Forest Management Under Fire Risk When Forest Carbon Sequestration Has Value

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Abstract

In this paper, we develop a multiple forest use model to determine the optimal harvest date for a forest stand producing both timber and carbon benefits under a risk of fire. The preferences of the representative non-industrial private forest (NIPF) owner are modeled though an expected utility specification. We introduce saving as a decision of the forest owner at any time. The problems of forest management and saving decisions are solved simultaneously using a stochastic dynamic programming method. A numerical programming method is used to characterize the optimal forest and saving policies. We apply this framework to model the behavior of a representative NIPF owner located in the Southwest of France. The empirical application indicates that a higher risk of fire will decrease the optimal rotation period. On the contrary, higher carbon prices increase the optimal harvesting age. To investigate the contradictory effects of fire risk and carbon price on forest rotation, we identify the set of carbon prices and fire risks that leads to given rotation age. We also show that the model can be used for deriving the NIPF owner's willingness to pay for a risk reduction and that this willingness to pay can be substantial (37.33 euros by ha and by year to reduce the annual fire risk from 1.26% to 0.07%). The quantification of NIPF owner's willingness to pay may help public authorities wishing to design and implement risk reduction programs.

Keywords: Forest Economics, Stochastic Dynamic Programming, Expected Utility, Fire risk, Carbon Sequestration.

1 Introduction

Traditionally, forests have been viewed as a source of timber used for producing goods and services. It has recently been emphasized, however, that forests also play a significant role in climate change mitigation, since carbon sequestration in forests is a major factor that affects the global carbon cycle, IPCC (2006). In the last few years, there has been an increasing tendency to consider forest ecosystems as a possible sink of carbon dioxide, recognizing forestry as carbon storage in the form of biomass. This non-timber service is produced jointly with timber, and vanishes with the standing forest stock. The decision to harvest timber therefore automatically affects the flow of stored carbon. In the absence of carbon forest markets, this joint production property is not reflected in the decision process of non-industrial private forest (NIPF) owners.

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As a consequence, effective forest carbon sequestration may be below the social optimum. Hence, it is important to determine the impact of internalizing carbon benefits into forest management decisions.

The timber and carbon relationship is complicated by the uncertainty associated with the risk of fire. By significantly damaging French or European forests, the recent exceptional fire events have reminded all forest owners that fire risk is an important component of forest management, Schelhaas *et al.* (2003). Indeed, a forest can be partially or totally destroyed by natural calamities, resulting in the release of sequestered carbon back into the atmosphere. It follows that the regulator needs to provide incentives to induce risk mitigation behaviors. Since fire risk impacts the outcome of public policies to sequester carbon in forest biomass, it is important to understand the influence of this risk on the behavior of a NIPF owner selling carbon credits. This raises several interesting issues: (i) What are the qualitative properties of timber supply when carbon service induces private revenue, (ii) What is the role of fire risk in this context and, finally (iii) How would the risk of carbon release influence the amount of carbon a NIPF owner is willing to supply for a given carbon price?

The purpose of this paper is then to develop a model that will provide some answers to these questions. As a prelude of the results, we show that a higher risk of fire decreases the optimal rotation duration, while higher carbon prices increase the optimal harvesting age. The effect of introducing benefits from carbon sequestration is mitigated by the presence of fire risk. The introduction of carbon benefits may not necessarily induce forest owners to systematically increase forest rotation, especially with high fire risks. To investigate the relative strengths of such contradictory effects, we identify a frontier in the carbon price / fire risk space that leads to a given rotation age. We then analyze how this carbon price / fire risk frontier is affected by NIPF owner's risk aversion. We also show that the model can be used for deriving NIPF owner's willingness to pay for a risk reduction and that this willingness to pay can be substantial (37.33 euros by ha and by year to reduce the annual fire risk from 1.26% to 0.07%). The quantification of NIPF owner's willingness to pay may help public authorities wishing to design and implement risk reduction programs.

The remaining of the paper is organized as follows. In section 2, we present a brief review of the relevant literature which provides an additional motivation for our analysis. In Section 3, we describe the forest management model and we present the stochastic dynamic programming (SDP) method we use. Section 4 deals with an empirical application to the case of a French representative NIPF owner. We conclude by a brief summary of our findings.

2 Relevant literature and our contributions

The implications for NIPF owners of introducing carbon services have been studied in many articles, usually using a Faustmann-type framework (or Net Present Value NPV approach). Several articles have considered a deterministic context (Englin and Callaway (1993); Englin and Callaway (1995); Hoen (1994); VanKooten *et al.* (1995); Romero *et al.* (1998); Gong and Kriström (2005); Diaz-Balteiro and Romero (2003); Benitez and Obersteiner (2006); Gutrich and Howarth (2007); Pohjola and Valsta (2007); Thompson et al. (2009)). These models, which are mainly a variation of the multiple use forestry modeling suggested by Hartman (1976), have added the non-timber value of forest to the Faustmann approach. In this deterministic context, the consensus is that carbon benefits tend to lengthen the rotation since they provide an incentive to leave the stand uncut for a longer period. Recently, the risk of fire has been introduced into such a context. Focusing on a forest stand producing timber and carbon sequestration benefits, Stainback and Alavalapati (2004) have found that fire risk decreases both the land expectation value and the optimal rotation age. Stollery (2005) has adapted the modified Hartman model, allowing the fire risk to change over time. In this extension of the Reed model, he shows that an increase in the risk of fire results in a downward trend in both the commercially and the socially optimal forest rotation periods. Adding carbon sequestration benefits into the model, Stollery (2005) shows that the socially optimal rotation length also declines with climate warming. Last, some other studies have analyzed the impact of carbon sequestration on the optimal rotation duration under price risk within a real options framework, but without fire risk.[1]

Our paper explores various issues associated with the harvesting behavior of an NIPF owner when carbon sequestration has value, and when there is a fire risk affecting timber production. More formally, we develop a model to characterize the optimal harvest date, the optimal quantity of sequestered carbon and the optimal timber supply for a forest stand that produces both timber and carbon benefits under risk of fire. We introduce saving as a decision of the forest-owner at any time, and we incorporate the fact that the forest-owner is responsible for repayment of all carbon released back into the atmosphere after harvesting.

We extend the existing literature in several important directions.

First, we introduce the possibility for the forest owner to diversify risk by investing in a risk-free asset.

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[1] See Chladná, (2007) for uncertain timber and carbon prices in a single rotation case or Guthrie and Kumareswaran (2009) for an uncertain timber price with multiple rotations. Chladná (2007) have found that rotation duration tends to be extended only under constantly high carbon prices whereas Guthrie and Kumareswaran (2009) find that the rotation duration are extended when including social benefits derived from carbon storage.

As a result, forest management decisions are intrinsically linked with saving decisions. The literature in forest management under uncertainty has considered for a long time that a forest stand could be viewed as a risky asset, see for instance Reed (1984), Englin et al. (2000) or Alvarez and Koskela (2006). Compared to this literature, we introduce explicitly the possibility of savings through a risk-free asset.[2] This point has also been examined by Alvarez and Koskela (2007) in a real option framework without carbon benefits, and by Koskela and Ollikainen (1997) in a two-period model under price risk and amenity services of forest stand. In the model we will consider, the timber production is viewed as a risky asset that must be managed to secure consumption over the long-run. Hence the consumption-saving tradeoff has an impact on forest management, and *vice-versa*.

Second, we analyze the impact of risk preferences on forest-owner decisions in a risky environment. While forest owner risk preferences are known to be an important determinant of forest decisions, previous works have shown that risk aversion has an ambiguous impact on forest management decisions.[3]

Third, we assess the impact of valuing carbon sequestration on the behavior of NIPF owner in a context of fire risk. Subsidizing forest owners for carbon sequestration is not new. More generally, the relationship between forest taxation, non-market amenities, and forest rotation has been studied in a deterministic context (Englin and Klan (1990), Koskela and Ollikainen (2001)) and in a stochastic context (Koskela and Ollikainen (1997), Englin et al. (2000)). In a deterministic context, if the forest owner internalizes forest amenities, then the change in the optimal rotation duration compared to the basic Faustmann case crucially depends on the nature of amenity valuation. For example, if the valuation increases with the age of the forest stand, then it can be shown that the Hartman rotation period is longer than the Faustmann one. In a stochastic context, Koskela and Ollikainen (1997) have proved, in a two-period decision model with price risk, that the effect is positive on current harvesting but ambiguous on future harvesting. Englin et al. (2000) have incorporated fire risk and forest amenities into the traditional NPV approach. Their results suggest that the amenity value will increase the optimal rotation period. Finally, some works have introduced carbon sequestration in a stochastic context within a real options framework (Chladná (2007), Guthrie and Kumareswaran (2009))

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[2] Savings has been shown to be an important driver of forest-owner decisions in the deterministic case, see for example Tahvonen (1998).

[3] Some models have incorporated forest-owner's risk aversion into stochastic control problems. The effect of forest owner's risk aversion on optimal rotation remains not clearly defined. Although the majority of previous works (Caufield(1988), Taylor and Fortson (1991), Alvarez and Koskela (2006), (2007), Couture and Reynaud (2008)) have concluded that risk aversion tends to shorten the optimal rotation, Gong and Löfgren (2005) show that the impact of risk aversion on optimal rotation depends both on the regeneration cost and on the interest rate, whereas Valsta (1992) shows that risk aversion leads to longer optimal rotation than under risk neutrality.

or NPV framework (Stainback and Alavalapati (2004) and Stollery (2005)). It should be noticed that the carbon storage service presents a strong specificity compared to the forest amenities usually considered (biodiversity preservation, recreational use, etc.). When a forest fire occurs, the amenity benefits are not simply stopped until the stand is replanted but, since the carbon is released back into the atmosphere, all the accumulated carbon benefits produced while the stand was growing are lost. If the forest-owner is responsible for repayment of carbon credits, then he will face an additional financial risk.

Lastly, in complement to the existing literature based on the Faustmann framework, we characterize the optimal harvesting decisions under fire risk and the risk aversion of the forest owner using a stopping method in a stochastic dynamic programming framework. Compared to the NPV framework, this approach explicitly incorporates into the decision program the possibility for the forest-owner to adjust cutting plans in response to stochastic events. By modeling fire risk with a Poisson process, we are then able to fully characterize the optimal consumption and harvesting dynamic paths.

3 The model

3.1 Specification of the model

We consider a NIPF owner managing an homogeneous, even-aged forest stand.[4] We denote by A the area of forest (in hectares). The problem of the forest owner is to determine the optimal sequence of harvest ages. Forest owner preferences are represented by a utility function denoted by $U(c)$ with $U' \geq 0$, $U'' \leq 0$ and where c represents the consumption of the forest owner.

Following the existing literature on forest management under fire risk (see Reed (1984) among others), we assume that the risk of fire can be described by a Poisson process. This assumption implies that the risk of fire is the same every year. Over a short interval dt of time, the probability of a fire is λdt, where λ is the mean arrival rate, with probability $(1 - \lambda dt)$ that there is no fire. We assume that if a fire occurs, no salvage is possible. If a fire occurs, then the forest owner is assumed to replant the forest stand with a planting cost μ per unit of surface.

We denote by $v(t)$ the volume of wood per unit of surface at age t. This volume is assumed to increase with

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[4] In several countries, non-industrial timber production represents a significant share of total timber supply (roughly 60% in most Scandinavian countries and the US). In France, NIPF owners constitute the main type of forest ownership, representing roughly 75% of forest areas.

time according to a growth function $g(t)$. We assume that the growth process of a tree is finite, that is: $\lim_{t\to\infty} g'(t) = 0$. We consider a multiple forest-use model. The harvested timber volume is used to produce several forest products such as sawtimber, pulpwood, fuelwood, etc. These forest products are indexed by k with $k = 1, \ldots, K$ and we denote by p_k the net price associated with product k. We assume that the share of timber products varies according to tree age.[5] We denote by $\delta_k(t)$ the share of timber volume used for product k for a tree of age t. By definition, we have$\sum_{k=1}^{K} \delta_k(t) = 1 \quad \forall t$. It follows that, by choosing a harvesting date, the forest owner chooses not only the volume of timber to be harvested, but also the breakdown of this volume between all possible forest products. Assuming that the forest is replanted after being harvested, the revenue per unit of surface (net of replanting cost) from harvesting a forest stand at age $\overline{t}$ is given by $\sum_{k=1}^{K} p_k \cdot \delta_k(\overline{t}) \cdot v(\overline{t}) - \mu$. The forest stand generates carbon sequestration which is valued by society. Society therefore pays a revenue for the carbon sequestered by the forest stand. We assume that, at date t, the forest owner receives a payment corresponding to the value of the incremental increase in carbon sequestration. Let us denote by p_c the price per metric ton of carbon. Then the payment at date t per unit of surface for carbon sequestration received by the forest owner is written $p_c \cdot \alpha \cdot v'(t)$ where α denotes the conversion factor to convert a wood volume into metric tons of carbon, and $v'(t)$ is the derivative of $v(t)$ with respect to t. The decay of forest products produces carbon emissions. Carbon emissions are modeled as a linear process where an equal amount decays at each instant until the whole carbon pool is released back into the atmosphere. However, forest products differ according to their lifespan. We denote by T_k the lifespan (in years) of forest product k. If the forest owner harvests a forest plot of age $\overline{t}$ at date t, then the instantaneous flows of carbon per unit of area due to product k will be $\frac{\alpha \cdot \delta_k(\overline{t}) \cdot v(\overline{t})}{T_k}$ over the period $[t, t + T_k]$. We assume that carbon emissions due to forest product decay give rise to payment by the forest owner at the harvesting date. This payment corresponds to the discounted value of future carbon releases from all forest products at date t. Denoting by r the rate of pure present preference of the NIPF owner, this discounted value for product k is written:

$$\int_{t}^{t+T_k} p_c \cdot \frac{\alpha \cdot \delta_k(\overline{t}) \cdot v(\overline{t})}{T_k} \cdot \exp^{-r(\tau - t)} d\tau = \Phi_k \cdot \delta_k(\overline{t}) \cdot v(\overline{t}) \qquad (1)$$

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[5] For instance, for young trees the share of the timber volume that can be used for sawtimber is generally low, with most of the timber volume serving as pulpwood in that case. As trees grow and become older, the share of timber volume used for sawtimber increases. We assume that this share is given to the forest owner, but it varies with the age of trees.

where:

$$\Phi_k = p_c \cdot \frac{\alpha}{T_k} \cdot \frac{1 - \exp^{-rT_k}}{r} \tag{2}$$

where Φ_k can be interpreted as an implicit unit carbon tax for product k paid by the forest owner.

Forest owners derive their utility from consumption. At each date, the forest owner therefore chooses the amount of wealth used for consumption, the remainder being invested in a financial asset with a risk free rate of return denoted by η. For simplicity, we assume that no loans are possible, that is $w(t) > 0$ where $w(t)$ represents the forest owner wealth invests in risk-free saving. In such a context, timber production for a NIPF owner can be viewed as a risky asset that must be managed to secure consumption over the long-run. The problem of the forest owner consists in optimizing a portfolio comprising a risky asset (forest) and a risk-free asset (saving) in order to maximize the expected utility of consumption over time. Hence, the consumption-savings tradeoff will have an impact on forest management, and *vice-versa*.[6] Without risk occurrence, the dynamic of wealth depends upon whether the forest stand is harvested or not. If the forest owner does not harvest the stand then the wealth is increased by the interests from saving $\eta \cdot w(t)$, and by the payment corresponding to the increase in carbon sequestration, $A \cdot p_c \cdot \alpha \cdot v'(t)$, but is reduced by the level of consumption, $c(t)$. If the forest owner harvests the stand, then his/her wealth increases from timber benefits $A \cdot \left[\sum_k p_k \cdot \delta_k(\bar{t}) \cdot v(\bar{t})\right]$ and from savings $\eta \cdot w(t)$ but decreases due to the carbon cost of product decay, $A \cdot \left[\sum_k \Phi_k \cdot \delta_k(\bar{t}) \cdot v(\bar{t})\right]$, the replanting cost $A \cdot \mu$ and the level of consumption, $c(t)$. If a fire occurs, all the timber production is lost. We assume that the forest owner does not pay for the carbon emissions.[7] In that case, wealth increases from savings $\eta \cdot w(t)$ but decreases due to the replanting cost $A \cdot \mu$ and to the level of consumption, $c(t)$

3.2 The optimality conditions

The forest owner's problem is to choose the flow of consumption $c(t)$ over time and the harvesting date $\bar{t}$ so as to maximize his/her discounted expected utility. In other words, the dynamic optimization problem at each period consists in determining if the forest stand must be harvested or not, and if the revenue flow must be used for consumption or saving. Since the problem is dynamic and stochastic, the optimal

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[6] Couture and Reynaud (2008) dealt with the consumption-savings tradeoff in the context of forest management within a maximizing utility framework under risk but they did not incorporate carbon benefits.

[7] Empirical evidence suggests that in case of fire, public authorities are very reluctant to impose carbon release payments on forest owners.

decision path depends on forest owner risk preferences. Risk preferences refer to the forest owner's desire to smooth consumption across states of nature. Applying the basic stochastic dynamic programming technique of optimal stopping (Dixit and Pindyck (1994)), the Bellman equation is defined as follows: At $t < \overline{t}^*(w)$,

$$J\big(t, w\big) = max_{\{c\}}\ U(c)dt + e^{-rt}\Big[(1 - \lambda dt)J\big(t + dt, w + (\eta w - c)dt + p_c\alpha A v'\big) + \lambda dt J\big(0, w + (\eta w - c)dt - A\mu\big)\Big] \tag{3}$$

where $J\big(t, w\big)$ is the expected current value of future utilities to be optimized by the forest owner, t is the age of the forest stand in years, and $\overline{t}^*$ represents the optimal age of harvest depending on wealth w. Notice that since the forest stand grows according to growth function $g(t)$, it is strictly equivalent to define the value function as $J\big(t, w\big)$ or as $J\big(v, w\big)$. In the application, we will use this property, and we may alternatively consider the optimal harvesting date or the optimal timber volume at the harvesting date. The right-hand side of the Bellman equation is made up of two terms: the instantaneous utility which only depends upon present consumption, and the continuation value which is stochastic. Without risk occurrence, the wealth of the forest owner during time interval Δ_t is increased by savings and carbon benefits obtained for the considered period, and decreases due to consumption. If a risk occurs, then forest owner wealth increases due to savings returns but decreases due to consumption and planting costs. The optimal harvesting age, $\overline{t}^*$, is first determined by a *value matching* condition:

$$J\big(\overline{t}^*, w\big) = J\Big(0, w + A \cdot \Big[\sum_k (p_k - \Phi_k) \cdot \delta_k(\overline{t}^*) \cdot v(\overline{t}^*)\Big] - A\mu\Big) \tag{4}$$

which means that at the optimal date, the forest owner must be indifferent as to cutting the forest stand or not. In other words, at the optimal harvesting age, the payoff received by the forest owner exactly compensates for exercise of the harvesting option. However, under uncertainty, the value matching condition is not sufficient to find the optimal stopping time, since it yields an infinite number of solutions. An additional *smooth pasting* condition must typically be specified. This condition requires the derivatives of both sides of the value matching condition to be equated. The smooth-pasting condition must be written:

$$J_t\big(\overline{t}^*, w\big) = J_w\big(0, w + A \cdot \Big[\sum_k (p_k - \Phi_k) \cdot \delta_k(\overline{t}^*) \cdot v(\overline{t}^*)\Big] - A\mu\Big)$$

$$\Big[Av'[\sum_k (p_k - \Phi_k) \cdot \delta_k(\bar{t}^*)]\Big] \quad \forall w. \tag{5}$$

This condition requires the derivatives of the value function to match at optimal date $\bar{t}^*$. The optimal harvest date and optimal consumption-savings path are defined by the three conditions (3), (4), and (5).

4 Application to maritime pine forests in southwest France

4.1 Specification of the empirical model

We apply this modeling to represent the behavior of a representative NIPF owner located in South-West France (Aquitaine region). The Aquitaine forest covers a total of 1.8 million hectares. It is the largest forest area of the EU. NIPF owners represent 92% of the total area. This natural forest has a dominating economic role. The main objective of Aquitaine forests is to produce wood. Aquitaine represents 20% of the French wood sector. The main tree species is maritime pine (70% of the total area). In this region, forest fires are one of the main problems for forests. These fires threaten the different functions of the forest: economic, social, and ecological. The protection policy of the Aquitaine forest against fire risks developed over several years is exemplary. Policy measures combining prevention and fines have resulted in very efficient control. Since 1985, the average size of burnt surface areas has diminished.

The average size of the observed private forest estates in Aquitaine is equal to 12 hectares ($A = 12$). Preferences of the forest owners are described by a strictly increasing and concave Von-Neumann Morgenstern utility function. As suggested by Eeckhoudt and Gollier (1992), we specify a Constant Relative Risk Aversion utility function: $U(c) = \frac{c^{1-\beta}}{1-\beta}$ with β being the Arrow-Pratt coefficient of absolute risk aversion. Initially we fix β at 0.5.[8] This level corresponds to a moderately risk-averse forest owner. The rate of pure present preference for the NIPF owner, r, is equal to 0.03. The no-risk rate η of the financial asset is also equal to 0.03.

The risk of fire is described as a Poisson process. In undisturbed temperate forests, the average rate of disturbance is between 0.5% and 2% per year, and this rate seems to hold over a number of different ecosystems (Stainback and Alavalapati (2004)). In Aquitaine, fires have burned on average 3,200 hectares per year over the period 1981 to 2003. This area represents roughly 0.17% of Aquitaine's forest land base

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[8] There is currently no available estimate for this parameter in the case of a NIPF owner facing a climate risk. More generally, there is neither no consensus on the level of the coefficient of risk aversion for economic agents. Static comparative exercises with respect to the level of the risk aversion will be conducted.

(1,890,481 hectares). It follows that the Poisson parameter for the risk of fire in Aquitaine is estimated to be $\lambda = 0.17\%$.

The estimate of the growth function parameters for the maritime pine in Aquitaine has been obtained using a nonlinear regression based on a data set derived from Vanniere (1984). Based on the existing literature (Koskela and Ollikainen (1999)), we have assumed that the growth process could be approximated by a logistic function. The resulting estimated function is: $g(t) = \frac{424}{1+40.6914e^{-0.1221t}}$ where t is the age of the stand (the number of observations is 11; and the R^2 coefficient is equal to 0.9977). We assume that the volume of wood in the forest stand can produce two outputs ($K = 2$), namely sawtimber and pulpwood. Wood products are indexed by $k = \{s, p\}$ respectively for sawtimber and pulpwood. As discussed previously and following (Bateman and Lovett(2000)), information regarding the proportion of wood allocated to each end use is required to incorporate carbon release into the carbon-timber forest management model. We assume that these proportions vary according to time. This variation reflects the fact that in the case of young trees, timber is only used for pulpwood. Then as the trees grow, the proportion of sawtimber increases with time. Using data for French timber production in 1999-2003, and a logistic equation, the proportion of sawtimber $\delta_s(t)$ as a function of tree age t is written: $\delta_s(t) = \frac{0.8028}{1+2748.8e^{-0.2636t}}$. The R^2 coefficient is equal to 0.9979. By definition, the proportion of pulpwood $\delta_p(t)$ as a function of tree age t is written: $\delta_p(t) = 1 - \delta_s(t)$.

Sawtimber and pulpwood prices are assumed to be constant and set at the following values: $p_s = 24.2$ euros per cubic meter and $p_p = 7.5$ euros per cubic meter (data from "Forêts de France"). For young tree generations, the gross value of forest sales is very low, since most of the timber volume is used for pulpwood. As the trees grow, then the share of the timber volume that can be used for sawtimber increases. Then the gross value of forest sales increases. Finally, the planting cost, D, is equal to 1000 euros per hectare (Couture and Reynaud (2008)).

The amount of carbon sequestered in a standing forest comes directly from the forest's biomass.[9] As suggested by Stavins (1999), carbon sequestration occurs in four components of the forest: trees, understory vegetation, forest floor, and soil. However, woody parts generally make up around 80% of a forest's total biomass.[10]

As discussed previously, the payment at date t per unit of surface for carbon sequestration received by

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[9] Forest biomass consists primarily of above-ground and below-ground tree components (stems, branches, leaves, and roots); other woody vegetation; and mosses, lichens and herbs.

[10] The share varies however greatly among forest types.

the forest owner is $p_c \cdot \alpha \cdot v'(t)$ where α denotes the conversion factor to convert a wood volume into metric tons of carbon. Following Stainback and Alavalapati (2004), the conversion coefficient is 0.3 metric tons of carbon per cubic meter. Incorporating net carbon sequestration implies choosing a price (p_c) for sequestered carbon. Previous studies have used a variety of methods to calculate this price. Some authors, for instance, have used econometric approaches based on observations of landowners' actual behavior confronted with the opportunity costs of alternative land uses (Stavins (1999); Newell and Stavins(2000)). For a survey of the literature, see Sedjo *et al.* (1997), and more recently Richards and Stokes (2004). The value of carbon varies in the literature and usually ranges from \$0 to \$200 per metric ton (Stainback and Alavalapati (2004)). Like Pohjola and Valsta (2007), we initially fix p_c to 10 euros per ton and we will conduct some sensitivity analysis on this parameter.

Sawtimber is modeled to decay over T_s years, and pulpwood over T_p years. As suggested by Stainback and Alavalapati (2004), the decay of sawtimber and pulpwood is modeled to be a linear process[11] in which an equal amount decays each year until all of the carbon pool has been released back into the atmosphere. Following Stainback and Alavalapati (2004) and Vallet (2005), we fix $T_s = 60$ years and $T_p = 5$ years.

The value of gross carbon benefits is maximized for a harvesting date around 35 years. One should, however, keep in mind that these values do not take into account the cost of forest-product carbon decay paid at the harvesting date.

4.2 Solving the stochastic dynamic programming problem

The method for solving the stochastic dynamic programming problem used here is based on the approach developed by Howitt *et al.* (2005) and is briefly presented here. The methodology comprises two stages. First, based on a value iteration approach, we numerically identify the value function, the solution to the problem. Second, the optimal decision rule is obtained by an algorithm based on the recurrence relation of the stochastic dynamic programming using the first stage value function approximation. In the first stage, we need to estimate the value function $J(.)$ defined by the three equations (3), (4), and (5). Since no analytical solution of this equation can be found, a numerical procedure was used. We use a value iteration approach, meaning that we seek a numerical approximation $\widehat{J}(.)$ to the infinite horizon value function that maximizes

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[11] Stollery (2005) assumes that at harvest, a proportion of the sequestered carbon is quickly transferred back to the atmosphere, and the remainder of the carbon goes into long-term product storage that decomposes at a exogenous and constant rate.

the value of the problem resulting from decisions made in the future. The main steps of the value iteration algorithm are presented in Judd (1998). It consists of assigning an initial value for the value function, and then recursively solving the maximization problem until the implied carry-over value function converges to an invariant approximation. The interested reader may also refer to Howitt *et al.* (2005) for an implementation of the value iteration algorithm with a recursive utility and to Couture and Reynaud (2008) for an extension of this approach to a multi-dimensional case. In order to solve the Bellman's equation, a specific functional form for $\widehat{J}(.)$ must be chosen to approximate the solution to the infinite-horizon problem. Howitt *et al.* (2005) have used a Chebychev Polynomial form, for instance, whereas Couture and Reynaud (2008) used a second-order polynomial approximation because they had a lot of state variables. Here we specify a second-order Chebychev Polynomial form which has good interpolation properties, see (Judd (1998)). The value function iteration program was written in GAMS. The quasi-stabilization of the value function was achieved after 100 iterations. Moreover, the residuals of the Bellman's equation at each discretized point were small enough to consider that the Chebychev polynomial form is a good approximation of the unknown value function.

4.3 Simulation results

4.3.1 Simulating the model without carbon prices

We consider the forest owner stochastic dynamic optimization problem without any carbon payments. This situation will be referred to as the benchmark case. The model has been simulated over a very long time horizon (200 periods). Those simulations allow us to focus first on the optimal forest management decisions and second, on the optimal consumption-savings strategies of the risk averse forest owner. In the two-use model, the optimal rotation time is 38 years. Every 38 years, the forest owner harvests his forest stand. At this optimal date, the optimal total volume of timber is 304.36 m^3/ha. 72% of this volume is used for sawtimber and 28% for pulpwood.

As shown in Figure 1, the carbon stored in the timber stand follows a pattern similar to the wood dynamics. At the optimal harvesting date, the quantity of carbon sequestered in the forest stand is equal to 102.48 tons/hectare, compared to the maximum absorption rate of maritime pine equal to 127.2 tons/hectare. From year 1 to year 38, the increase in global carbon sequestration only results from the forest growth process.

Both the global carbon sequestration curve and the timber stand carbon sequestration curve coincide exactly. The first harvest occurs at date 38, when the timber carbon sequestration level is equal to 91.31 t/ha. Since 72% of the timber is used for sawtimber at this harvest date, 65.8 t/ha of carbon are transferred to sawtimber, the remainder being stored in pulpwood. Then, the carbon stored in products starts to be released at an annual rate of 1.10 ton/ha for sawtimber and 5.11 ton/ha per year for pulpwood. In the same way, since the forest is replanted immediately after being harvested, living trees start to store carbon again. The global carbon sequestration decreases up to the date when the marginal increase in carbon stored in the forest stand becomes greater than the marginal release in carbon from sawtimber and pulpwood. From this date, the global level of stored carbon increases up to the next rotation age. The curve of carbon stored in sawtimber presents an inflexion point (after the second harvest). This is due to the fact that the optimal rotation age (38 years) is shorter than the lifespan of sawtimber (60 years). Finally, the global carbon sequestration path converges cyclically toward a value equal to 88 t/ha.

As shown in Figure 2, the dynamic path of wealth appears to be cyclical. First, the wealth of the forest owner decreases up to the optimal harvesting date, when it increases due to the revenue from timber sale. Then the wealth starts to decrease again. This process is repeated cyclically. The fact that the saving level is null at the harvesting dates (38 years, 76 years,...) is not surprising. Harvesting dates correspond to dates at which the forest stock is high. For a high level of timber, the value function is not affected by the level of savings. Hence it is optimal to reduce savings as the forest stock grows. The expected wealth takes into account the probability of forest stand loss due to fire. Since the optimal level of consumption depends on this expected value, it is interesting to analyze how the expected wealth of the forest owner evolves over time. Both forest and saving stocks are managed by the forest owner in order to stabilize consumption over time. This objective is correctly achieved since, as shown in Figure 2, fluctuations in consumption are relatively small compared to wealth fluctuations, for instance. Notice that the forest owner determines the level of consumption at each date on the basis of the expected wealth composed of savings and the expected value of the timber volume at this period. Hence, the consumption flow decreases as the expected wealth decreases, and increases in the opposite case.

4.3.2 Introducing a price for carbon sequestration

We now explore the model incorporating a carbon price equal to 10 euros per ton of sequestered carbon. When such a carbon price is implemented, forest owners face a more complex optimization problem. They must determine the optimal rotation length, not only with respect to the profit gained from selling wood products (namely sawtimber and pulpwood), but also taking into account the net payoffs generated by carbon sequestration. Obtaining revenues from carbon sequestration during the growing phase of the forest leads to an increase in its value. Hence, delaying the date of harvesting is favorable, not only because more wood volume can be sold, but also because more gains can be obtained from carbon sequestration. Moreover, postponing the harvest date also means that carbon taxes, which reflect the carbon release at harvest time, must be paid later (in other words, due to discounting, there is a difference in the discounted value of carbon payments). Carbon prices generate a net positive benefit for the forest owner for this reason. This benefit is balanced, however, by the increase in fire risk exposure when the forest owner chooses to delay the harvesting date.

Table 1 presents the optimal rotation duration for several carbon prices. As expected, introducing a carbon price extends the optimal rotation period, compared to the case without a carbon value. This effect is stronger with a high carbon price. For example, for $p_c = 10$ euro/ton, the optimal rotation duration is 42 years, compared to 38 years in the benchmark case without carbon value. With a carbon price equal to $p_c = 80$ euro/ton, the length of optimal rotation exceeds 50 years. Doubling the price of carbon increases the rotation length almost linearly, with an average increase of 3.5 years. The changes in optimal rotation length increase the carbon storage during the rotation period, by 12 $tons/ha$ with a carbon price of 10 $euro/ha$ up to 29 $tons/ha$ with a carbon price of 100 $euro/ha$. The increases in stand volume are between 38 m^3/ha and 95 m^3/ha. Carbon prices induce longer rotations, thereby increasing the amount of carbon stored on the stand. The amount of carbon sequestered is greater for higher carbon prices. With carbon prices, the proportion of sawtimber increases considerably due to longer rotation periods, whereas the proportion of pulpwood decreases because it has a shorter life span than sawtimber. The carbon tax/subsidy programme is found to increase the income of forest owners considerably. As the net revenue from carbon sequestration only consists of the total revenue, the forest owner can, however, affect total revenue greatly by changing stand management. For example, with a carbon price of 10 $euro/t$, the joint revenue would increase from

5917 *euro/ha* to 6286 *euro/ha* even without any change in silviculture. As applying optimal silviculture would increase the total revenue to 7394 *euro/ha*, the additional benefit from modifying silviculture would be 1108 *euro/ha*. Silviculture that includes the joint revenue from both timber production and carbon sequestration differs from timber-based silviculture. Both timber revenue and total revenue increase.

As shown in Table 2, the greater the size of forest, the shorter the optimal rotation period. One possible explanation is that when the size of the forest increases, the optimal rotation is shortened because the increase in gain due to the size is compensated by a lower harvest age in order to maintain a certain level of wealth. Another explanation is related to the risk aversion of the forest owner. As the size of forest increases, then the loss in the case of fire is also greater. As a result, risk aversion induces the forest owner to shorten the optimal rotation duration.

Since global warming results in an increase in the frequency of extreme climatic events, it is interesting to analyze how optimal forest owner decisions are modified by changes in the risk of fire. In Table 2, we report the optimal rotation duration for several values of the fire risk arrival rate, λ. As expected, increasing the risk of fire reduces the rotation age. With a low fire probability, it is optimal to accumulate carbon in trees and in forest products with a long lifespan. With a high risk of fire, it is optimal to reduce the optimal harvest age and to store the carbon in forest products. More specifically, the forest owner reduces the final timber stock exposed to fire risk as the fire risk increases. The rotation duration for maritime pine falls gradually from 42 years with a 0.17% fire risk to about 38 years with a 1.7% fire risk, which corresponds to the optimal rotation age obtained in the benchmark case, that is without any carbon price.

In order to evaluate the impact of risk preferences on optimal rotation age, the stochastic dynamic recursive equation was solved for various levels of the Arrow-Pratt constant relative risk-aversion coefficient (CRRA), β. We considered three levels of the CRRA coefficient: almost risk neutrality: $\beta = 0.0001$; moderate risk aversion: $\beta = 0.5$; and extreme risk aversion: $\beta = 1.5$. As can be seen in Table 2, the optimal rotation age decreases as risk aversion increases. In the case of quasi risk neutrality ($\beta = 0.0001$), the forest owner does not use saving to smooth consumption; at each period during stand growth, consumption only depends upon carbon benefits. On harvesting, it depends on timber benefits. Intuitively, when risk aversion increases, the forest owner tries to avoid exposing its assets to risk. This can be achieved though earlier harvesting, which means a lower value being attributed to the forest stand. The forest owner can stabilize his consumption, first by having a high level of savings, and second by having a lower final timber stock exposed to fire risk. This second mechanism is made possible by harvesting sooner. In other words, higher saving together with a lower harvesting age provide better insurance against the variation in consumption due to catastrophic events. The higher the parameter of risk aversion, the higher the savings and the lower the cutting age. This can be explained intuitively. A risk averse forest owner wants to secure income. This can be achieved by having a higher level of wealth (with a certain return) and a lower timber level (asset with a high risk of loss). Implications of this result in terms of carbon sequestration are the following. As the cutting age decreases, less carbon will be sequestered in the stand. In spite of the positive carbon price, a risk-averse forest owner facing fire risks prefers to reduce the optimal rotation time. Hence risk preferences have important impacts on the effect of carbon policy. This factor needs to be integrated in the analysis in order to estimate the real effects of such policies on optimal forest management. It follows that forest owners' risk preferences should be viewed as important features of any framework aiming to analyze private forest owner decisions facing fire risk.

4.3.3 Carbon price and fire risk levels

Most carbon price scenarios predict a significant price increase over the next decades (Capoor and Ambrosi (2007), Chladná (2007)). Upwardly trending carbon prices will strengthen the rationale for storing carbon in trees. As shown previously, this will result in delaying the optimal date of harvesting. Global warming due to climate change is expected to increase the frequency of extreme events such as fires (Stollery (2005)). There has been a lot of research aimed at establishing the relationship between climate change and increased forest fires. There is now a broad consensus that fire risks are expected to increase over time as the climate warms. Increased fire risk will lessen the rationale for storing carbon in trees. As shown previously, this will result in shortening the optimal rotation. The joint impact of fire risk and carbon price increase is a priori ambiguous since a forest owner will react to an increase in fire risk by reducing rotation duration. On the contrary, if carbon prices increase, then it is optimal to extend the rotation length. Increasing carbon prices has an opposite effect to the impact of an increase in fire risks, and without specific knowledge of the relative strengths of these effects, one cannot predict which of them will dominate. This is the question we will now investigate.

In what follows, we identify a frontier in the carbon price × fire risk space such that all points along this frontier result in the same optimal rotation age. More specifically, Figure 3 represents the carbon price/risk frontier maintaining the optimal rotation age at $\overline{t}^* = 42$. Region I corresponds to couples (carbon price × fire risk) in which that the optimal rotation age is less than 42 years. On the contrary, for couples located in region II, the optimal rotation period is greater than 42 years. In Figure 3 we have represented the (carbon price × fire risk) frontier for three levels of forest owner risk aversion. The plain line corresponds to the frontier in the benchmark case where the forest owner is supposed moderately risk averse, $\beta = 0.5$. As expected, the frontier is increasing: an increase in fire risk must be compensated by an increase in carbon price in order to keep the rotation duration the same. The relationship between the risk of fire and the carbon price is almost linear along the frontier. Finally, we analyze how the risk preferences of the forest owner modify the location of the (carbon price × fire risk) frontier. As can be seen in Figure 3, for a given level of risk, the higher forest owner risk aversion, the higher the carbon price must be to maintain the rotation length at 42 years. For instance, for a risk of fire equal to 1%, the carbon price inducing a 42 year rotation is 10.6, 13.8 and 16.6 euro/ton respectively in the case of a risk neutral, risk averse and extremely risk averse forest owner. The economic intuition of this result is straightforward. For a given level of fire risk, increasing the forest owner's risk aversion implies a reduction in the optimal rotation age. To maintain the optimal rotation length at 42 years for a very risk-averse forest owner, one must make the non-harvesting strategy very profitable. This is only possible with a very high carbon price which implies large benefits from keeping old tree generations. This result suggests that the rotation length appears to be very sensitive to the level of risk aversion. Any model aiming to provide policy implications must investigate the risk aversion of forest owners carefully.

4.4 Willingness to pay for fire risk reduction

Some recent works in forestry indicate that the way a forest is managed can influence fire risk occurrence. Yoder (2004) considers fire prevention actions (conjointly self-protection and self-insurance activities) within a modified Faustmann rotation model. He shows that some management activities (reduction of surface fuels, and suppression of dead and dying trees below the canopy) can efficiently reduce fire risk occurrence. Curtis et al. (1998) report that thinning can also be used to mitigate fire risk since thinning reduces high fuel loads and may improve wind stability in order to prevent large trees from being blown down. Those forest management practices targeted toward reducing fire risk occurrence are costly.[12] Defining the efficient

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[12] Amacher et al. (2005) indicates that good estimates of these costs are usually not available. For US, they however suggest that the costs may range from \$11/acre for prescribed burning or \$27/acre for brush clearing to \$45/acre for precommercial

level of fire risk prevention expenses requires also a valid measure of the benefits to be expected for the forest owners. In what follows, we show that the stochastic dynamic program we have developed can be used to assess forest-owner's willingness to pay for reducing fire risk and then to derive the private benefit from risk reduction.

As mentioned previously, the SDP model has been simulated over a long time horizon (200 years). We then denote by $c^*(t,\lambda)$ the optimal instantaneous consumption at date t with a fire risk equal to λ. The optimal intertemporal utility corresponding to this consumption path writes :

$$\int_0^T \exp^{-rt} U(c^*(t,\lambda))dt. \tag{6}$$

We wish to compute the certainty equivalent (in terms of consumption) corresponding to the consumption path $c^*(t,\lambda)$. By definition the certainty equivalent, denoted by $CE(\lambda)$, is such that:

$$\int_0^T \exp^{-rt} U(CE(\lambda))dt = \int_0^T \exp^{-rt} U(c^*(t,\lambda))dt. \tag{7}$$

which corresponds to the fact that the intertemporal utility derived from a constant consumption flow $CE(\lambda)$ must be equal to the optimal intertemporal utility. Straightforward computations give:

$$CE(\lambda) = \Big[\Big(\frac{r(1-\beta)}{1-\exp^{-rT}}\Big)\cdot\Big(\int_0^T \exp^{-rt} U(c^*(t,\lambda))dt\Big)\Big]^{\frac{1}{1-\beta}} \tag{8}$$

We can then easily compute the forest-owner instantaneous willingness to pay for reducing the fire risk[13]. The willingness to pay for reducing the risk from λ_2 to λ_1 is simply equal to the difference between the two certainty equivalents for these two risk levels, i.e. $CE(\lambda_1) - CE(\lambda_2)$. A fire risk reducing program that allows to reduce the risk from λ_2 to λ_1 should be implemented if and only if the instantaneous cost is lower than the forest-owner willingness to pay. Naturally, the willingness to pay depends on the carbon price as it will be shown in the next paragraph.

In Figure 4, we have plotted the certainty equivalent as a function of the fire risk for two carbon price levels (10 and 20 euros per ton). As a benchmark, we have also considered a situation without any carbon

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thinning.

[13] Reed (1984) has used a cost-benefit analysis to study fire protection expenditure as self-protection. In his NPV framework, the author has defined the benefit of prevention action as the change in maximum expected present value.

price.

As expected, the certainty equivalent decreases with the fire risk: the higher is the fire risk, the lower are the expected utility and the certainty equivalent. Whatever the carbon price, we observe significant differences across certainty equivalents depending on the level of fire risks. For instance, without any carbon price the certainty equivalent is 311.58 euros/ha/year respectively for a fire risk equal to 0.07% compared to 274.25 euros/ha/year for a fire risk equal to 1.26%. This means that the forest-owner is ready to reduce the certain consumption level by 37.33 euros each year (representing a 12% reduction) to reduce the fire risk from 1.26% to 0.07%. This information is useful for a public authority that has to decide, on a cost/benefit basis, which fire risk reduction program should be implemented.[14]

It should be noticed that the relationship between the certainty equivalent and the fire risk appears to be non linear. For very low fire risks, the relationship is very flat, but it tends to be steeper for higher risks. For instance, the willingness to pay for a program that would reduce the fire risk from 0.17% to 0.07% is only 0.46 euros/ha/year compared to 26.53 euros/ha/year for a program allowing to reduce the risk from 0.27% to 0.17%. This result suggests the existence of thresholds effects that should be taken into account by public authorities wishing to implement fire risk reduction programs.

Lastly, a higher carbon price implies a higher certainty equivalent. For instance, for a fire risk equal to 0.17% the certainty equivalent represents 311.40, 327.29 and 339.50 euros/ha/year respectively without a carbon price and with a carbon price equal to 10 and 20 euros per ton. This means, that the carbon price will play a significant role in the adoption of fire risk prevention programs. An increase of the carbon price induces a rise of the total revenue of the forest owner. Consequently, the certainty equivalent also increases but not proportionally due to risk aversion. For example, when the carbon price of 10 euros is doubled, the variation of the total revenue is equal to +14% whereas the change in the certainty equivalent is just +3,5%. This result confirms the presence of threshold effects.

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[14] In the area we have considered, public authorities have implemented a program called DFCI-Aquitaine that aims at preventing the risk of forest fires. This program includes prevention efforts, infrastructure works, education and information campaigns, innovation actions in order to reduce the level of fire occurrence. Each private forest owner that participates to this program pays a minimal contribution of €2.3/ha/year. This contribution seems low compared to the calculated WTP but such prevention programs are also active thanks to subsidies it receives from the European Union, the French government and local authorities.

4.5 Forest management and policy implications

Efficient management of forests requires information on the magnitude of the global warming that has occurred or is likely to occur in the future, and estimates of the impact of global warming on factors including fire frequency and tree growth rates. At present, there is considerable uncertainty about the magnitude of climate change and its impact on forest management. One of the reasons for this uncertainty is that there is natural variability in climate, fire frequency and tree growth rates. The profitability of forest product increases when the benefit of sequestration of atmospheric carbon in forest biomass is taken into consideration. Benefits will have several important forest management implications. It will be optimal to increase rotation ages. With a carbon fee, the economic value of forests will increase dramatically because of their potential as a carbon sink. Today there is no market to stimulate investments in this good. One important challenge will be to introduce policy means that make it possible to reach an optimal investment level regarding forests as carbon sinks. For private forests, one can in principle use regulation by law or economic policy means, or combinations of these. It would, however, be very difficult in most countries to make forest owners willing to increase forest biomass without economic compensation of some sort i.e. subsidies or tax incentives. Subsidies for carbon capture in forest biomass have to be followed by corresponding taxes on the release of carbon from the end-use of the forest biomass.

The numerical simulations provided here have some important implications for the ability of forests to act as carbon sinks. The countries ratifying the Kyoto agreement have obtained the right to use forests as absorbers of carbon to offset their need to reduce fossil fuel emissions. However, our findings indicate that if the climate warms, then fire risk will rise, reducing the carbon-absorbing ability of forests. The use of existing forests as carbon sinks may be a temporary option compared to increasing forest area or switching from forests with faster-growing species. Forest carbon sinks need to be preserved by preventive action to mitigate the effects of natural disturbances. Therefore, to keep forests' carbon absorption, some mitigation activities need to be implemented jointly with forest management activities. Policy measures can induce fire protection strategies to reduce fire risk. There is a greater incentive for forest owners to reduce risk or purchase insurance. In a carbon market the concern for mitigating risk will become even greater. Policies to help forest owners to manage and reduce their risk would therefore have a positive impact on the amount of carbon stored by forests. In the same way, policy can affect the supply of sawtimber or pulpwood by changing the prices of timber products or limiting the fluctuations in such prices. In our model we assume that timber prices are constant over time, but it is clearly proved that these prices are volatile and uncertain. Some policy measures could reduce variations in timber prices. For example, imposing a minimum guaranteed threshold price in the sawtimber market to reduce the volatility of prices at some future time could impact on the supply of sawtimber by maintaining a high price for sawtimber. This policy could encourage longer rotations and more sawtimber production, increasing the carbon storage by forests and products. Another policy could affect pulpwood supply by taxing the revenue from such products. This measure could induce less pulpwood production and more sawtimber supply, encouraging carbon sequestration in growing trees and final products.

5 Conclusion

Public financial incentives for forest carbon sequestration might influence individual forest owners to postpone optimal rotation and protect their forest from natural damage. Catastrophic events can impact optimal forest management decisions because such events significantly alter the optimal production decision of a stand, and release some or all of the stored carbon back into the atmosphere. In this paper, we use an expected utility approach to analyze saving and forestry management jointly, in a stochastic intertemporal framework with a risk-averse forest owner. We have applied this framework to the management of a representative forest owner located in the south-west of France. Our results show that the forest owner prefers shorter rotations as the fire risk increases and longer rotations when the price of carbon rises. To investigate the contradictory effects of fire risk and carbon price on forest rotation, we identify the set of carbon prices and fire risks that leads to given rotation age. We also show that the model can be used for deriving the NIPF owner's willingness to pay for a risk reduction and that this willingness to pay can be substantial (37.33 euros by ha and by year to reduce the annual fire risk from 1.26% to 0.07%). The quantification of NIPF owner's willingness to pay may help public authorities wishing to design and implement risk reduction programs.

This research could be extended in many ways such as introducing insurance activities or public support funds. It is possible to introduce fire-risk coverage choices into the decision program of the NIPF owner. These decisions could impact optimal multiple-use forest management. Very often, in the case of catastrophic disasters in France, forest owners' financial losses are partially covered by compensation from public programs that guarantee a minimum value for damaged property. Introducing public program funds into the optimization program of the forest owner may significantly modify the optimal forest and consumption strategies.

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Table 1: Optimal rotation length, timber volume, and net revenues from timber production, carbon sequestration as a function of carbon price

	Carbon price (*euro/ton*)					
	0	10	20	40	80	100
Rotation length (years)	38	42	45	49	52	53
Timber volume (m^3/ha)	304	342	363	385	396	399
Sawtimber	218	263	286	307	317	320
Pulpwood	86	79	77	78	79	79
Change relative to the benchmark case (no value for carbon) (%)						
Sawtimber		20.6	31.2	40.8	45.4	46.8
Pulpwood		-8.1	-10.5	-9.3	-8.1	-8.1
Total		12.5	19.4	26.6	30.3	31.2
Carbon storage (ton/ha)	91	103	109	116	119	120
Timber revenue (euro/ha)	5917	6954	7502	8005	8260	8327
Carbon net revenue (euro/ha)	0	440	954	2040	4213	5310
Carbon benefit	0	1025	2180	4615	9500	11967
Carbon tax	0	585	1226	2574	5287	6657
Total revenue (euro/ha)	5917	7394	8455	10045	12473	13637
Change relative to the benchmark case (no value for carbon) (%)						
Timber revenue (%)		17.5	26.8	35.3	39.6	40.7
Total revenue (%)		25.0	42.9	69.8	110.8	130.5

Table 2: Optimal rotation length, timber volume, and revenues as a function of the forest size, or of risk, or of risk preferences

	Optimal rotation length (*years*)	Volume (/*ha*)		Revenue (*euro*/*ha*)		
		Timber (m^3)	Carbon (*ton*)	Timber	Carbon[a]	Total
Forest size						
6	46	369	111	7649	487	8136
12	42	342	103	6954	440	7394
24	40	324	97	6484	408	6892
Risk λ(%)						
0.17	42	342	103	6954	440	7394
0.34	41	333	100	6731	425	7156
0.68	40	324	97	6484	408	6892
1.26	39	315	94	6213	390	6603
1.7	38	304	91	5917	369	6286
Risk preferences β						
0.0001	45	363	109	7502	477	7979
0.5	42	342	103	6954	440	7394
1.5	41	324	97	6731	425	7156

[a] with a carbon price of 10 *euro*/*ton*.

Figure 1: Dynamics of carbon stored in the timber stand and products

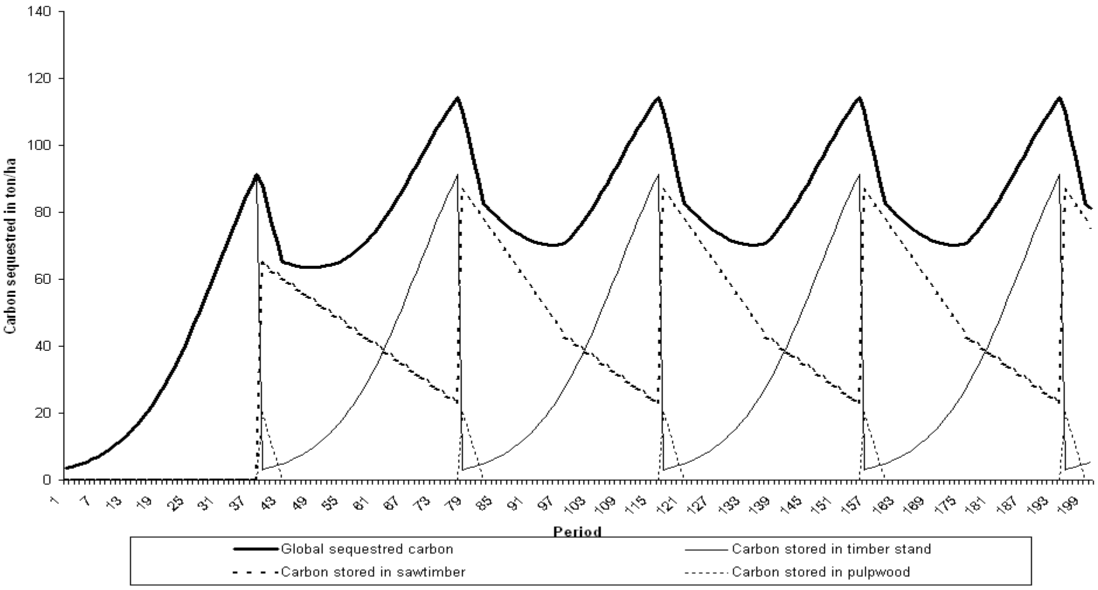

Figure 2: Dynamics of consumption-wealth-expected wealth

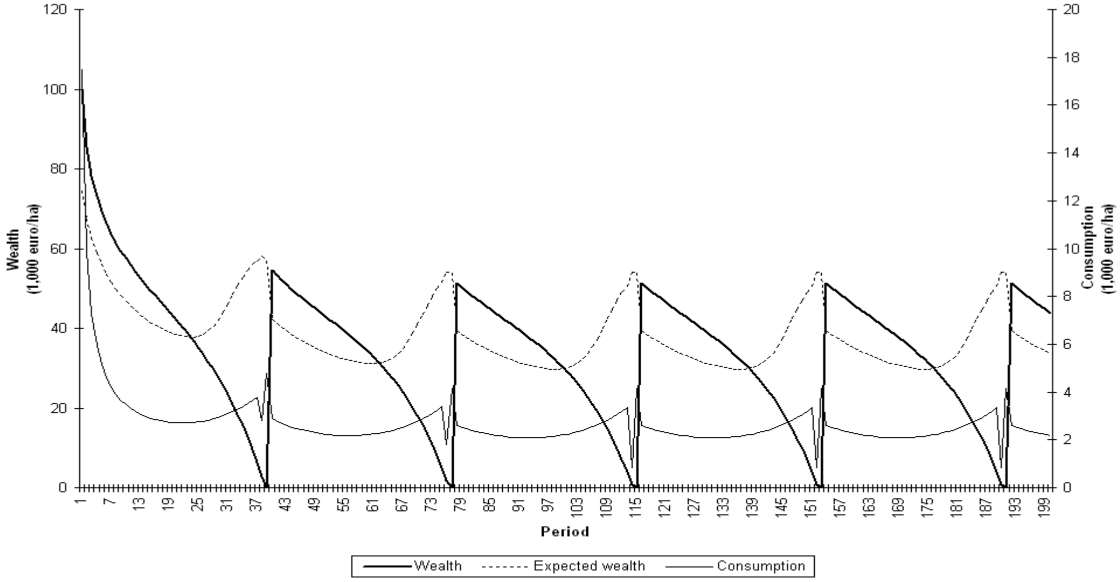

Figure 3: Carbon price/risk frontier

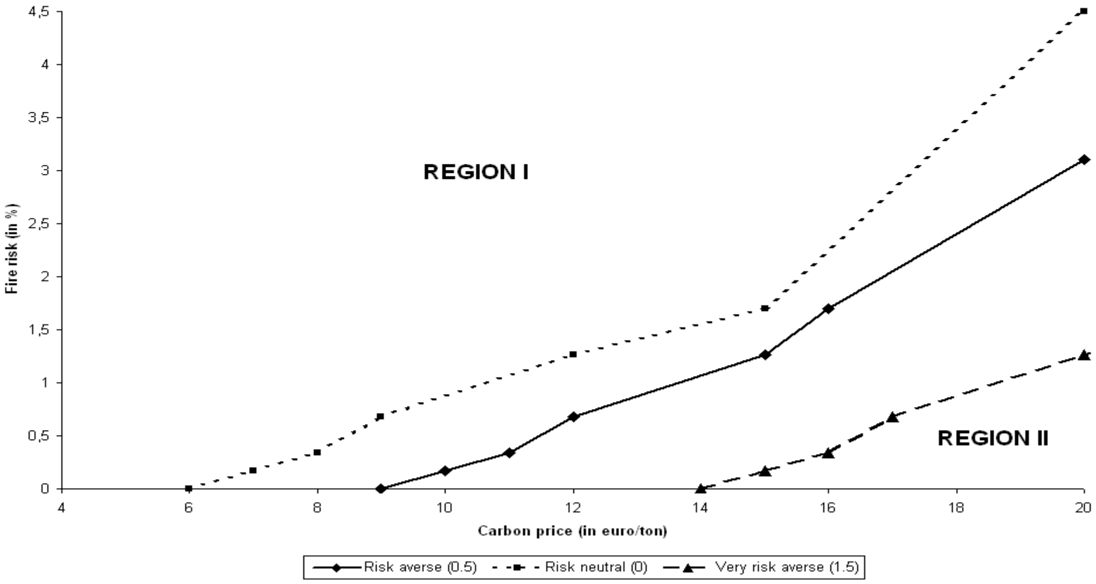

Figure 4: Certainty equivalents without and with carbon prices (10 and 20 euros/t)