# Statistics and learning

## Multivariate statistics 2 and clustering

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ISAE SupAero

Wednesday 2nd October 2013

# Link to the previous session

Goal of multivariate (exploratory) statistics: understanding high-dimensional data sets, reducing their 'useful' dimensions, representing them, seeking hidden or latent factors ...
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- introduce Correspondence analysis (CA): for $2$ qualitative variables with several (many) levels.
- introduce clustering methods like hierarchical clustering or Kmeans-like algorithms.

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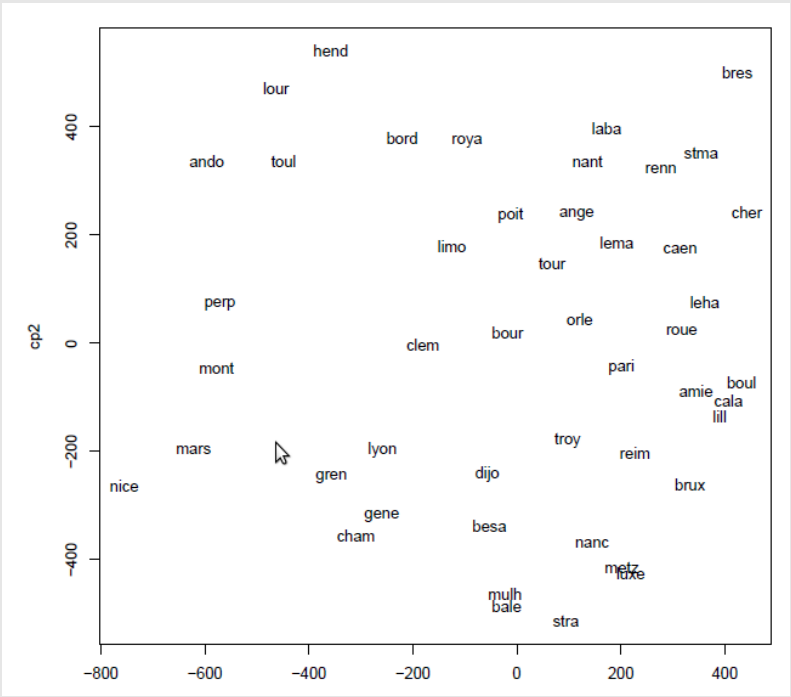
## Easy example

Road distances between $47$ French cities. Is it Euclidian ?



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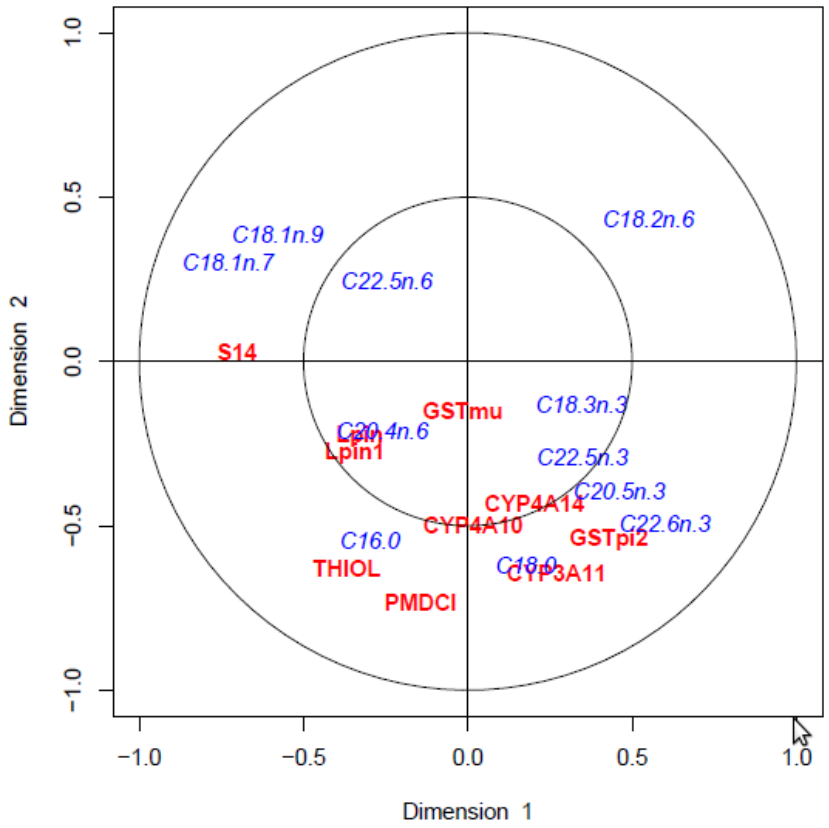
- Variables can be represented in either basis, it does not change the interpretation.

# CCA (cont'd)

Need to have $p, q \leq n$. We kept $10$ genes and $11$ fatty acids.




More interpretation ? $\rightarrow$ Practical session

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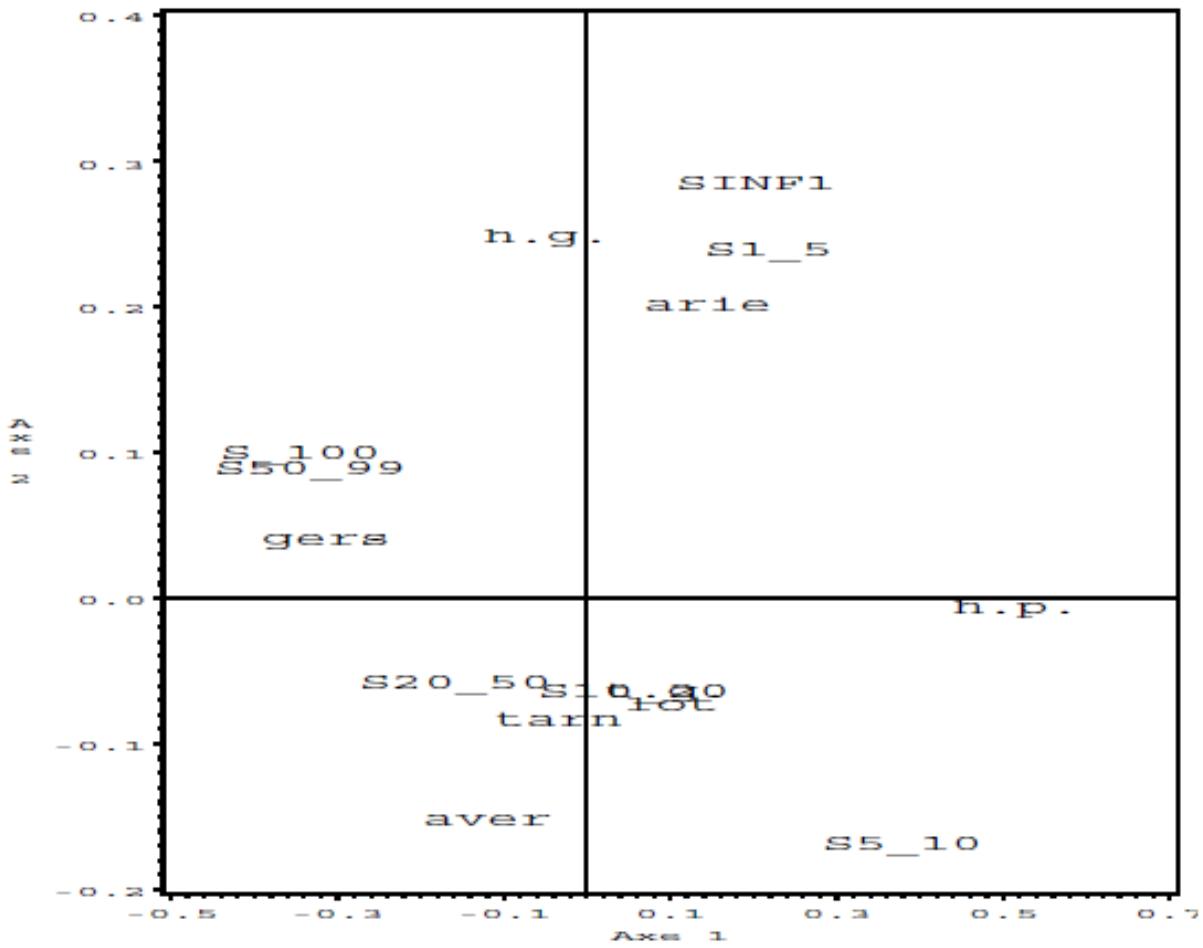
- Note that $\chi^2$ writes $n \sum_i \sum_j \tilde{f_{i,j}} x_{i,j}^2$

# CA: an example

Cultivated area in the Midi-Pyrénées region

Simultaneous representation of *département* and farm size (in 6 bins).



# Clustering: grouping into classes

Ever heard of that in your background ??

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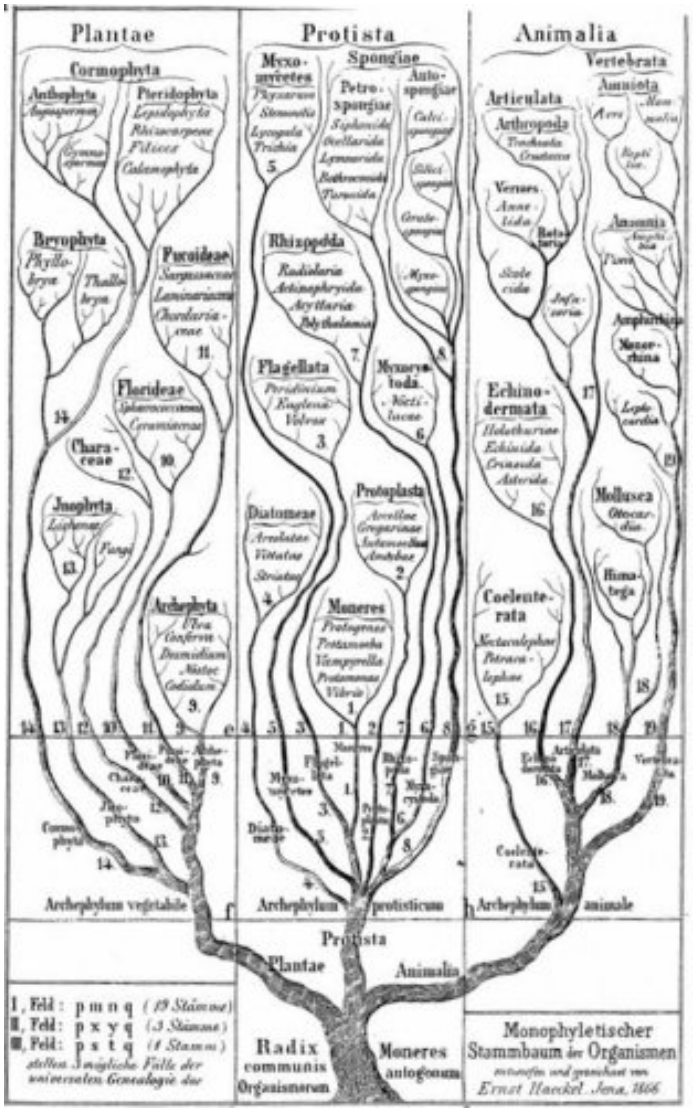

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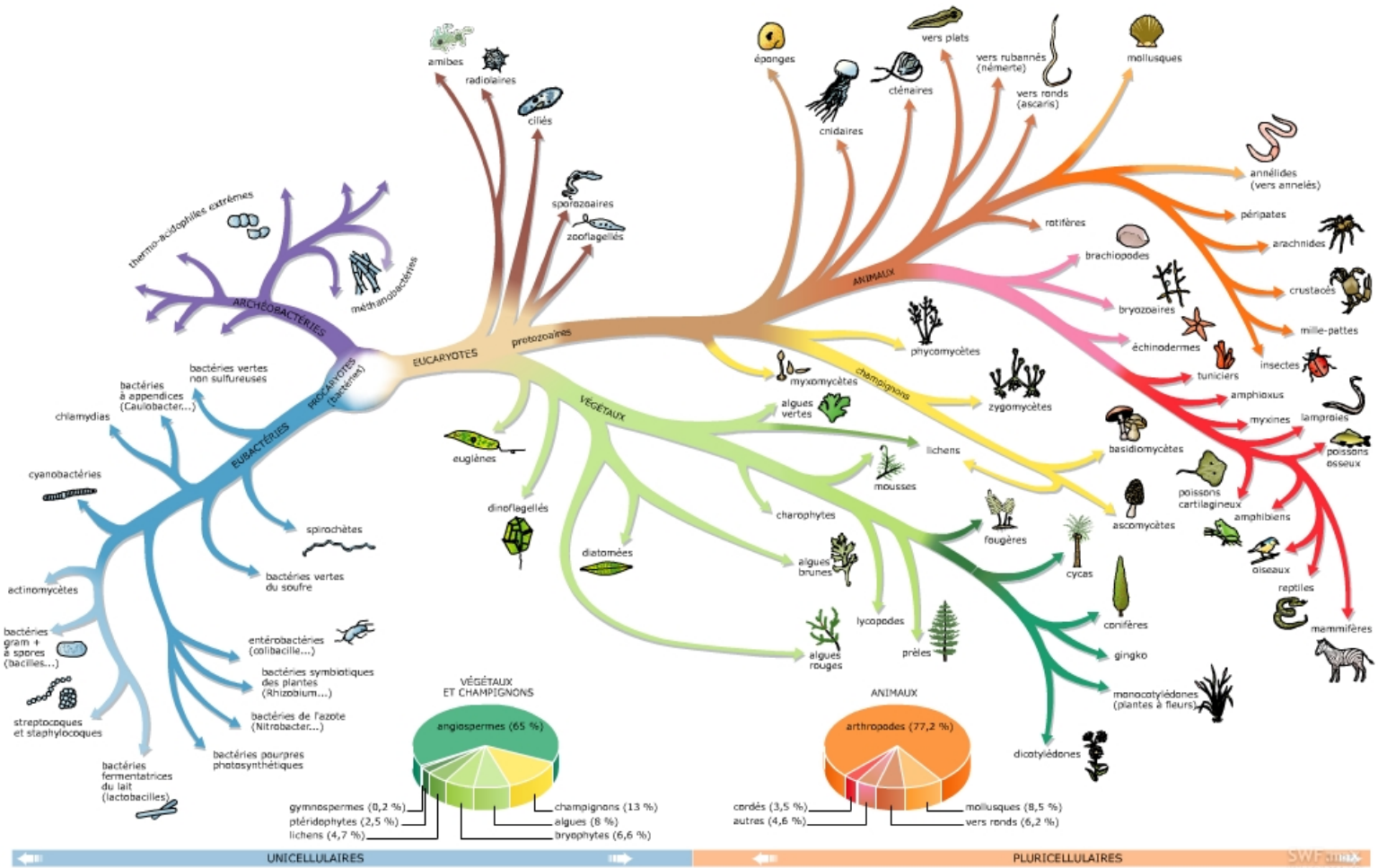

# Cluster analysis or clustering

- Task of **grouping** objects so that objects belonging to the same group are 'more similar' to each other than to those in any other group $\rightarrow$ multiobjective optimisation task.

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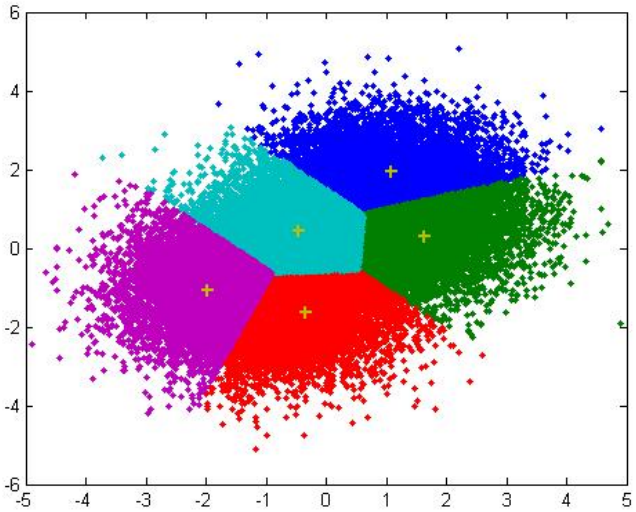

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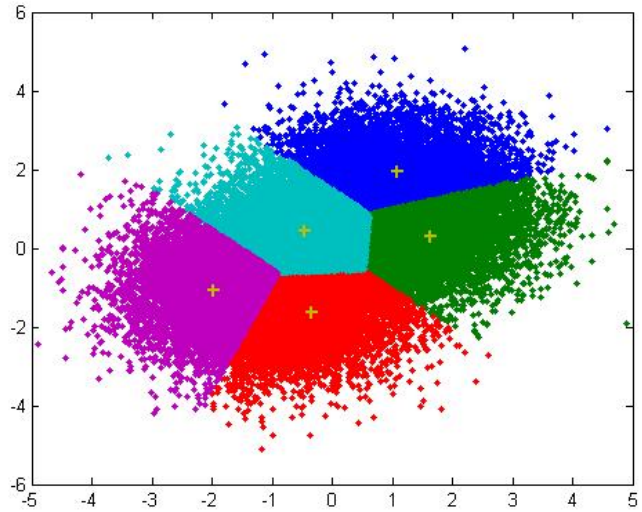


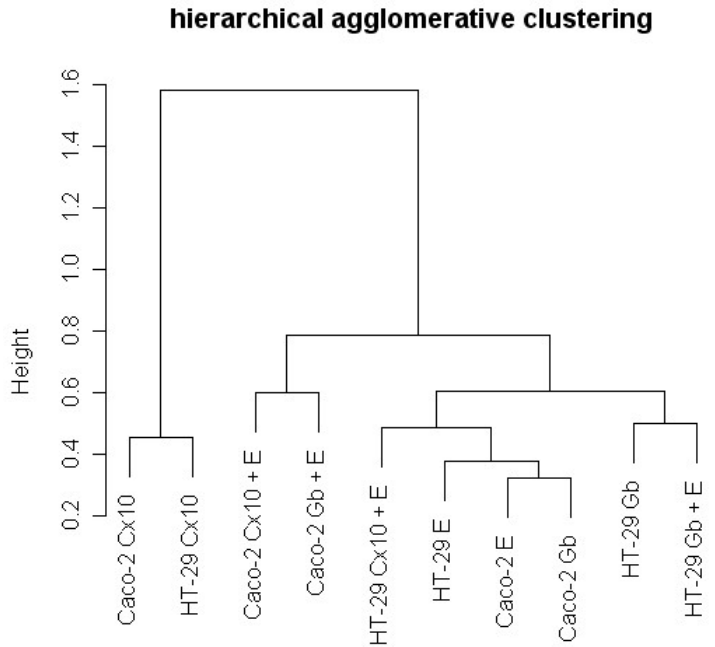

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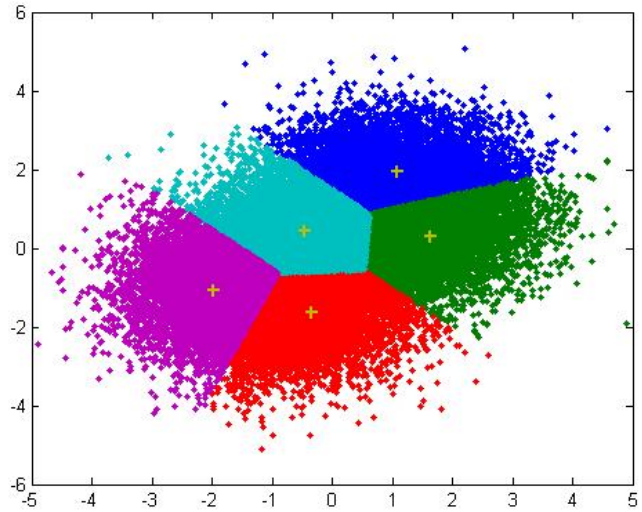


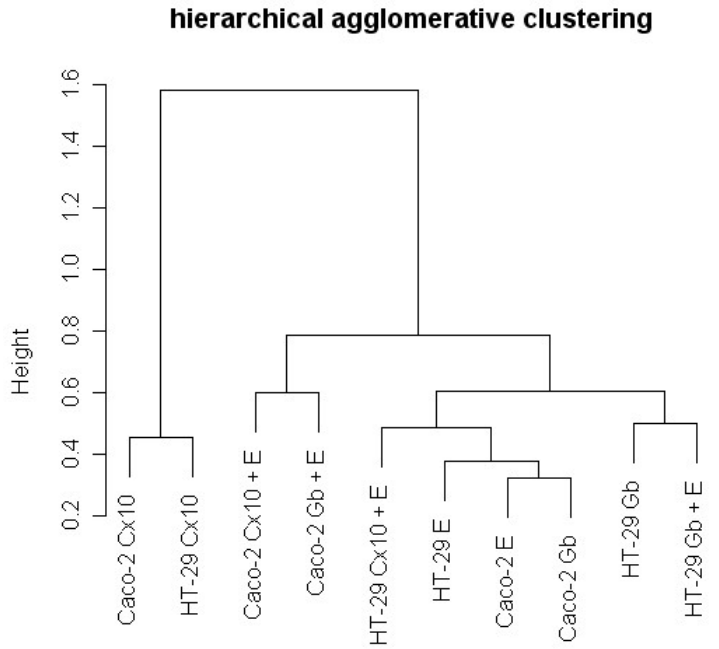


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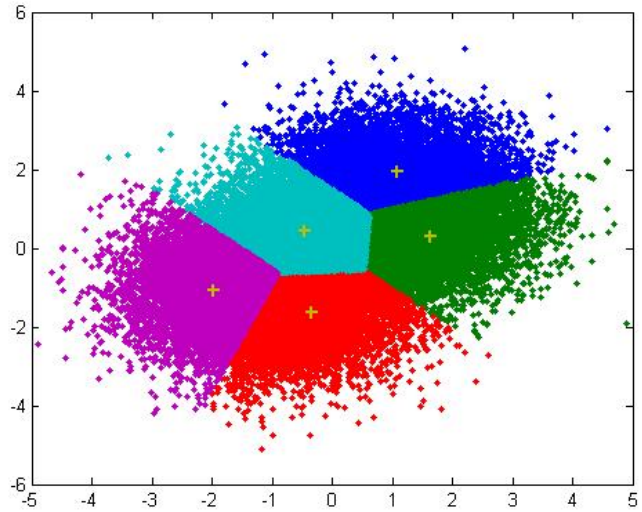


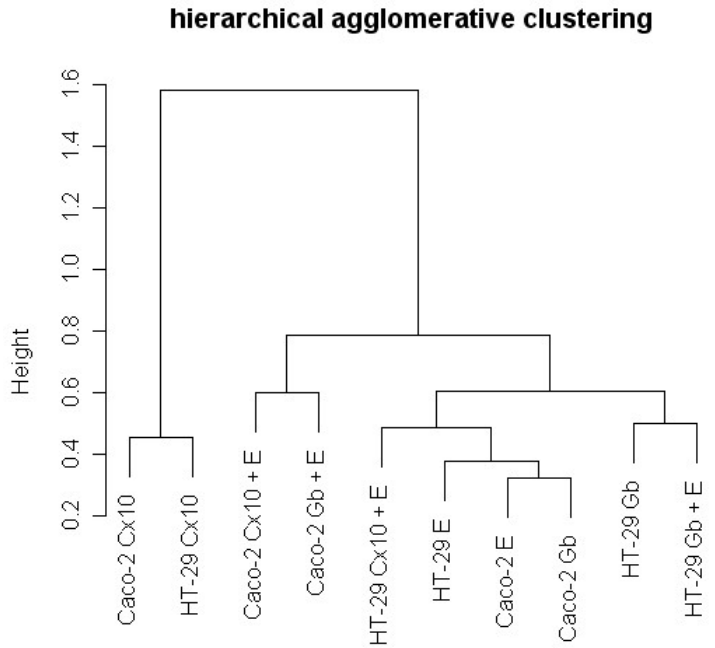


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- Possibly, different parameters (initialisation, distance used, ending criterion ...) lead to different representations.

# Clustering algorithms

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Let's quote only few of widespread clustering algorithms:

- hierarchical clustering (single, complete, average linkages)
- centroid models (*e.g.* Kmeans clustering)
- distribution models (statistical definition *e.g.* multivariate Gaussian distribution)
- graph or density models (*e.g.* clique)
- ...

# Clustering: some formalism

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A goodness-of-fit of partitions can be defined: (i) external: TP, FP ... $\rightarrow$ precision, sensitivity or Rand/Jaccard index or (ii) internal: Dunn index $D = \min_i \min_{j \neq i} \frac{d(i,j)}{\max_k d'(k)}$.

# Homework

What do students choose after French baccalauréat ?

First describe and then represent this (simple) data set in some informative way.

Hint: CA...

| origin | counselling | | | |
|---|---|---|---|---|
| | université | prep. clas. | other | Total |
| bac lit. | 13 | 2 | 5 | 20 |
| bac éco. | 20 | 2 | 8 | 30 |
| bac scient. | 10 | 5 | 5 | 20 |
| bac tech. | 7 | 1 | 22 | 30 |
| Total | 50 | 10 | 40 | 100 |

# Finished

Next time: tests