Multi-Objective Constraint Optimization with Lexicographic Preference Models

Anne-Marie George, Abdul Razak and Nic Wilson
Insight Centre for Data Analytics,
School of Computer Science and IT
University College Cork, Ireland
{annemarie.george,abdul.razak,nic.wilson}@insight-centre.org

In this paper, we consider algorithms for solving Multi-Objective Constraint Optimization Problems (MOCOP) with tradeoffs, which consist of elicited or observed preferences over decisions (feasible solutions) as described in [1]. In MOCOP, each decision is evaluated on a number of criteria, and can thus be associated with a vector of objective values, each component corresponding to a different criterion. The comparison between decisions is based on a comparison between objective vectors. The optimal decisions are those that are not dominated by any other decision due to some order relation, which respects the restrictions given by the tradeoffs. We focus on a branch-and-bound algorithm to find optimal solutions, which uses a mini-buckets algorithm for generating the upper bound at each node, as proposed in [2].

A key issue is how one orders the objective vectors. The two most common methods are using a weighted sum, or a Pareto ordering. With a weighted sum model, one chooses a weight for each criterion in the objective vectors. The weights determine the tradeoffs between different objectives, i.e., how much of some objectives the decision maker would be happy to tradeoff against some amounts of other objectives. With the weighted sum model, it can be hard to decide on the appropriate weights. They can be time-consuming to elicit, or there can be more than one person involved in the decision making process. With the Pareto or product ordering, two objective vectors are often incomparable, which leads to a rather weak pre-order on objective vectors and hence on decisions, especially if there are several objectives. This can lead to a sometimes extremely large number of optimal (undominated) decisions, which is not helpful for the decision maker.

In this paper, we consider a situation in which objective vectors are compared using a lexicographic order. Thus, objective vectors are compared on the most important component; only in case of equality the next most important component is considered and so on. For instance, in a 3-objective decision making problem, (1, 2, 4) is lexicographically better than (1, 3, 2) if one considers the components to be in decreasing order of importance and we are minimising. Here, the lexicographic model, i.e., the importance ordering on the objectives, by which the user expresses her preferences is unknown. However, the number of possible lexicographic orders that model the user's preferences is restricted by the set of tradeoffs. For instance, in a bi-objective case, the input preference "(3, 2) is better than (2, 3)" implies that the first objective is more desirable than the second. Preference Inference Problems involves inferring additional user preferences from elicited or observed preferences, based on assumptions regarding the form of the user preference relation; in our case lexicographic orders. We use this concept to perform dominance checks by the polynomial time algorithm described in [3]. Works related to lexicographic preferences include the following [4, 5, 6].

Our implementation indicates that when using a lexicographic model-based approach, the number of solutions is drastically reduced in comparison with other approaches described in [1, 2]. This is because the dominance relation based on lexicographic models is much stronger. However, for the problems we considered, the lexicographic model-based approach was slower.

References

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