}(\hat{\mathbf{y}} - \mathbf{y}) \geq N - \hat{\mathbf{y}}_i^T \mathbf{y}_i - \xi, \forall \mathbf{y} \in \mathcal{Y}$$

Max-Margin Learning

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Exponential no. of constraints!!
 $(2^N - 1)$

Max-Margin Learning

- Formulated as SVM learning problem

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$$\text{subject to } \boldsymbol{\theta}^T \mathbf{F} \hat{\mathbf{y}} - N + \xi \geq \max_{\mathbf{y} \in \mathcal{Y}} \boldsymbol{\theta}^T \mathbf{F} \mathbf{y} - \hat{\mathbf{y}}_i^\top \mathbf{y}_i$$


Inference problem to find the **most violated constraint**
e.g., graph cuts

Max-Margin Learning

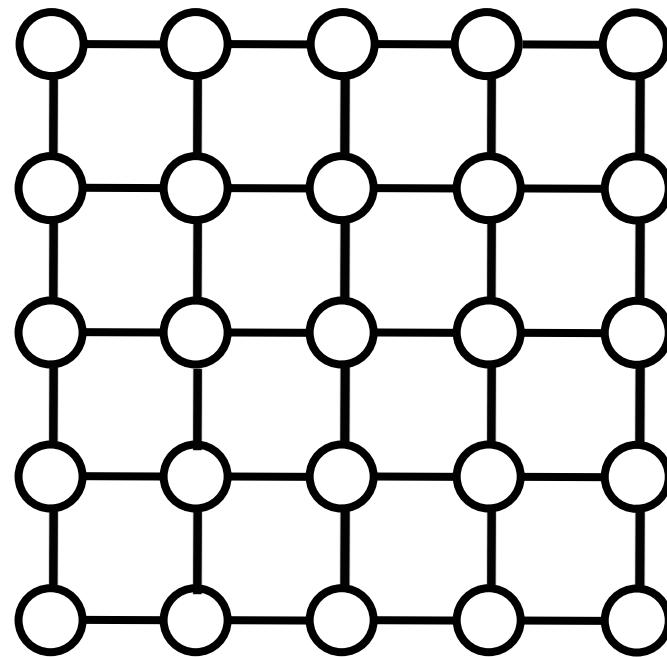
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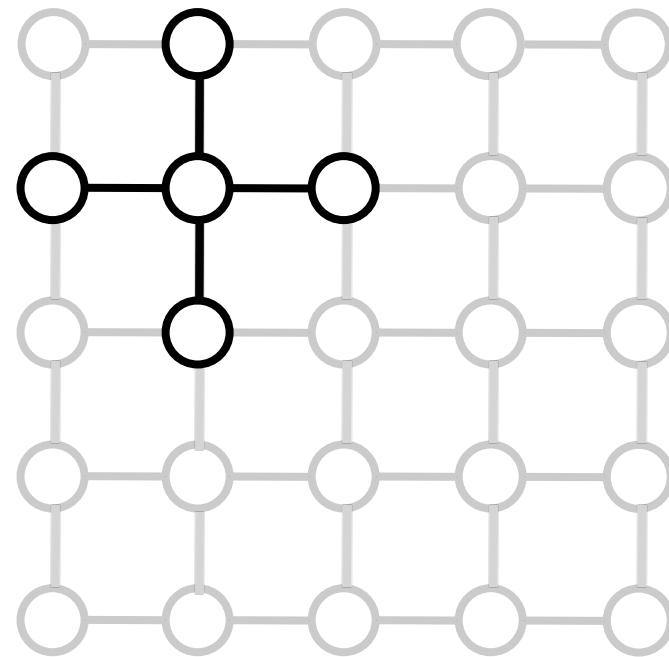
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- ✓ Eliminates the partition function
- ✗ Limited by the inference step

Our Piecewise Model

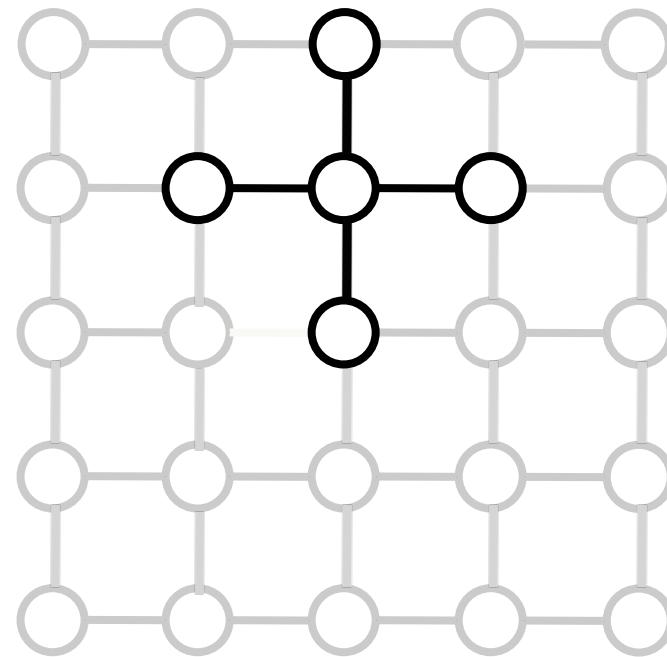


Our Piecewise Model



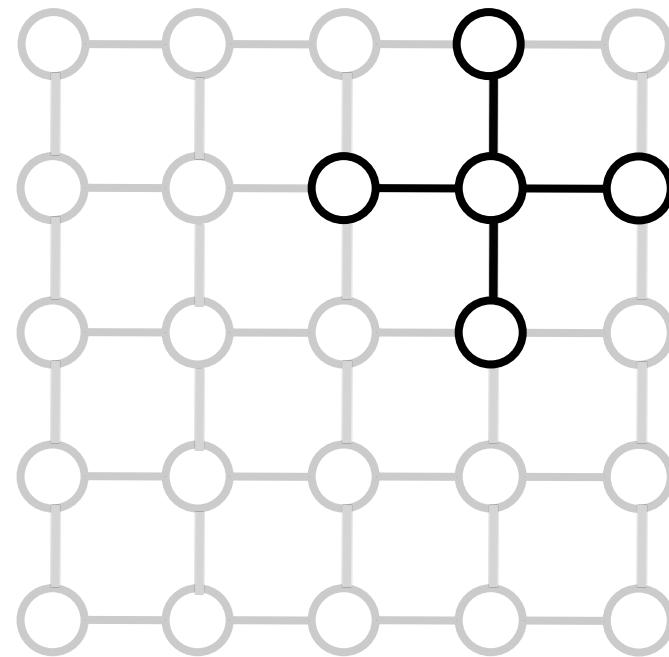
Training Exemplar 1

Our Piecewise Model



Training Exemplar 2

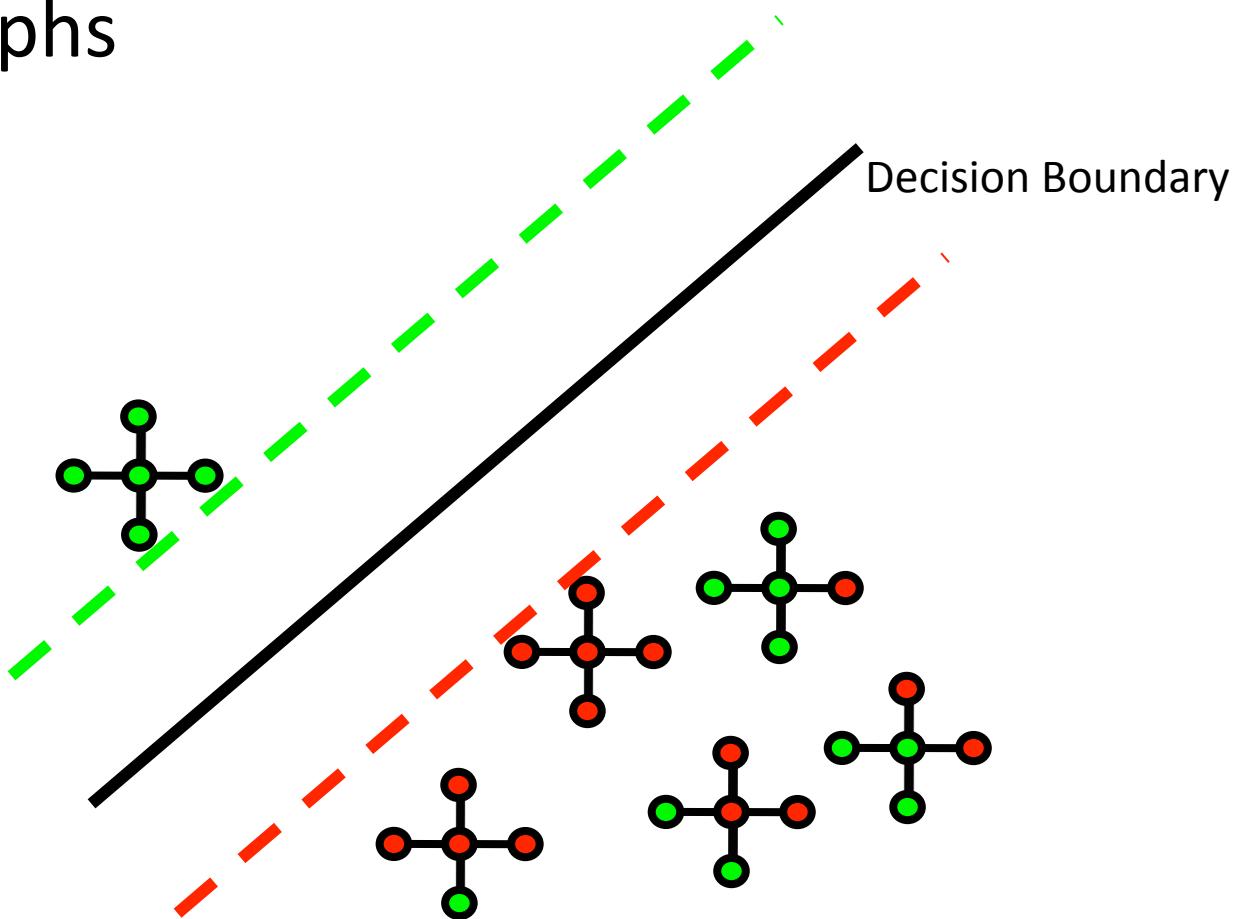
Our Piecewise Model



Training Exemplar 3

Our Piecewise Model

- Formulate a max-margin problem on these sub-graphs



Our Piecewise Model

- Formulate a max-margin problem on these sub-graphs

$$\min \quad \frac{1}{2} \|\boldsymbol{\theta}\|^2 + C\xi_s$$

subject to $\boldsymbol{\theta} \mathbf{F}_s (\hat{\mathbf{y}}_s - \mathbf{y}_s) \geq N - \hat{\mathbf{y}}_s^\top \mathbf{y}_s - \xi_s, \forall \mathbf{y}_s \in \mathcal{Y}_s$

Our Piecewise Model

- Formulate a max-margin problem on these sub-graphs

$$\min \quad \frac{1}{2} \|\boldsymbol{\theta}\|^2 + C\xi_s$$

Slack variable

subject to $\boldsymbol{\theta} \mathbf{F}_s (\hat{\mathbf{y}}_s - \mathbf{y}_s) \geq N - \hat{\mathbf{y}}_s^\top \mathbf{y}_s - \xi_s, \forall \mathbf{y}_s \in \mathcal{Y}_s$

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Exponential # constraints again!

Our Piecewise Model

- Formulate a max-margin problem on these sub-graphs

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subject to $\boldsymbol{\theta} \mathbf{F}_s (\hat{\mathbf{y}}_s - \mathbf{y}_s) \geq N - \hat{\mathbf{y}}_s^\top \mathbf{y}_s - \xi_s, \forall \mathbf{y}_s \in \mathcal{Y}_s$

- Can be solved efficiently using BP messages

Piecewise: Results

- Middlebury Stereo

Method	Art	Books	Dolls	Laundry	Moebius	Reindeer	Average
Li and Huttenlocher	14.66	19.12	12.70	19.16	10.88	11.72	14.71
Our method	12.94	16.24	12.21	16.72	10.82	11.10	13.34

- Man-made Structures

Method	FP per image	DR %
MRF	2.36	57.20
DRF1	1.37	70.50
DRF2	1.76	72.54
Our method	1.40	72.60

So far ...

- In the context of labelling problems

- Learning parameters of the function

Next:

- Modelling image priors & performing inference

Modelling image priors & inference

- Energy function: $E(\mathbf{y}) = \sum_{i \in \mathcal{V}} \psi_i(y_i) + \sum_{(i,j) \in \mathcal{E}} \psi_{ij}(y_i, y_j)$
- How to minimize this efficiently?
- New image priors, e.g., higher-order term: $\sum_{\mathbf{c} \in \mathcal{S}} \psi_{\mathbf{c}}(y_{\mathbf{c}})$

Example: Image Segmentation

$$E(\mathbf{y}) = \sum_{i \in \mathcal{V}} \psi_i(y_i) + \sum_{(i,j) \in \mathcal{E}} \psi_{ij}(y_i, y_j)$$

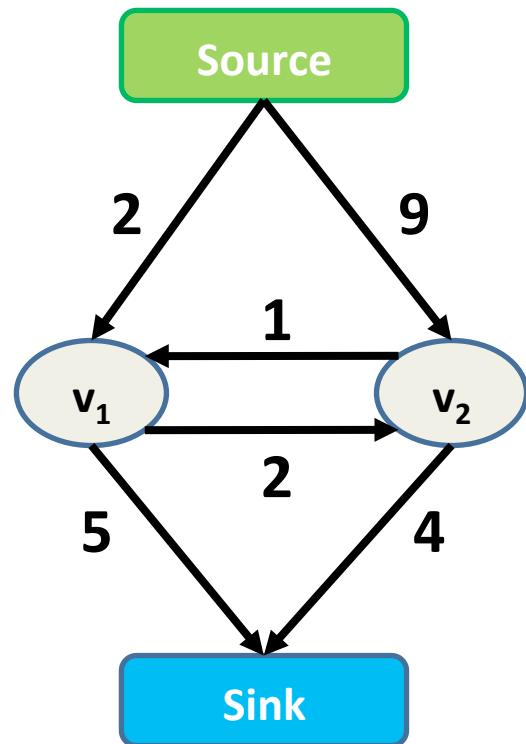
$$\begin{aligned} E: \{0, 1\}^N &\rightarrow \mathcal{R} \\ 0 &\rightarrow fg \\ 1 &\rightarrow bg \end{aligned}$$

N: number of pixels

How to minimize $E(y)$?

st-mincut algorithm, belief propagation, α -expansion

The st-Mincut Problem



Graph (V, E, C)

Vertices $V = \{v_1, v_2 \dots v_n\}$

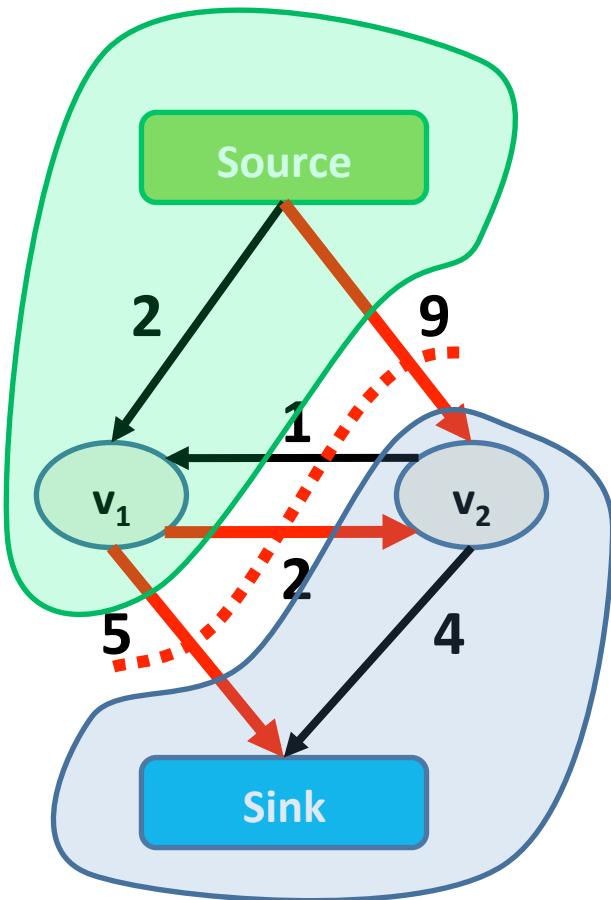
Edges $E = \{(v_1, v_2) \dots\}$

Costs $C = \{c_{(1, 2)} \dots\}$

The st-Mincut Problem

What is an st-cut?

An st-cut (S, T) divides the nodes between source and sink.



What is the cost of an st-cut?

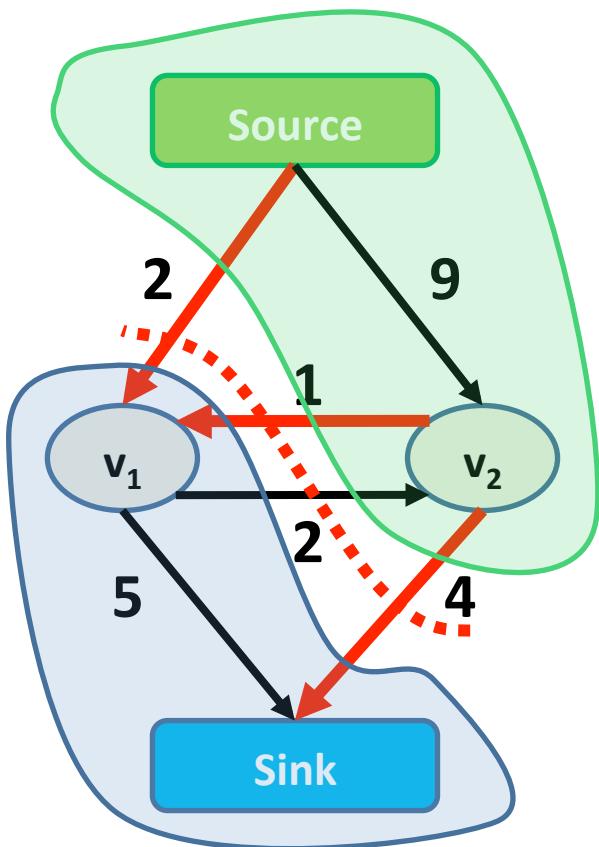
Sum of cost of all edges going from S to T

$$5 + 2 + 9 = 16$$

The st-Mincut Problem

What is an st-cut?

An st-cut (S, T) divides the nodes between source and sink.



$$2 + 1 + 4 = 7$$

What is the cost of an st-cut?

Sum of cost of all edges going from S to T

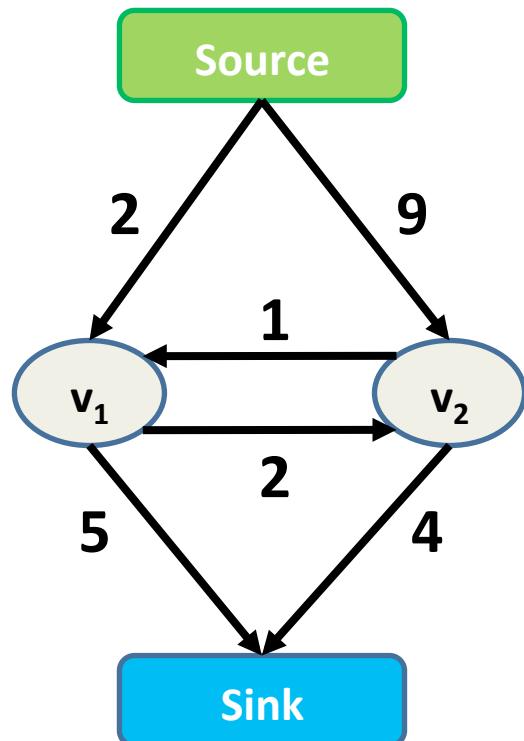
What is the st-mincut?

st-cut with the minimum cost

How to compute the st-mincut?

Solve the dual **maximum flow** problem

Compute the maximum flow between
Source and Sink



Constraints

Edges: Flow < Capacity

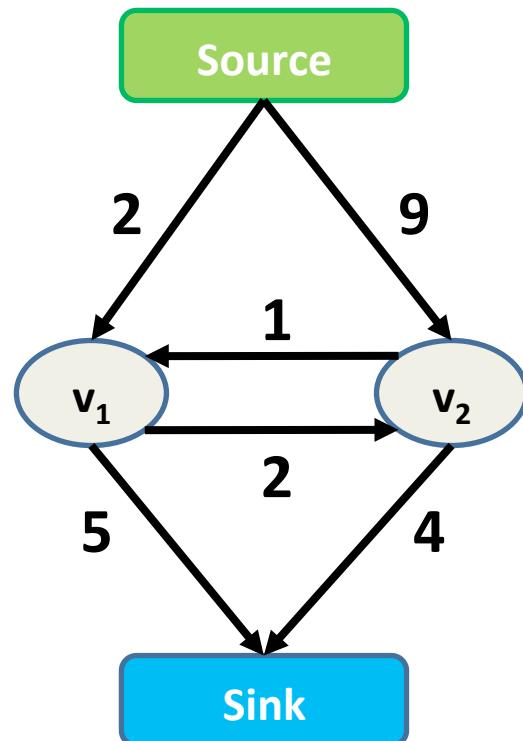
Nodes: Flow in = Flow out

Min-cut/Maxflow Theorem

In every network, the maximum flow
equals the cost of the st-mincut

Maxflow Algorithms

Flow = 0



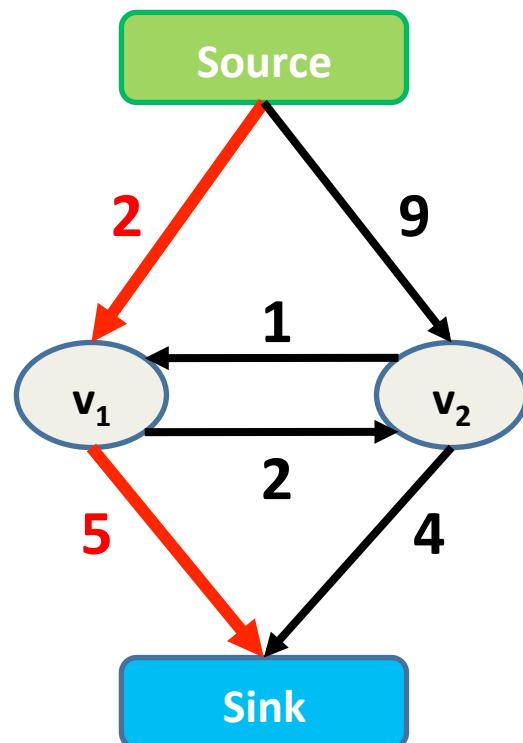
Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 0



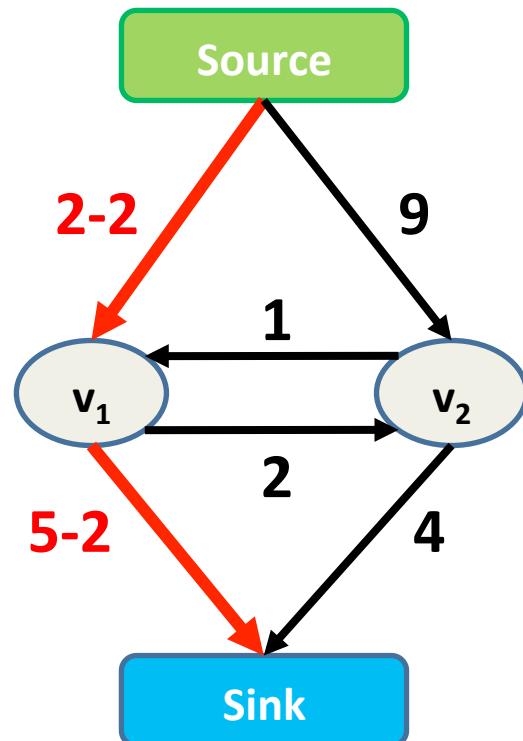
Augmenting Path Based Algorithms

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Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 0 + 2



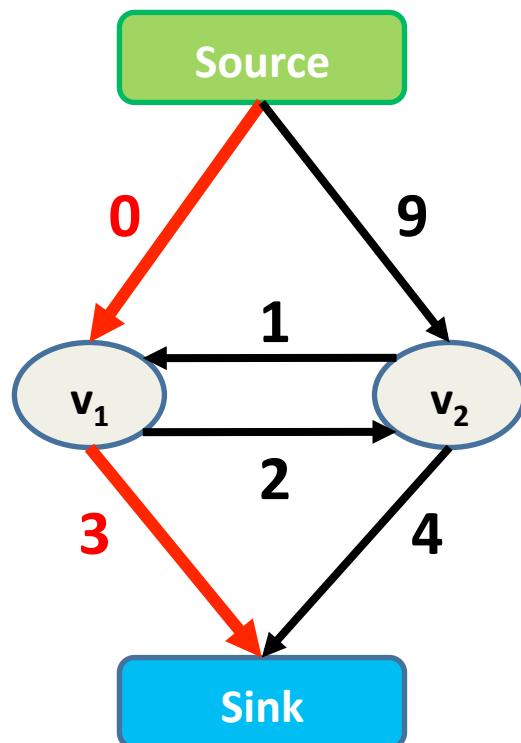
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Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 2



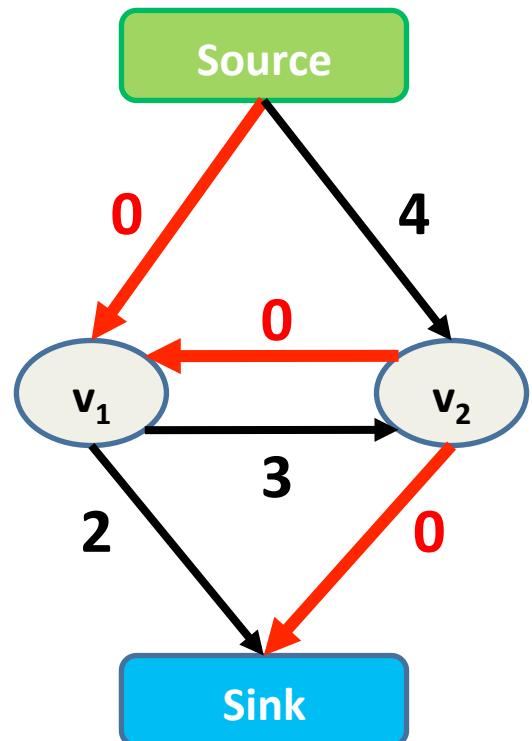
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Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 7

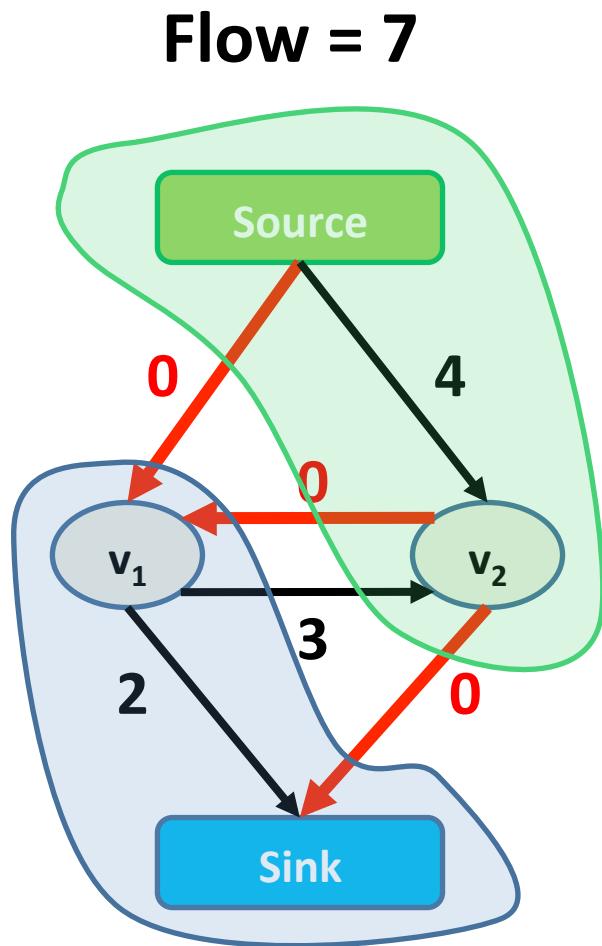


Augmenting Path Based Algorithms

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Algorithms assume non-negative capacity

Maxflow Algorithms



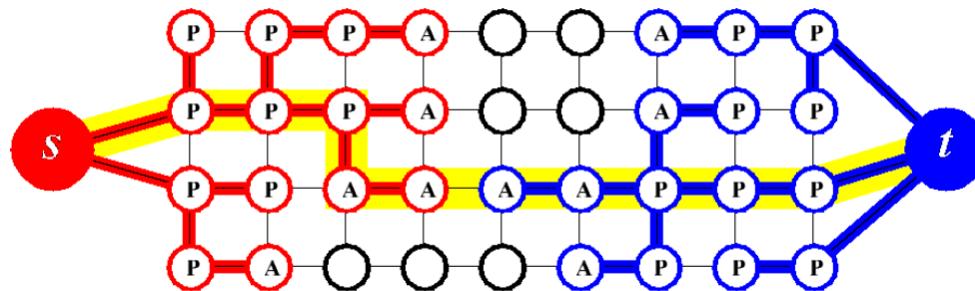
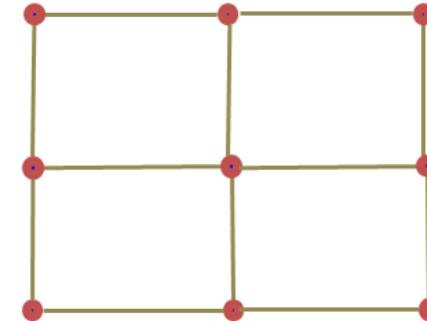
Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

Maxflow in Computer Vision

- Specialized algorithms for vision problems
 - Grid graphs
 - Low connectivity ($m \sim O(n)$)
- Dual search tree augmenting path algorithm [Boykov & Kolmogorov PAMI'04]



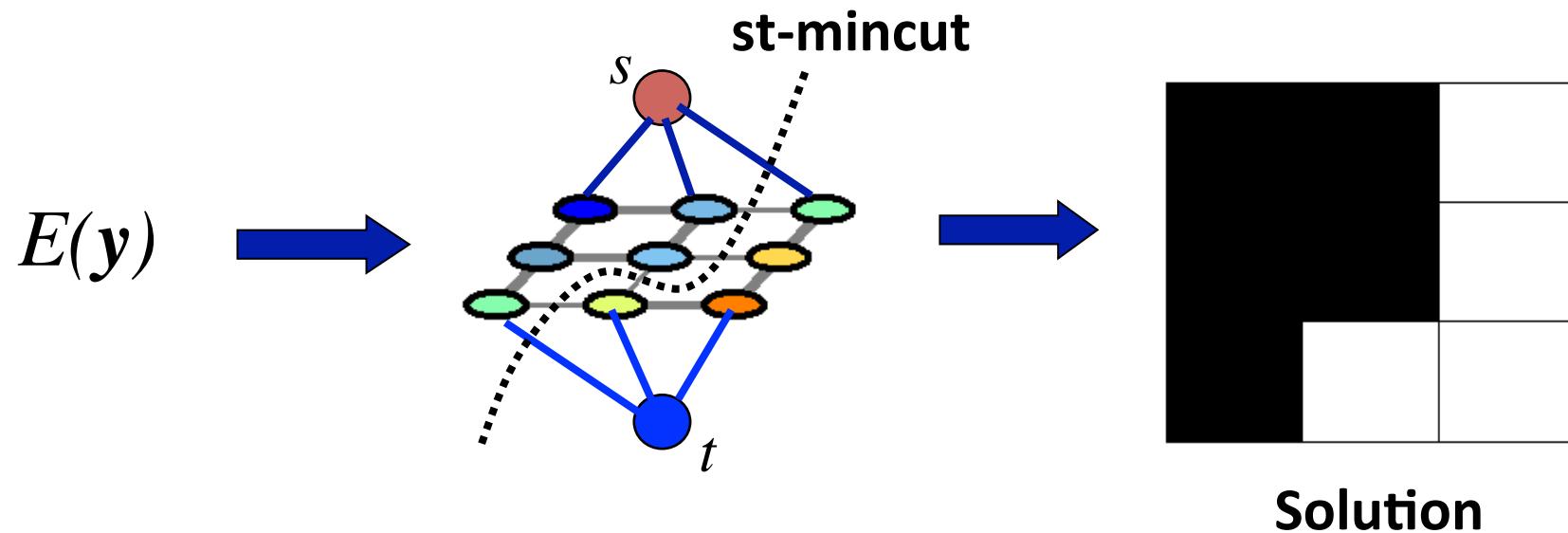
- Finds approximate shortest augmenting paths efficiently
- High worst-case time complexity
- Empirically outperforms other algorithms on vision problems

Slide courtesy: Pushmeet Kohli

st-mincut \leftrightarrow Labelling

Construct a graph such that:

1. Any st-cut corresponds to an assignment of y
2. The cost of the cut is equal to the energy of $y : E(y)$



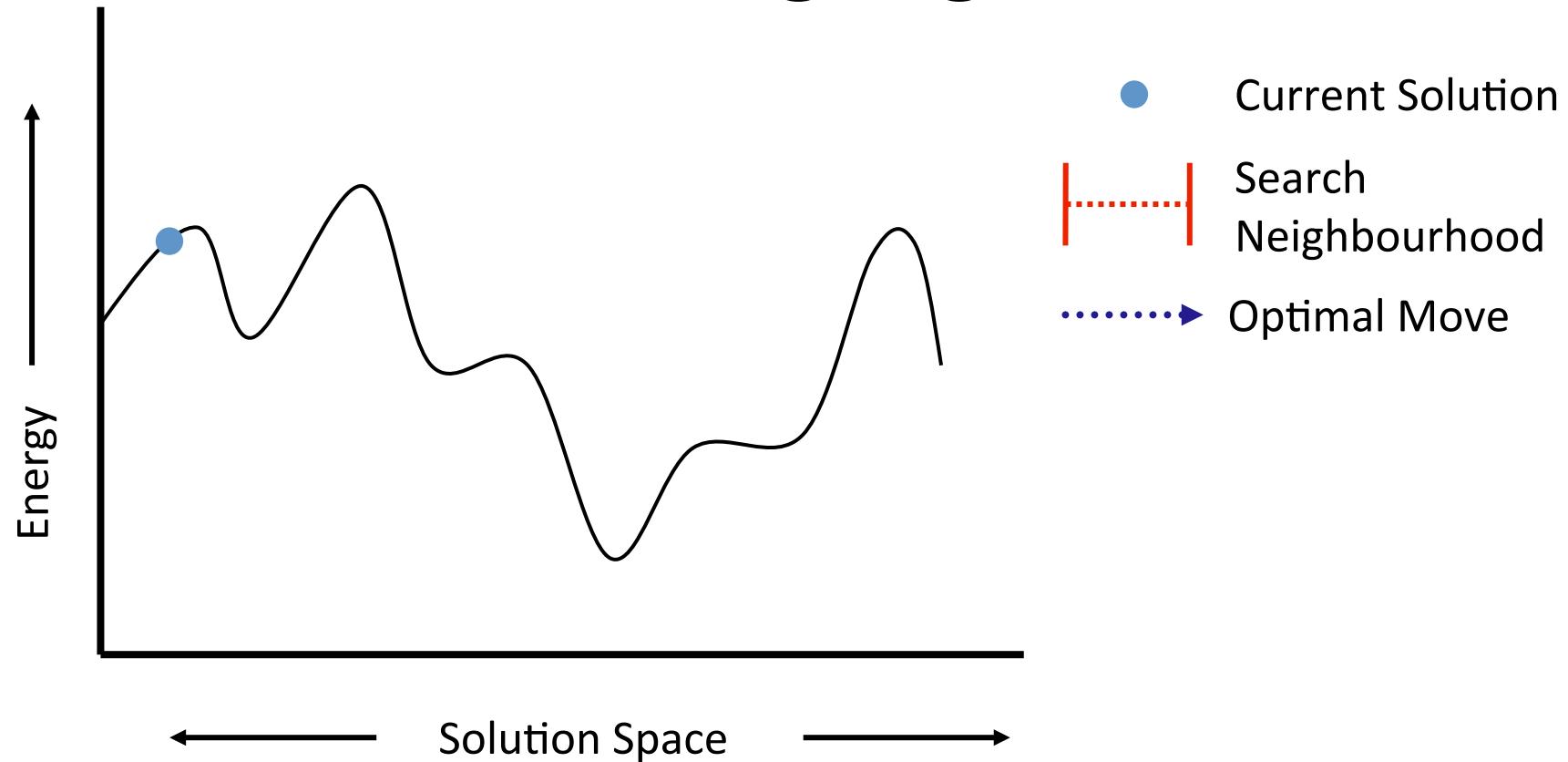
St-mincut based Move algorithms

$$E(\mathbf{y}) = \sum_{i \in \mathcal{V}} \psi_i(y_i) + \sum_{(i,j) \in \mathcal{E}} \psi_{ij}(y_i, y_j)$$

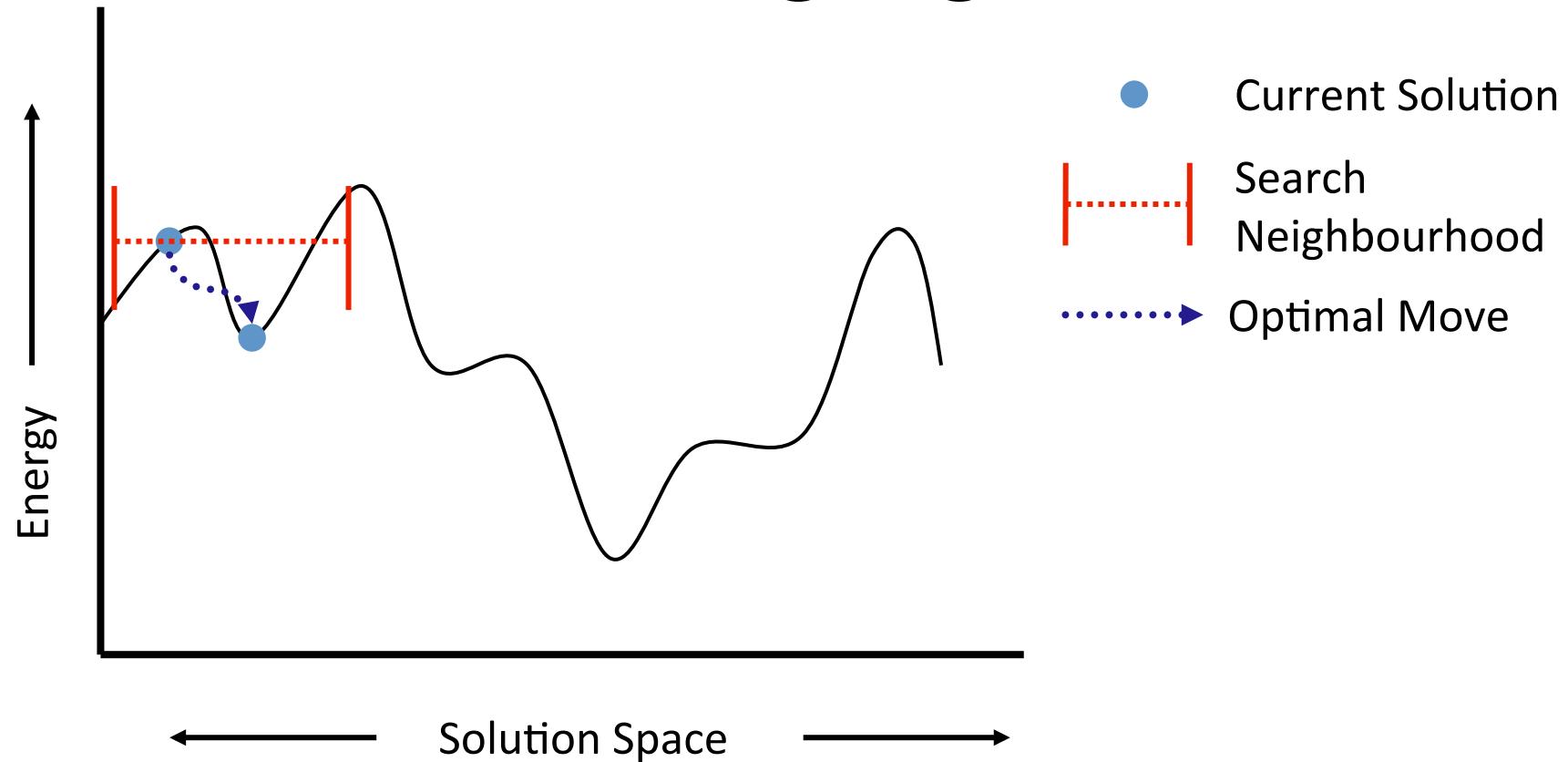
$$\mathbf{y} \in \text{Labels } \mathcal{L} = \{l_1, l_2, \dots, l_k\}$$

- Commonly used for solving multi-label problems
- Extremely efficient and produce good solutions
- Not Exact: Produce local optima

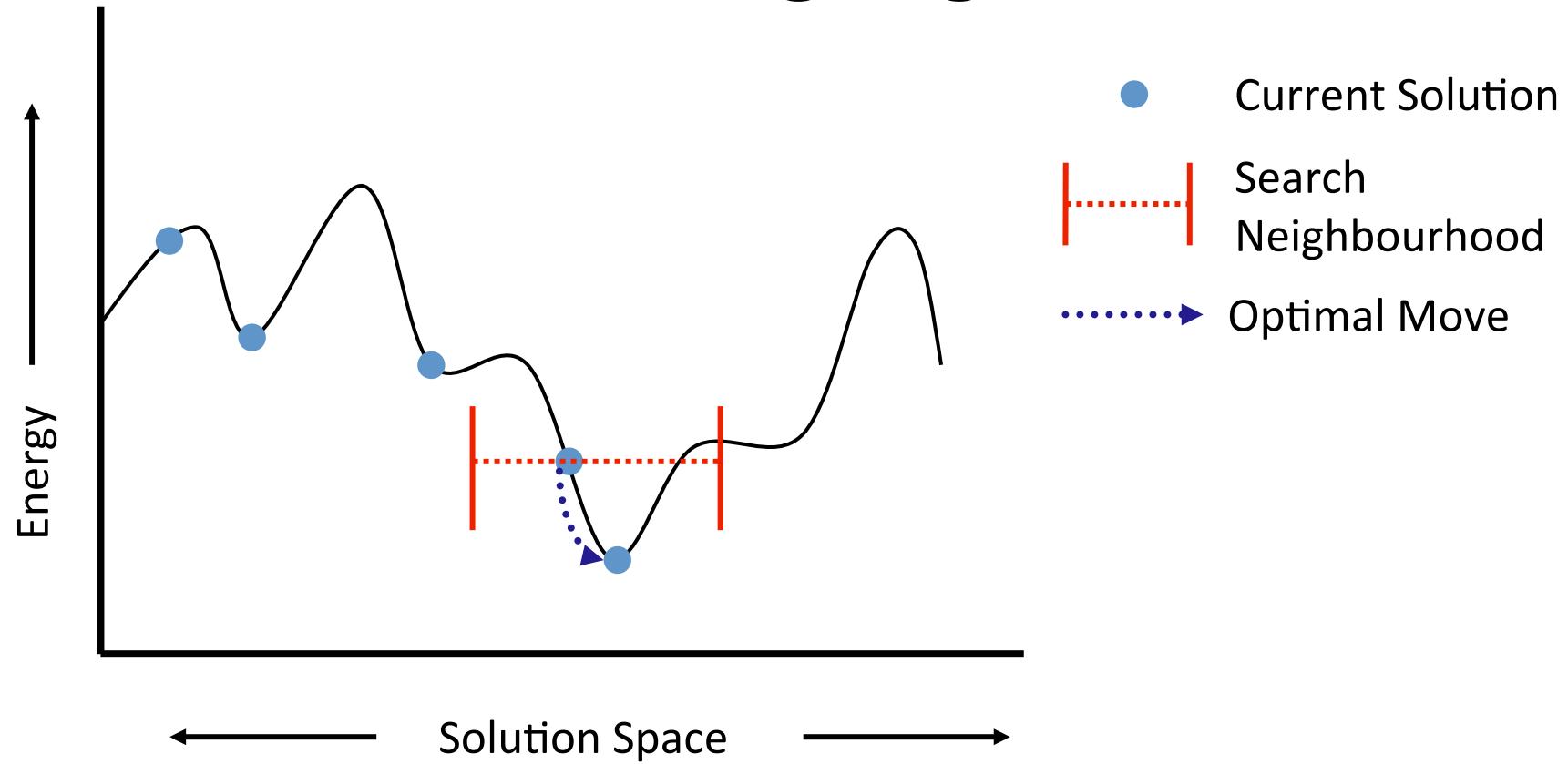
Move Making Algorithms



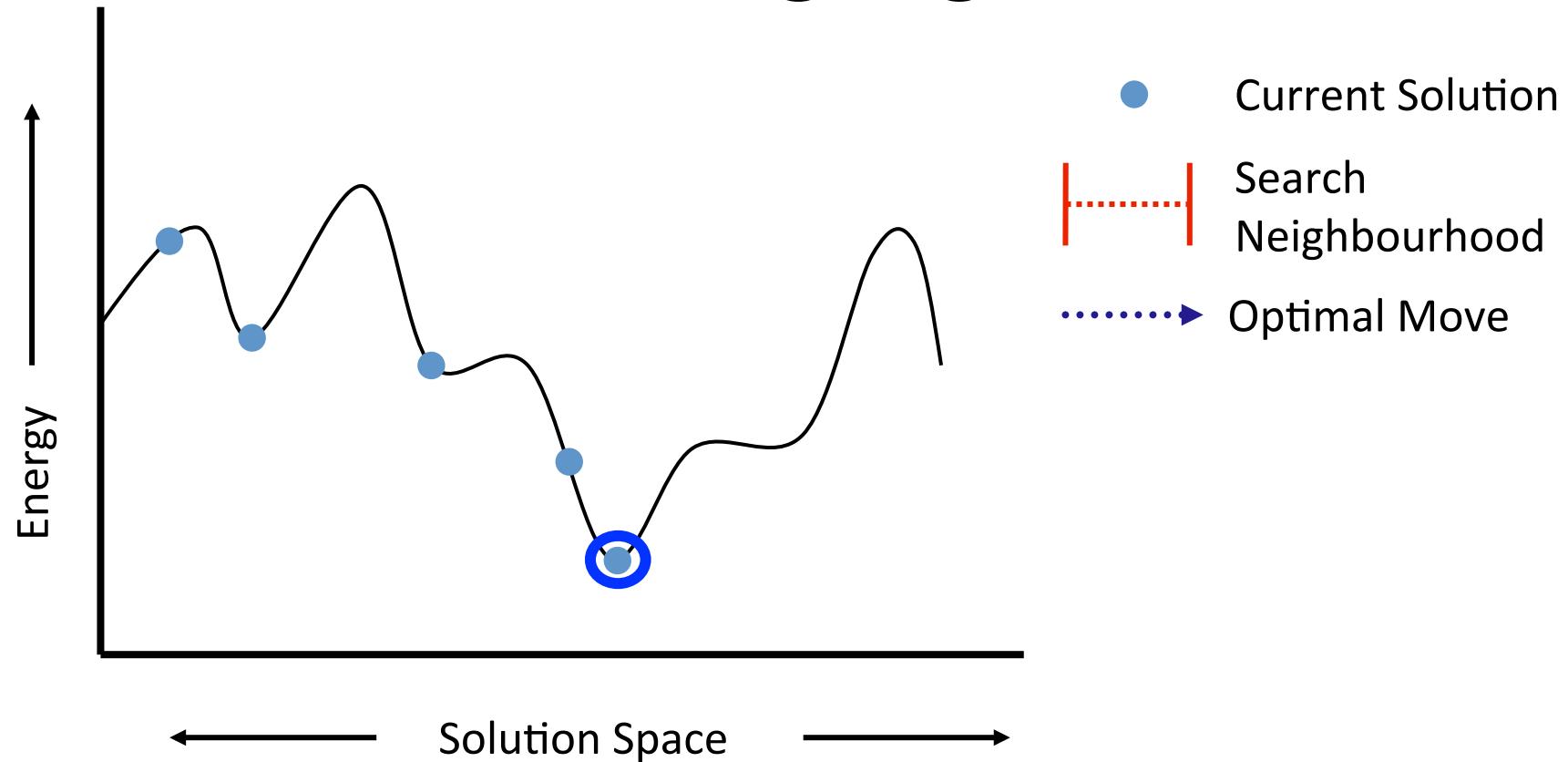
Move Making Algorithms



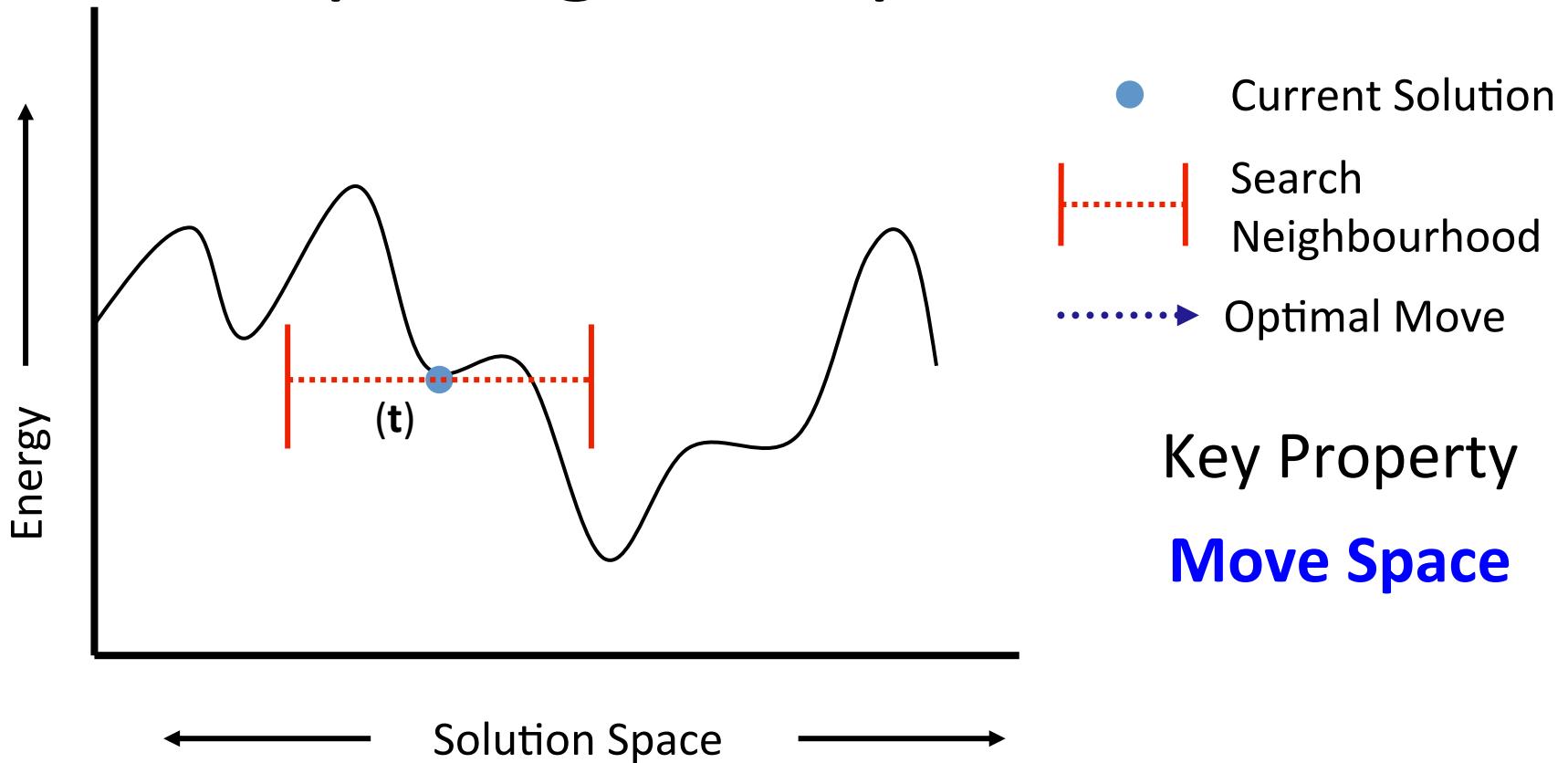
Move Making Algorithms



Move Making Algorithms



Computing the Optimal Move



Bigger move
space

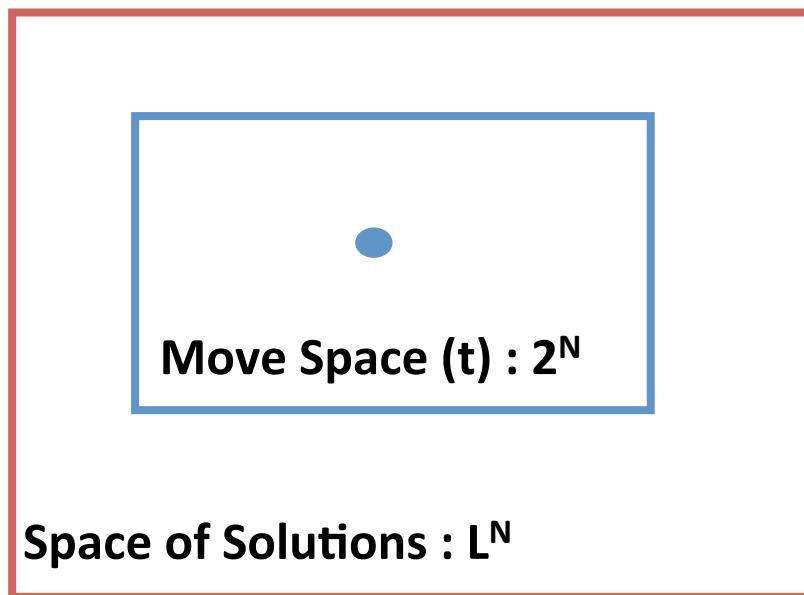


- Better solutions
- Finding the optimal move hard

Moves using Graph Cuts

Expansion and Swap move algorithms [Boykov et al., PAMI'01]

- Make a series of changes to the solution (moves)
- Each move results in a solution with smaller energy



- Current solution
- Search neighbourhood
- N Number of variables
- L Number of labels

How to minimize move functions?

Expansion Move

- Variables take label α or retain current label



Status:

■	Tree
■	Ground
■	House
■	Sky

Expansion Move

- Variables take label α or retain current label



Status: Initialize with Tree



Expansion Move

- Variables take label α or retain current label



Status: Expand Ground



Expansion Move

- Variables take label α or retain current label



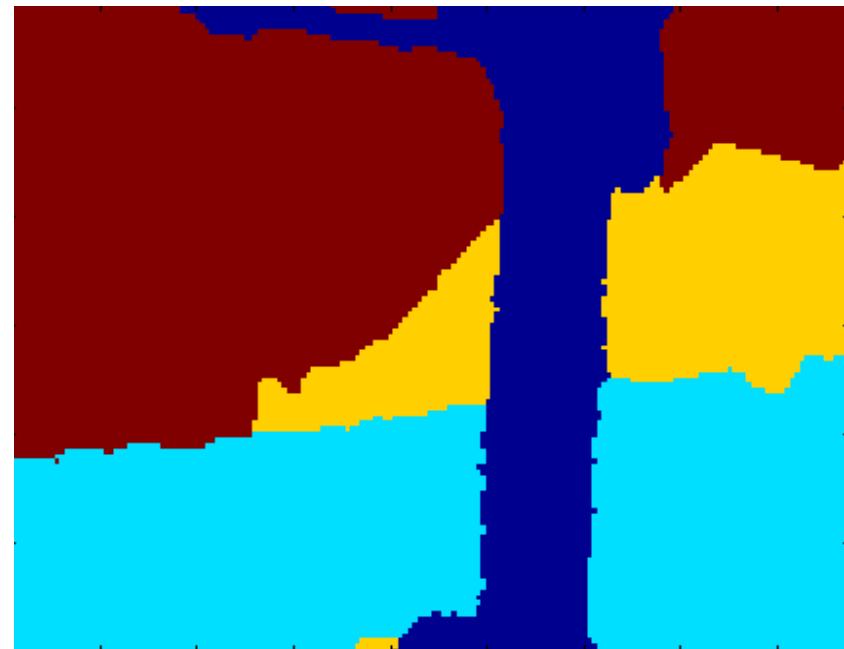
Status: Expand House



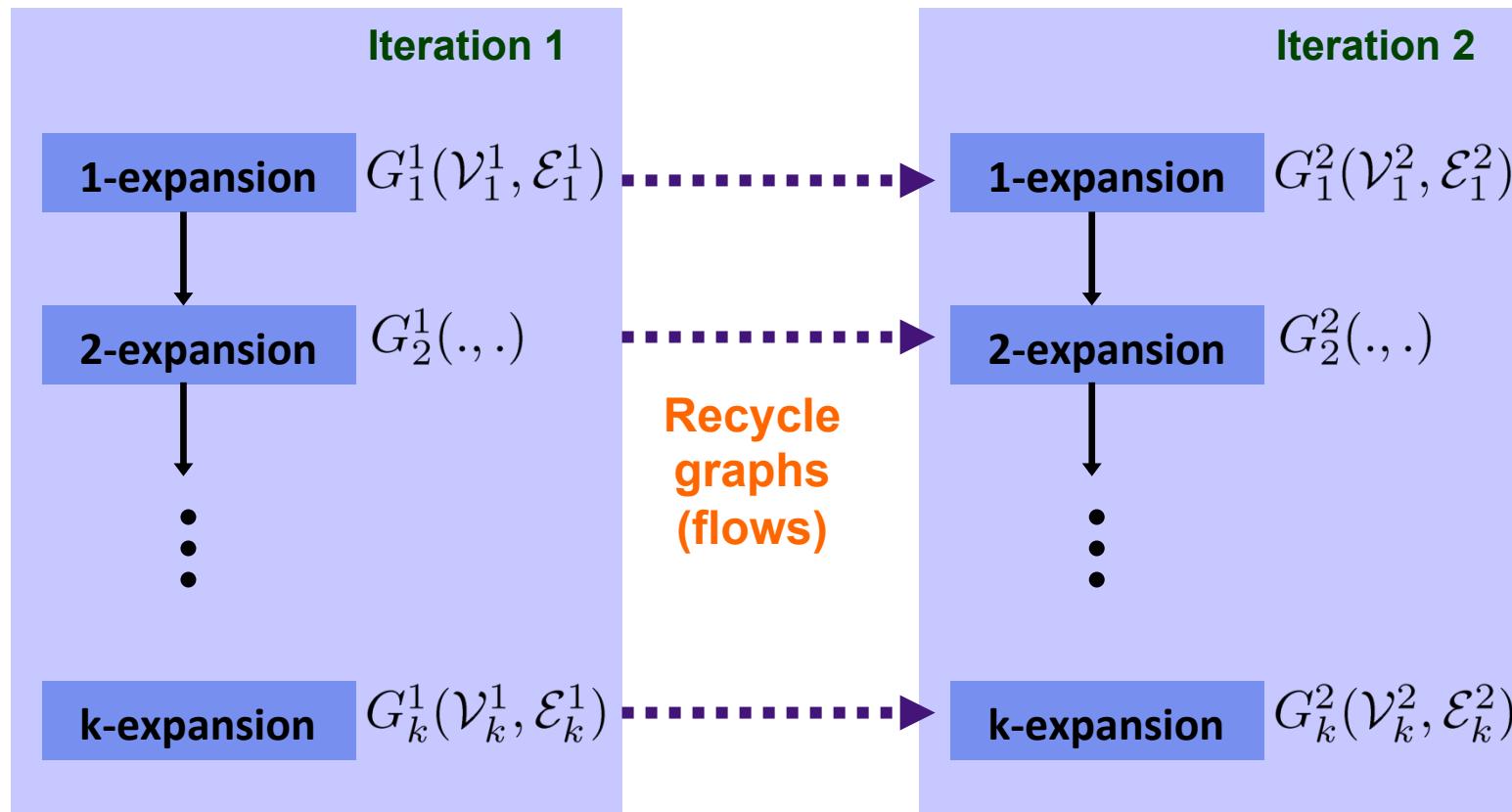
Expansion Move

- Variables take label α or retain current label

Status: Expand Sky



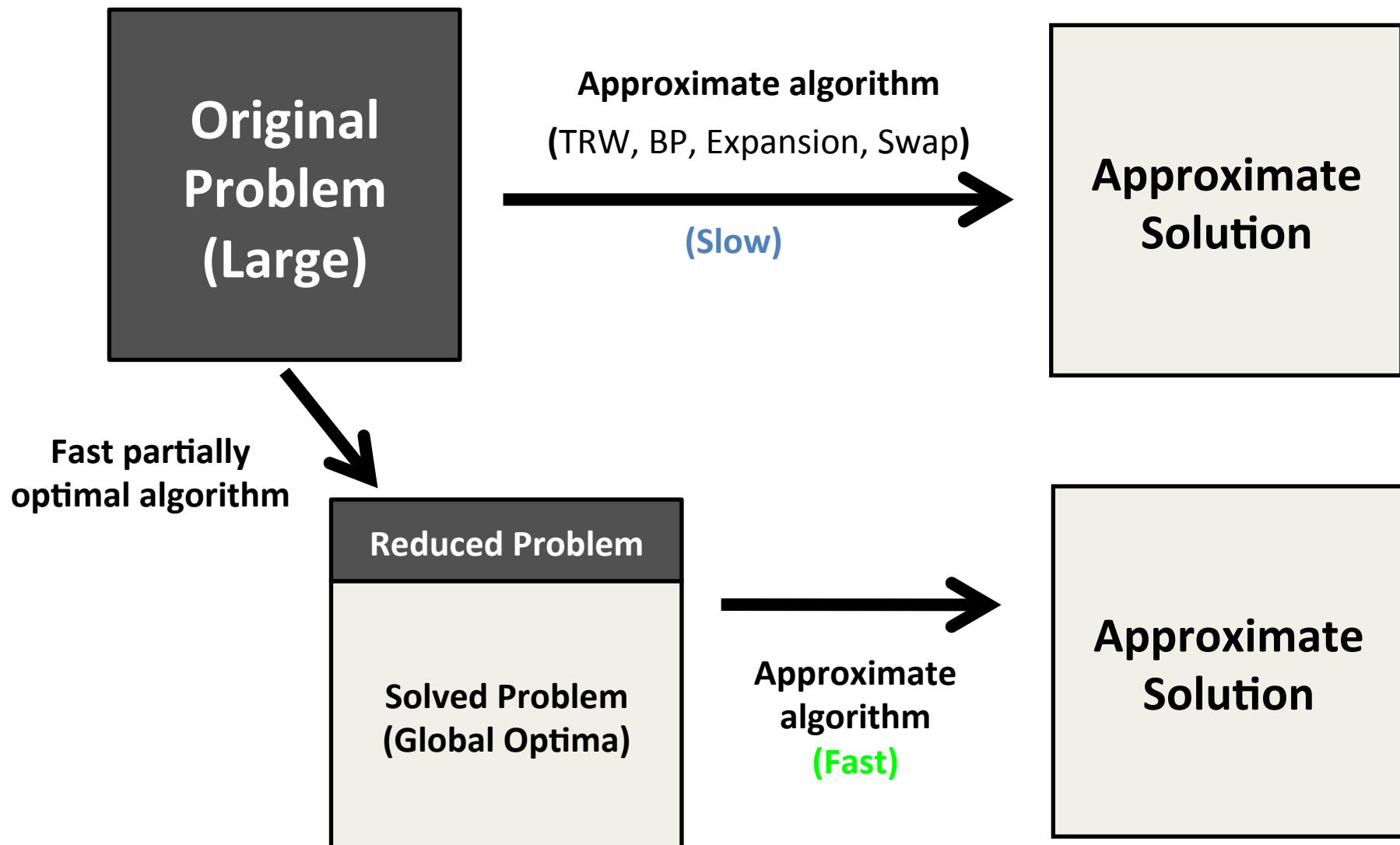
Efficient Inference: Recycling



Build a new graph for
each expansion move

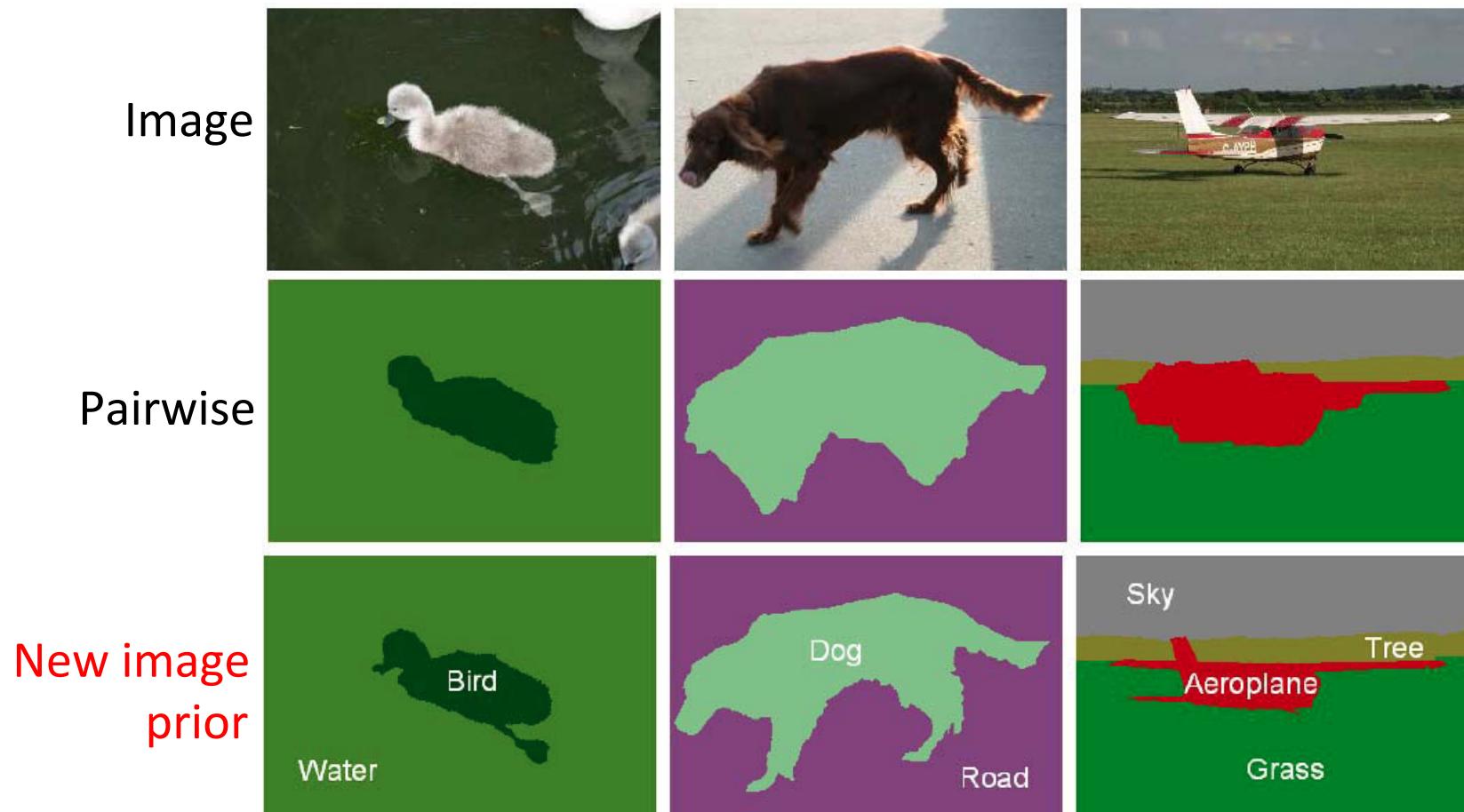
Build a new graph for
each expansion move

Efficient Inference: Reduction



Modelling image priors & inference

- Pairwise energy functions are limited



Kohli et al., IJCV'09, Ladicky et al., ECCV'10

Modelling image priors & inference

- Pairwise energy functions are limited



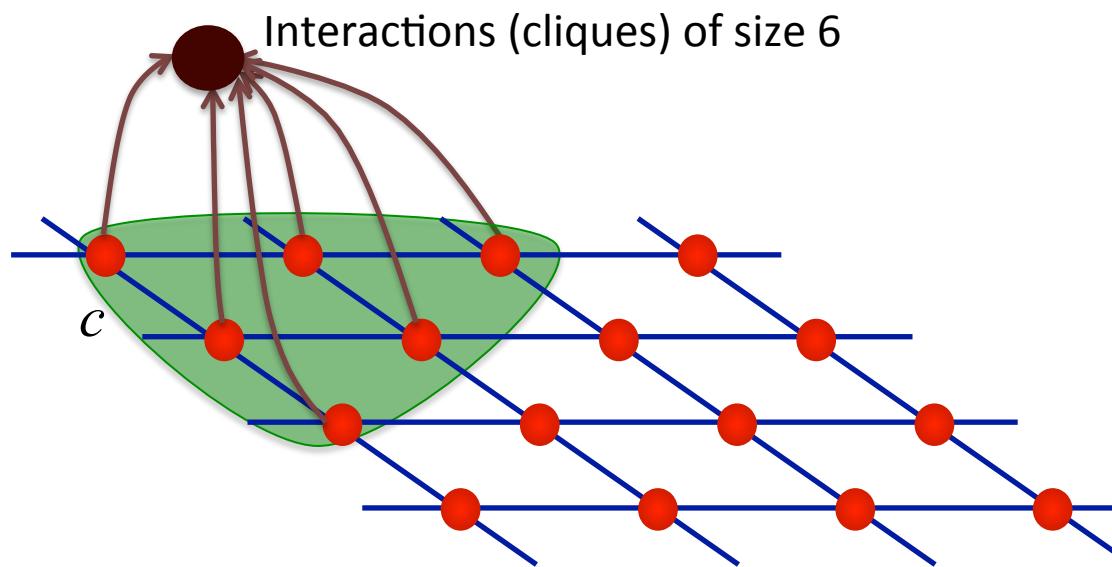
- Need **new energy terms** to capture **fine contours**

Modelling image priors & inference

- New energy terms to capture fine contours

$$E(\mathbf{y}) = \sum_{i \in \mathcal{V}} \psi_i(y_i) + \sum_{(i,j) \in \mathcal{E}} \psi_{ij}(y_i, y_j) + \sum_{\mathbf{c} \in \mathcal{S}} \psi_{\mathbf{c}}(y_{\mathbf{c}})$$

- Higher-order terms, involving several nodes, e.g.,



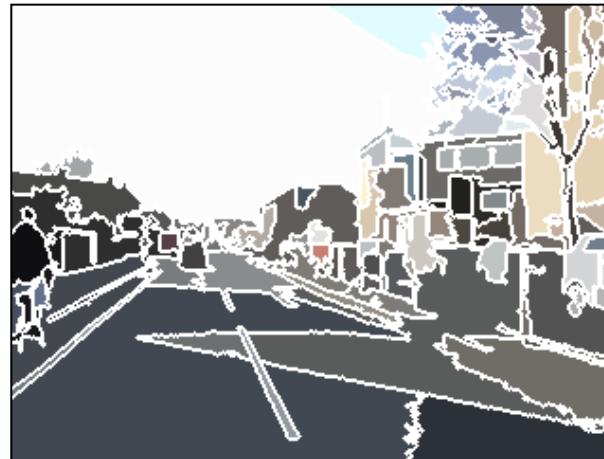
Kohli et al., IJCV'09, Ladicky et al., ECCV'10, Alahari PAMI'10

Modelling image priors & inference

- Higher-order terms, e.g.,
 - grouping “similar” pixels



Input Image



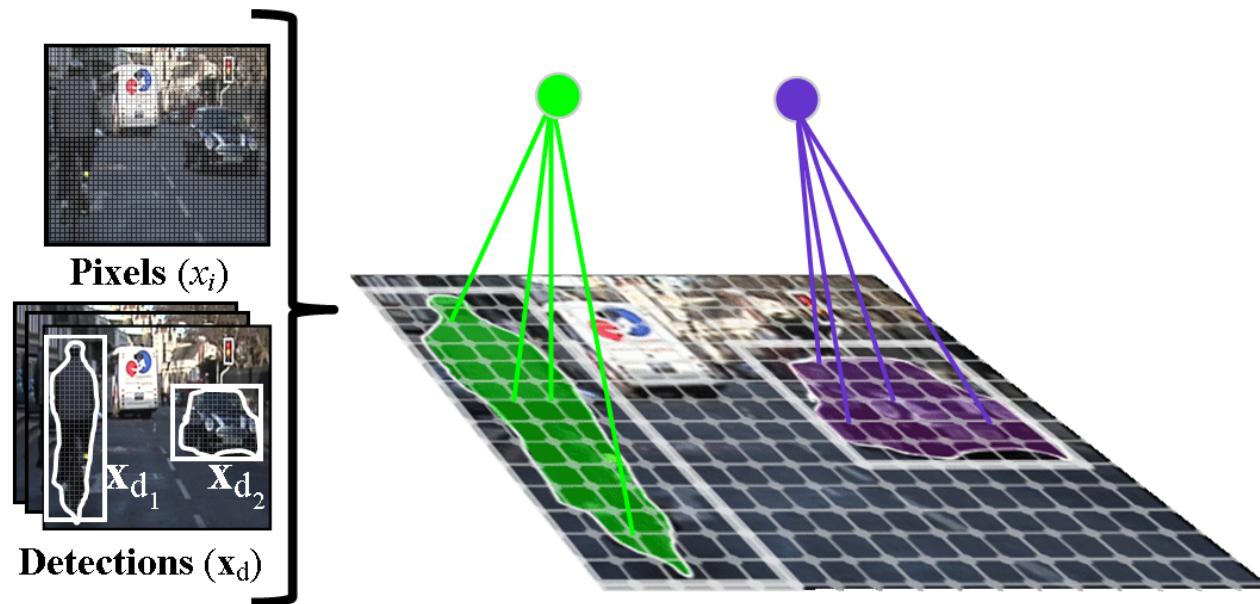
Meanshift segments [Comaniciu & Meer '02]



$$\psi_c(\mathbf{y}_c) = \min \left\{ \min_{k \in \mathcal{L}} ((|c| - n_k(\mathbf{y}_c))\theta_k + \gamma_k), \gamma_{\max} \right\}$$

Modelling image priors & inference

- Higher-order terms, e.g.,
 - Using “detected” objects



$$\psi_h(\mathbf{y}) = \min(\gamma_{max}, \min_l(\gamma_l + k_l \sum_{i \in \mathbf{y}} \delta(y_i \neq l)))$$

Modelling image priors & inference

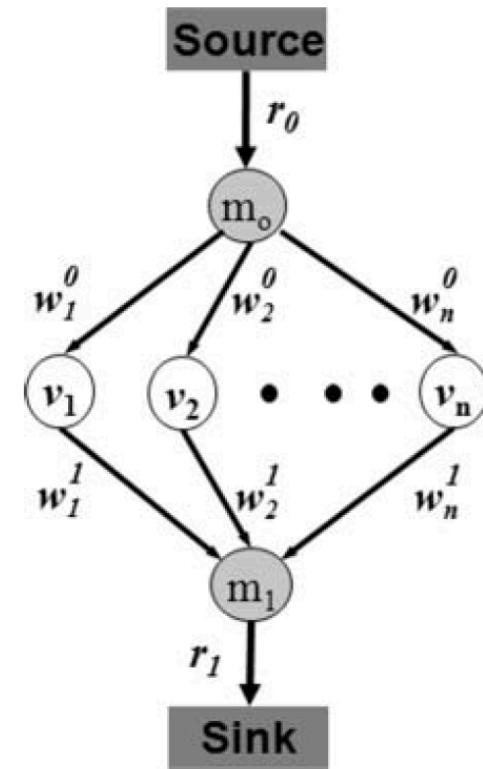
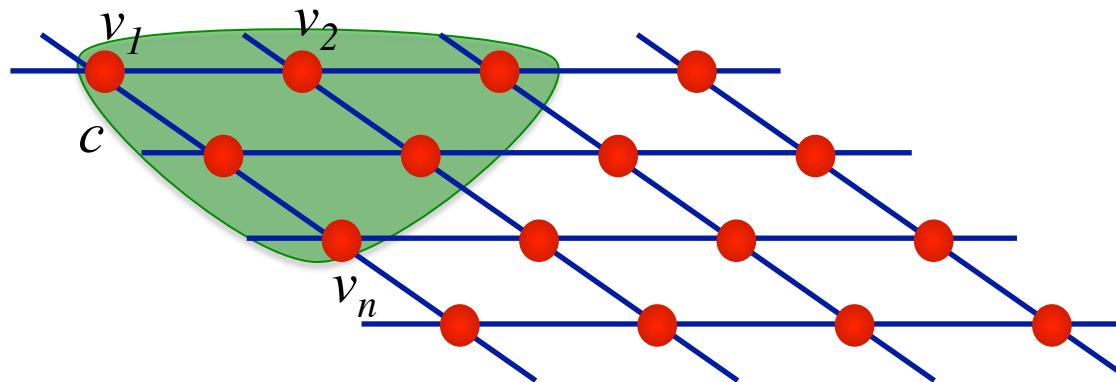
- Higher-order terms of the form:

$$\psi_h(\mathbf{y}) = \min(\gamma_{max}, \min_l(\gamma_l + k_l \sum_{i \in \mathbf{y}} \delta(y_i \neq l)))$$

- Can be minimized efficiently with move-making algorithms

Modelling image priors & inference

- Graph construction with auxiliary nodes



Modelling image priors & inference

- Evaluation: CamVid dataset [Brostow et al. '08]

	Building	Tree	Sky	Car	Sign-Symbol	Road	Pedestrian	Fence	Column-Pole	Sidewalk	Bicyclist	Average
Recall ⁶												
Brostow et al. '08	46.2	61.9	89.7	68.6	42.9	89.5	53.6	46.6	0.7	60.5	22.5	53.0
Sturgess et al. '09	84.5	72.6	97.5	72.7	34.1	95.3	34.2	45.7	8.1	77.6	28.5	59.2
Without detectors	79.3	76.0	96.2	74.6	43.2	94.0	40.4	47.0	14.6	81.2	31.1	61.6
Our method	81.5	76.6	96.2	78.7	40.2	93.9	43.0	47.6	14.3	81.5	33.9	62.5
Intersection vs Union ⁷												
Sturgess et al. '09	71.6	60.4	89.5	58.3	19.4	86.6	26.1	35.0	7.2	63.8	22.6	49.2
Without detectors	70.0	63.7	89.5	58.9	17.1	86.3	20.0	35.8	9.2	64.6	23.1	48.9
Our method	71.5	63.7	89.4	64.8	19.8	86.8	23.7	35.6	9.3	64.6	26.5	50.5

So far ...

- In the context of labelling problems

- Learning parameters of the function

- Modelling image priors & inference

Next:

- Modelling temporal constraints & inference

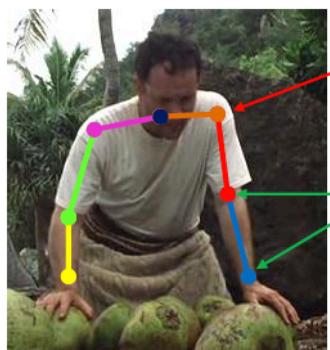
Human Pose Estimation



Poses in the Wild dataset, CVPR 2014

Human Pose Estimation (in an image)

- Formulated as a graph optimization problem



ϕ_u : unary potential

$\psi_{u,v}$: pairwise potential

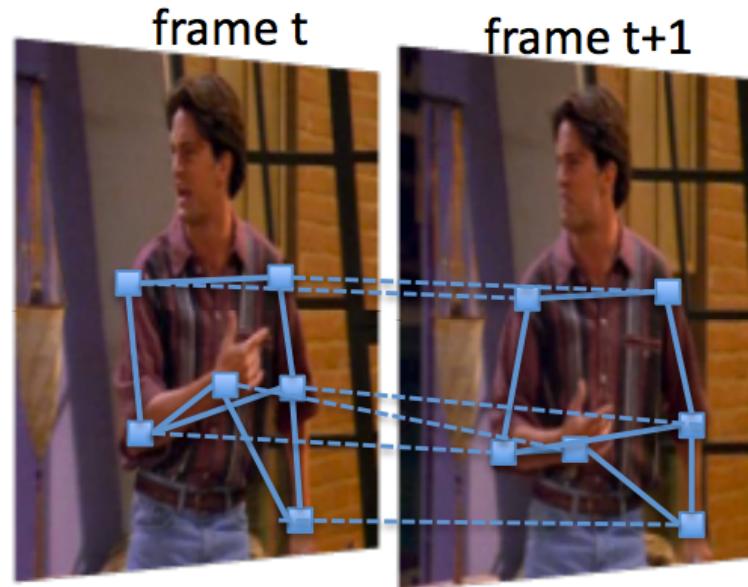
For an image I , pose model $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, and

$$p = \{p^u = (x^u, y^u) \in \mathbb{R}^2 : \forall u \in \mathcal{V}\}$$

$$\min C(I, p) := \sum_{u \in \mathcal{V}} \phi_u(I, p^u) + \sum_{(u, v) \in \mathcal{E}} \psi_{u,v}(p^u - p^v)$$

Human Pose Estimation

- Extension to videos: introduce temporal links
- Inference is now computationally expensive – requires approximate methods



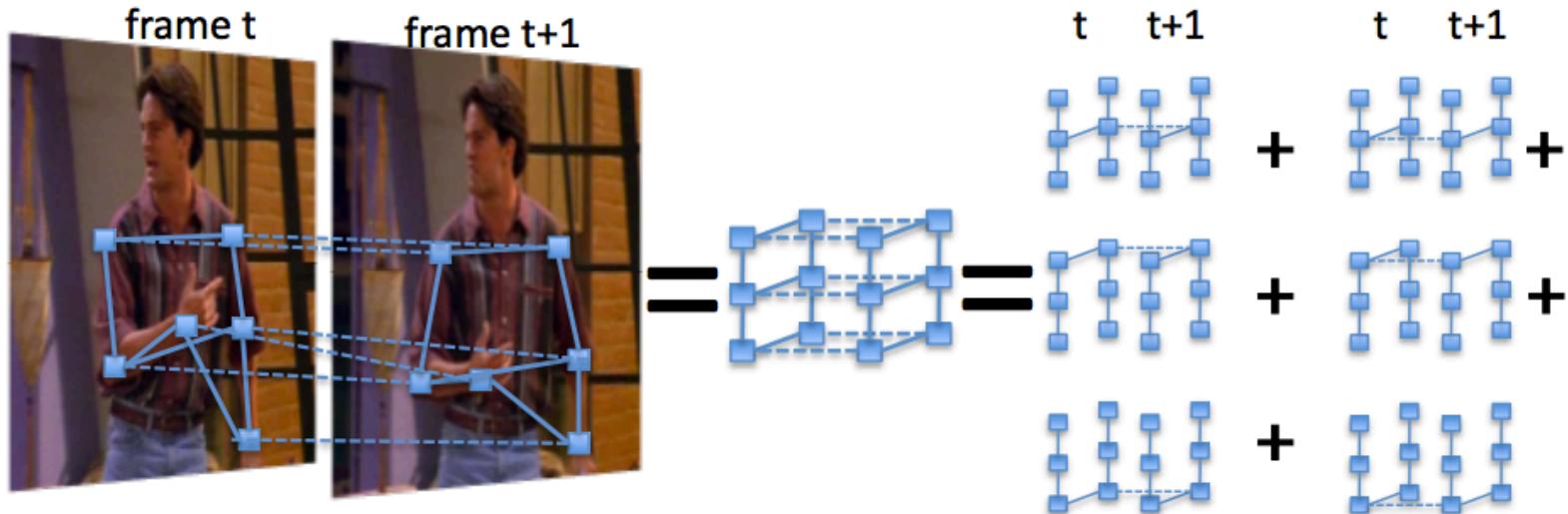
e.g., Sapp et al., '11, Tokola et al., '13

Human Pose Estimation

- Extension to videos: introduce temporal links
- Inference is now computationally expensive – requires approximate methods
- e.g.,
 - Change graph structure [Sapp et al. '11, Weiss et al. '11]
 - Use approximate inference [Ferrari et al. '08, Wang et al. '08, Park & Ramanan '11, Tokola et al. '13]

Human Pose Estimation (Video)

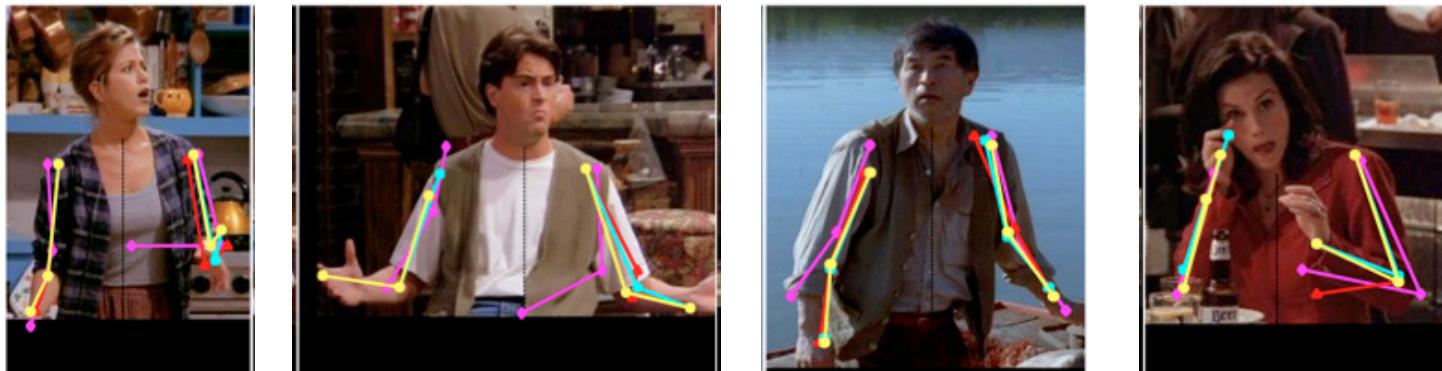
- Approximate the graph as combination of trees



- Several useful interactions are discarded

Human Pose Estimation (Video)

- Compute a candidate set of poses in each frame
- Then, track (entire pose or pose-parts) over time



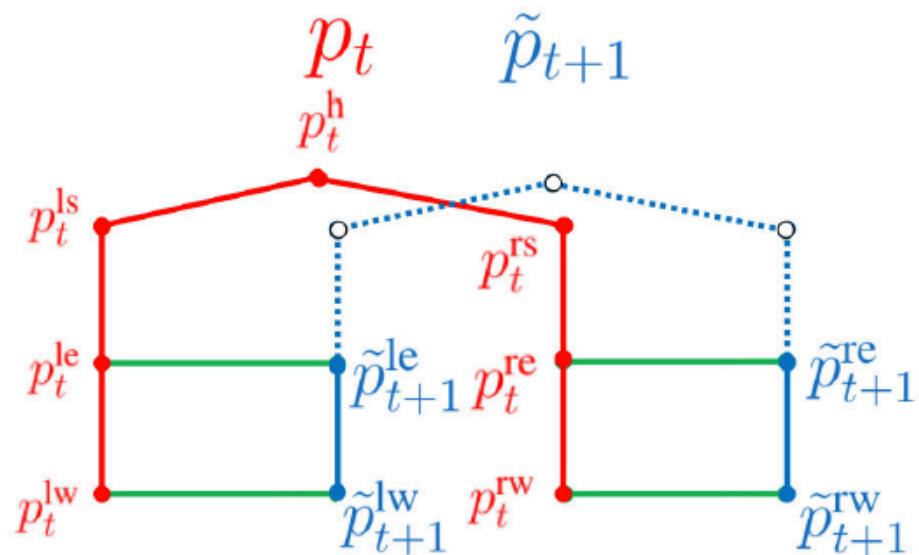
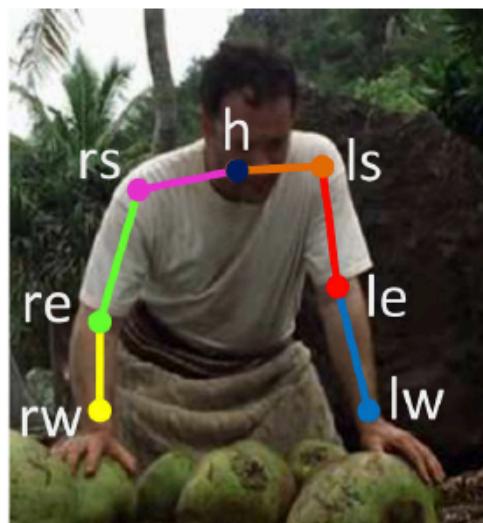
- Limited by the no. of candidates or regularization

Our Pose Estimation Approach

- Combines
 - 1. Candidate pose set
 - Generate better candidates
 - 2. Decomposition strategy
 - Generate limb sequences and recompose the pose

Better Candidate Poses

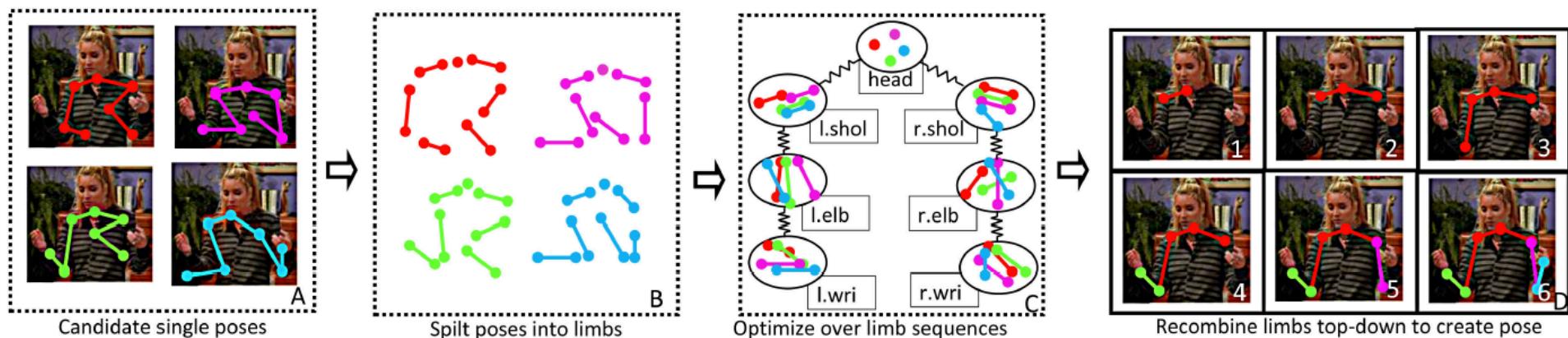
- Stabilize the lower-limb pose estimates



$$C(I_t, p_t) + \tilde{C}(I_{t+1}, \tilde{p}_{t+1}) + \tilde{\lambda}_1 \sum_{u \in \mathcal{W}} \|\tilde{p}_{t+1}^u - p_t^u - f_t(p_t^u)\|_2^2$$

De/Re- composition

- Decompose poses and perform limb-tracking



Evaluation: Poses in the Wild Dataset

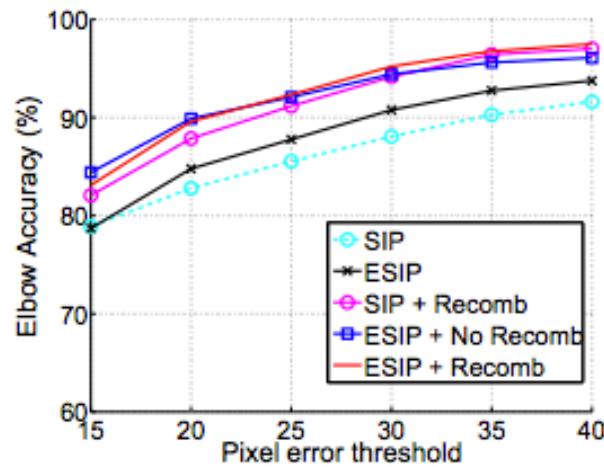


Our new public dataset: Poses in the Wild

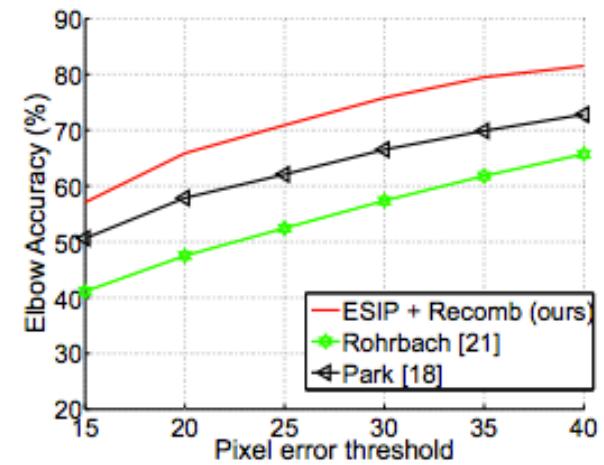
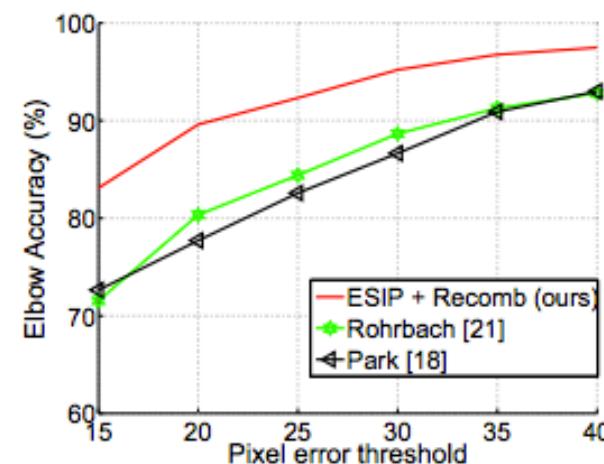
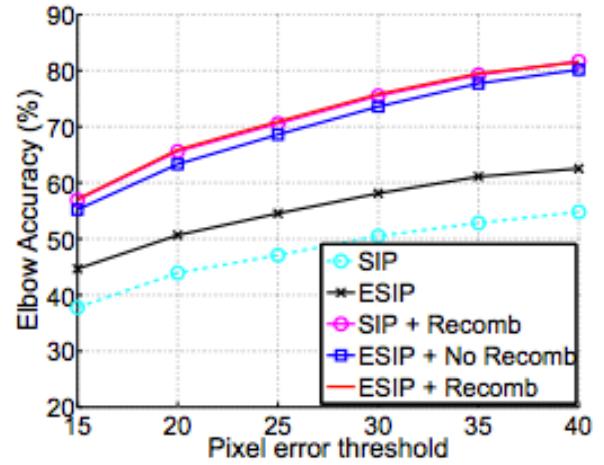
Available at: <http://lear.inrialpes.fr/research/posesinthewild/>

Human Pose Estimation: Elbows

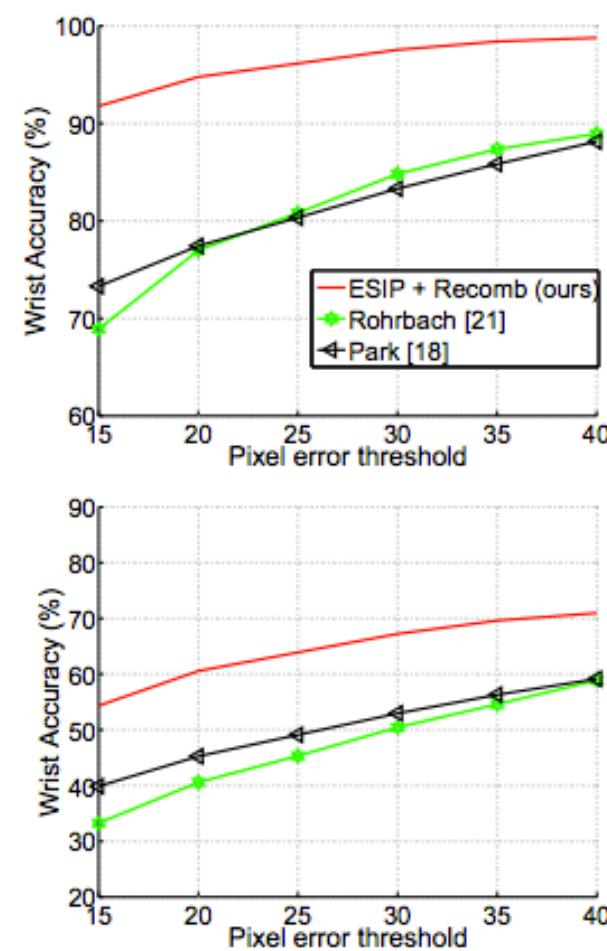
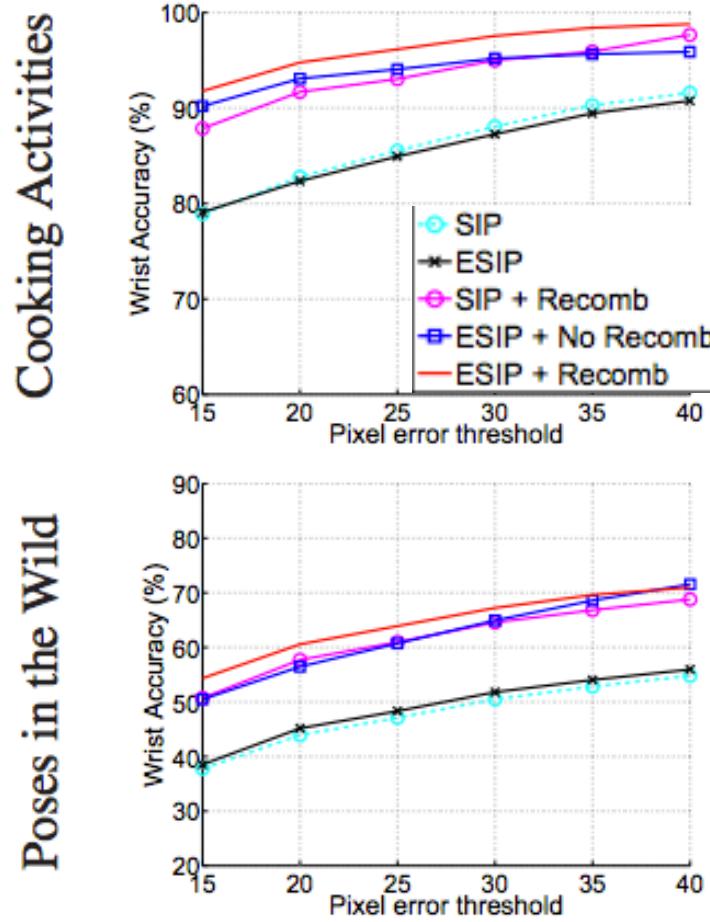
Cooking Activities



Poses in the Wild



Human Pose Estimation: Wrists



Summary

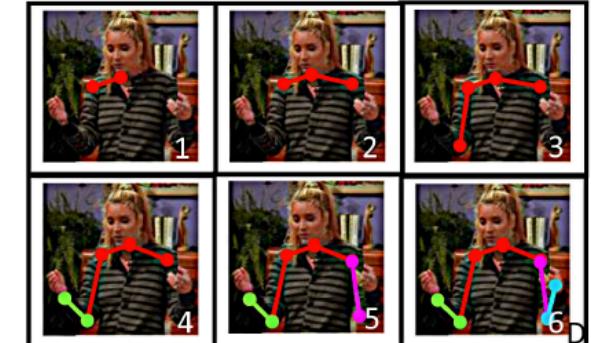
- In the context of labelling problems



Semantic segmentation



Person recognition



Pose estimation

- Learning parameters of the function
- Modelling image priors & inference
- Modelling temporal constraints & inference