Computational issues surrounding the management of an ecological food web

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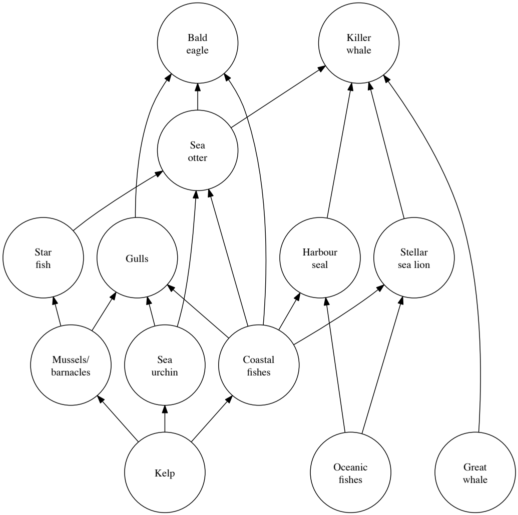
Rationale

- Ecology has many complex problems.
- Conservation is management within ecology.
- Management is allocation of resources.
- Ecological systems as food webs.
- How should we allocate resources to these systems through time?

The problem - managing a food web - a DAG

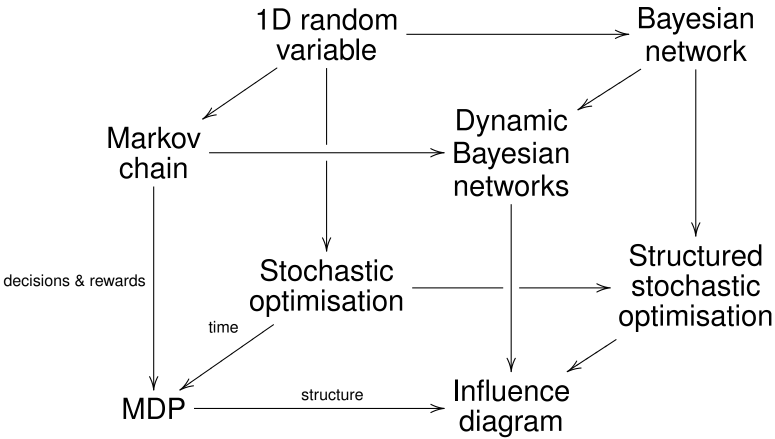


prey $\longrightarrow$ predator

Managing a food web

- Graph metrics provide a suitable set of heuristics.
- If we're going to manage then we need to quantify the management problem.
- What is the problem? Management of graph, $G = (V, E)$ over time.
- Has been done over one time step using a Bayesian network.
- Compare heuristics with optimal solution.

Diagram of modelling techniques

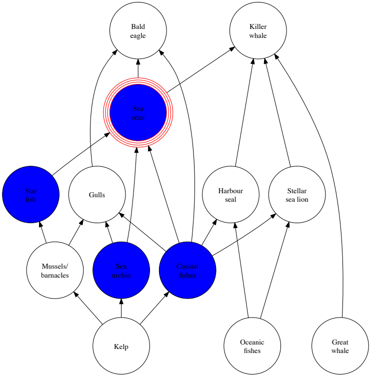


MDP framework, $\langle \mathcal{X}, \mathcal{A}, P, R \rangle$, **G**

- **Finite time horizon:** $t = \{1, \cdots, T\},\ T < \infty$
- **Species states:** extinct or extant, $x_i^t \in \{0, 1\}$
- **Species-level actions:** to protect or not, $a_i^t \in \{0, 1\}$

Budget constraint: $$\sum_{i \in V} c_i a_i^t \leq B^t$$

Neighbourhoods



A species' neighbourhood includes all prey and itself

$$N(i) = \{j \in V \mid (j, i) \in E\} \cup \{i\}$$

Allows decomposition on a local scale of transition probabilities (into a product) and rewards (into a sum).

Transition probabilties

$$P^t(x^{t+1} \mid x^t, a^t) = \prod_{i=1}^{n} P_i^t(x_i^{t+1} \mid x_{N(i)}^t, a_i^t)$$

Subject to:

- Probability of survival is p_i^0 times proportion of alive prey.
- $P_i(x_i^{t+1} = 1 \mid x_i^t = 1, x_{N(i)\setminus i}, a_i^t = 0) = p_i^0 \left(\frac{f^{\star,t}}{f}\right)$
- For basal species, survival probability is just p_i^0.
- Extinction (death) is an absorbing state.
- A species must have at least one prey species extant.
- A species will survive if protected and the above hold.

Rewards

Final time-step reward function: number of extant species,

$$R^T(x^T) = \sum_{i=1}^{n} x_i^T$$

and per-time-step rewards are zero, $R^t(x^t) = 0, \ t < T$.

Total expected reward of a policy, δ

$$\upsilon_\delta^T(x^1) = \mathsf{E}\left[\sum_{t=1}^{T} R^t(x^t, a^t) \,\middle|\, x^1, \delta\right]$$

Various reward functions can be investigated.

Optimal solution - Backwards Induction Algorithm

1. Set the current time-step to $t = T$ and the value in the final time-step to $\upsilon_*^T(x^T) = R^T(x^T)\ \forall x^T \in \mathcal{X}$
2. Set $t = t - 1$ and calculate $\upsilon_*^t(x^t)$ for each state using

$$\begin{aligned} v_*^t(x^t) &= \max_{a^t \in \mathcal{A}} Q^t(x^t, a^t) \\ a_*^t &= \arg\max_{a^t \in \mathcal{A}} Q^t(x^t, a^t) \end{aligned}$$

   where

$$Q^t(x^t, a^t) = R^t(x^t) + \sum_{x^{t+1}} P(x^{t+1} \mid x^t, a^t)\, \upsilon_*^{t+1}(x^{t+1})$$

3. If $t = 1$ then stop, otherwise return to step 2.

Metrics policies, $\delta = (d^1, \cdots, d^{T-1}), d^t : x^t \rightarrow a^t$

Manage species in descending order of metric until B^t is exhausted

Metric policies

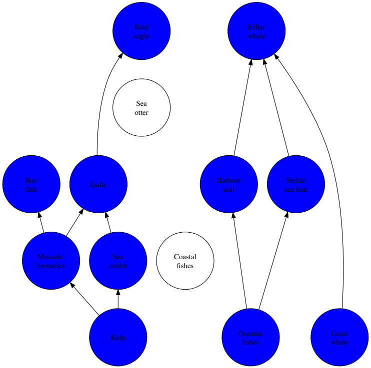
- Degree centrality
- Betweenness centrality
- Closeness centrality
- Keystone index
- Trophic level

Other policies

- Bottom-up index
- Return on Investment
- None
- Random

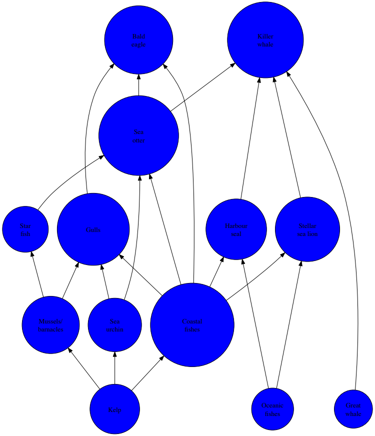
Metrics policies

- Isolates have metric values of zero (managed last)
- Ties use randomisation
- Disconnected graphs calculate relative measures on each subgraph.



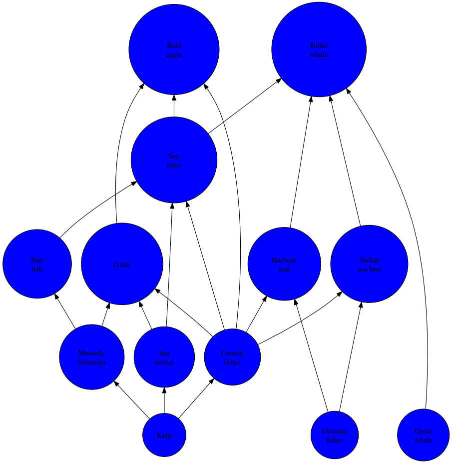
Metrics - degree centrality

$$D_i = D_i^{\leftarrow} + D_i^{\rightarrow}$$



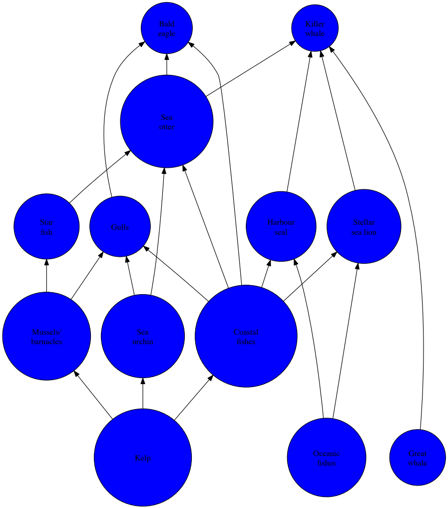
Metrics - prey degree

$D_i^{\leftarrow}$ is the size of the set $V_i^{\leftarrow} = \{j \in V : (j, i) \in E\}$ of all prey of species i

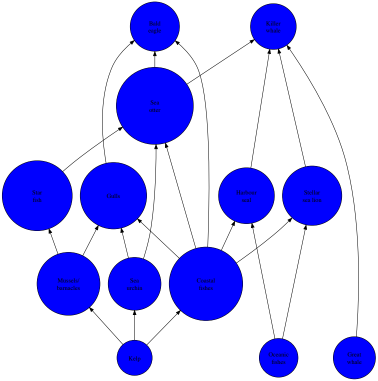


Metrics - predator degree

$D_i^{\rightarrow}$ is the size of the set $V_i^{\rightarrow} = \{j \in V : (i,j) \in E\}$, the set of all predators of species i



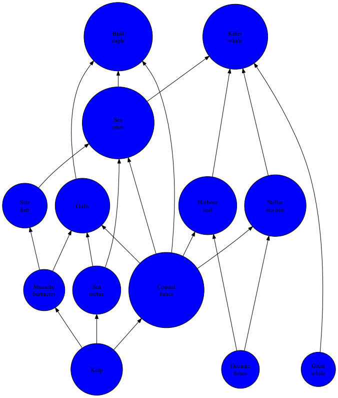
Metrics - betweenness centrality



$$BC_i = \frac{\sum\limits_{j<k} \frac{g_{jk}(i)}{g_{jk}}}{(\mid V \mid -1)(\mid V \mid -2)},$$

$g_{jk} =$ number of shortest paths between species j and k,
$g_{jk}(i) =$ number of shortest paths between species j and species k which pass through species i.

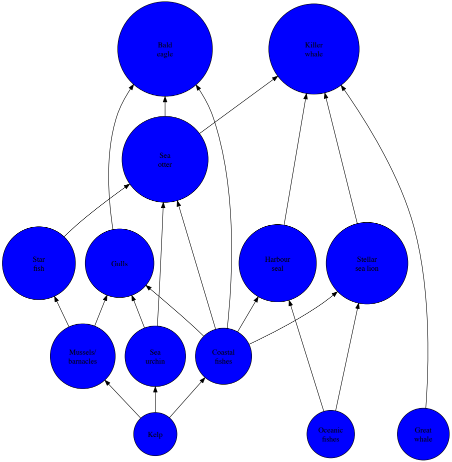
Metrics - closeness centrality



$$CC_k = \left[\frac{\sum_{i=1}^{n} d(i,k)}{n-1} \right]^{-1}$$

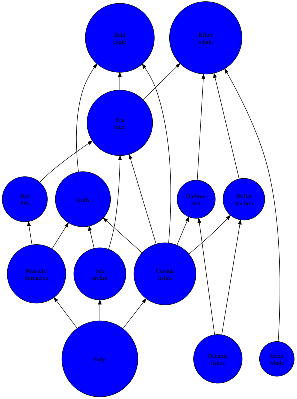
$d(i,k) =$ distance from species i to k

Metrics - trophic level



Related to Bottom-up prioritisation

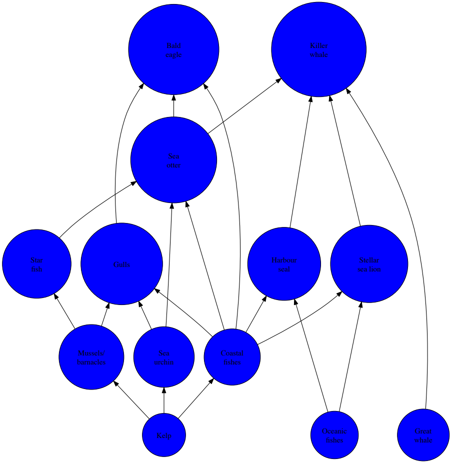
Metrics - keystone index



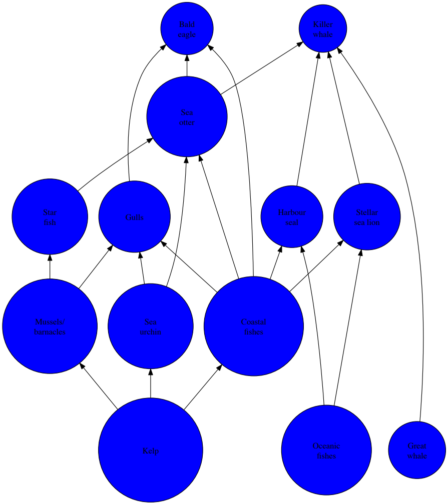
$$K_i = K_i^{\downarrow} + K_i^{\uparrow}, \quad \text{top-down + bottom-up}$$

$$K_i^{\downarrow} = \sum_{c \in V_i^{\leftarrow}} \frac{1}{D_c^{\leftarrow}}(1 + K_c^{\downarrow}), \qquad K_i^{\uparrow} = \sum_{e \in V_i^{\rightarrow}} \frac{1}{D_e^{\rightarrow}}(1 + K_e^{\uparrow})$$

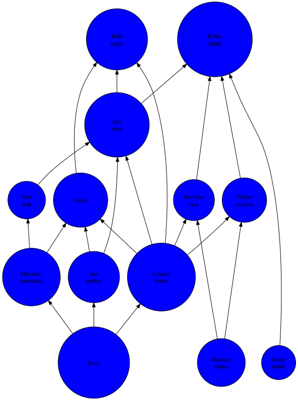
Metrics - keystone top-down index



Metrics - keystone bottom-up index



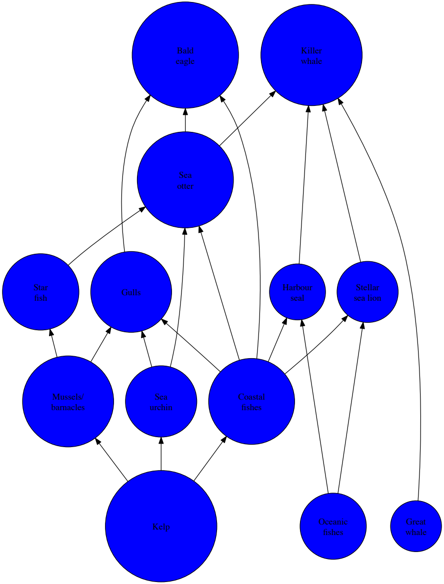
Metrics - keystone directed index



$$K_i = K_i^{\text{dir}} + K_i^{\text{undir}} \quad \text{(direct + indirect)}$$

$$K_i^{\text{dir}} = \sum_{c \in V_i^{\leftarrow}} \frac{1}{D_c^{\leftarrow}} + \sum_{e \in V_i^{\rightarrow}} \frac{1}{D_e^{\rightarrow}}, \quad K_i^{\text{indir}} = \sum_{c \in V_i^{\leftarrow}} \frac{K_c^{\uparrow}}{D_c^{\leftarrow}} + \sum_{e \in V_i^{\rightarrow}} \frac{K_e^{\downarrow}}{D_e^{\rightarrow}}$$

Metrics - keystone indirected index



Finite horizon metric policy evaluation

1. Set the current time-step to $t = T$ and the terminal rewards in the final time-step to $\upsilon_\delta^T(x^T) = R^T(x^T) \; \forall \; x^T \in \mathcal{X}$
2. Set $t = t - 1$ and calculate $\upsilon_\delta^t(x^t)$ for each state using

$$v_\delta^t(x^t) = Q^t(x^t, d(x^t))$$

   where

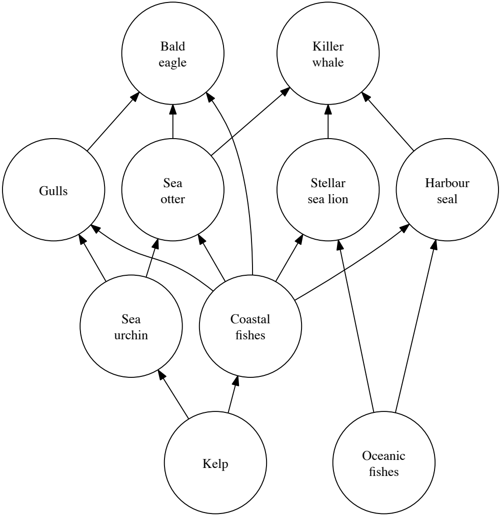
$$Q^t(x^t, d(x^t)) = R^t(x^t) \quad + \sum_{x^{t+1}} P(x^{t+1} \mid x^t, d^t(x^t)) \, \upsilon_\delta^{t+1}(x^{t+1})$$

3. If $t = 1$ then stop, otherwise return to step 2.

Experiments

- For 25 species, $B = 8$, we've more than 1.2×10^{21} transition probabilities
- Various transitions can be set to zero based on the conditions of the transition probabilities.
- $\mathbf{M} = \mathbf{SG}$
  - $\mathbf{S} : \mathcal{X} \to 2^{\mathcal{X}}$, is a $2^n \times n$ Boolean matrix that indicates for each possible state which species is extant.
  - $\mathbf{G}$ is adjacency matrix. $\mathbf{G}_{ij} = 1$ if species i is a prey of species j and otherwise 0. No cannibalism.
- $\mathbf{M}_{i,j} =$ Number of extant prey of species j when the state is $\mathbf{S}_{i,:}$
- $\mathbf{Q} = \mathbf{M} \odot \mathbf{S}$
- $\mathbf{P}_{i,i',a} = 0 \;\text{ if }\; \mathbf{Q}_{i,j} = 0 \; \forall j \text{ s.t. } \mathbf{S}_{i',j} > 0.$
- For 10 species Alaskan web, $> 95\%$ of state transitions are invalid.

Preliminary results - 10 species



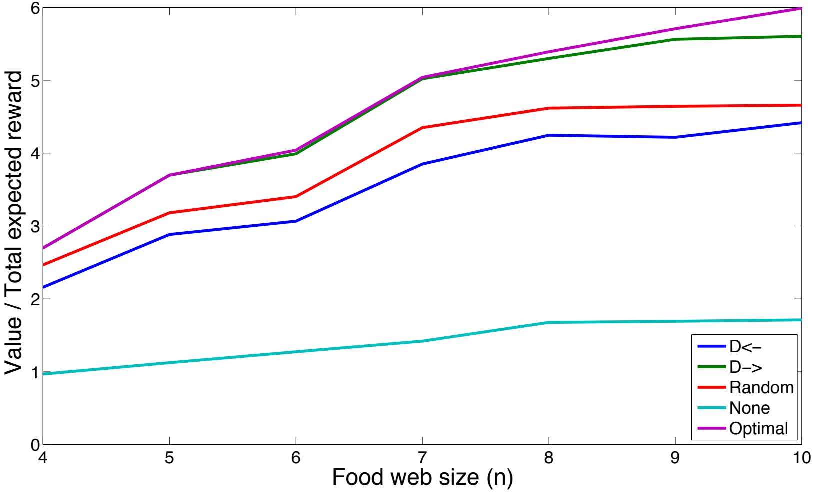
Start with solution to 10 species, $B = 4$, $T = 10$.

Preliminary results - 10 species

Policy	$v_\delta(x^1)$
Optimal	5.92
$K^{\uparrow}$	5.52
BUP	5.52
$D^{\rightarrow}$	5.51
K	4.99
K^{indir}	4.97
K^{dir}	4.76
BC	4.00
D	3.84
CC	3.72
Random	3.66
$K^{\downarrow}$	3.49
$D^{\leftarrow}$	3.45
None	1.10

Parameters: 10 species, $B^t = 4$, $T = 10$

Results



Parameters: $B^t = [2, 3, 3, 4, 4, 4, 4]$ respectively, $T = 10$, 25 random food webs, connectance = 0.1

Future work

- Find exact solution for webs with n up to 20.
  - Code transition probabilities in a faster language
  - Use POMDP solver, eg Perseus
- Extract decision tree from optimal policy
- Simulate management using heuristics on large webs
- Investigate alternative approximating solutions