Statistics and learning

Neural Networks

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- Computational developments had to wait until the 90s.
- Neural network theory benefited the study of the functioning of the brain and provided the basis to create artifical intelligence.

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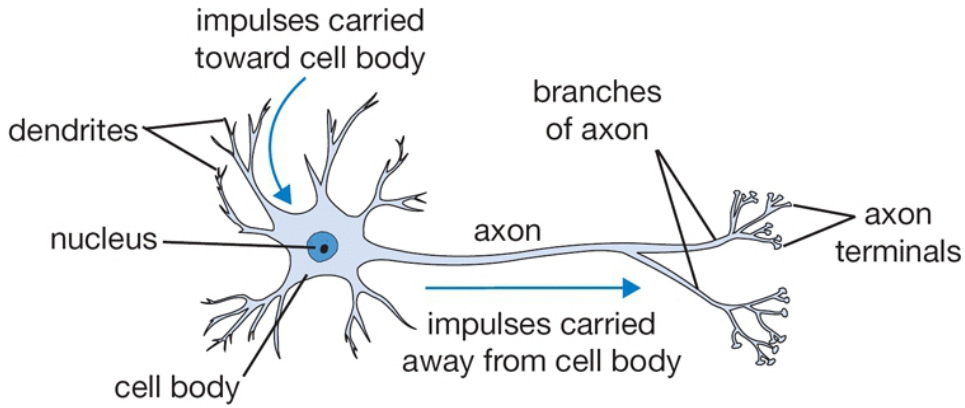
- Application: system identification and control (vehicle or process control), game-playing with decision making (chess), pattern/sequence recognition (radar systems, face identification, object recognition, spam filters, speech or handwriting recognition), medical diagnosis, financial applications, data mining (knowledge discovery in databases), visualisation.

Biological vs artifical neuron



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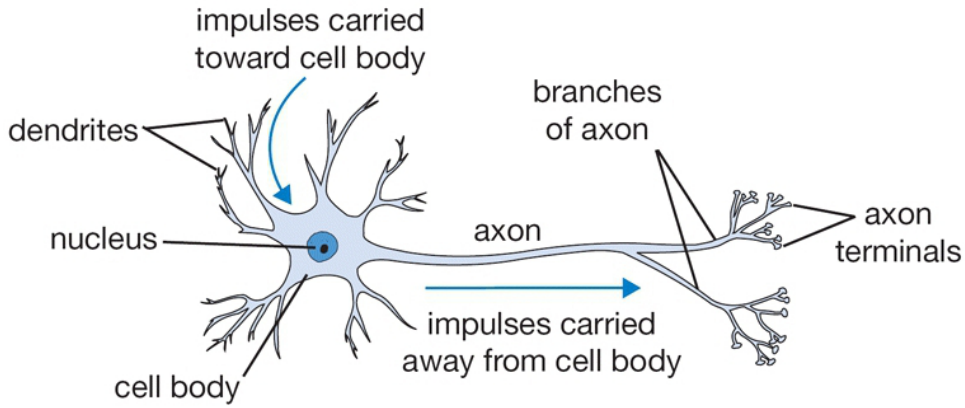

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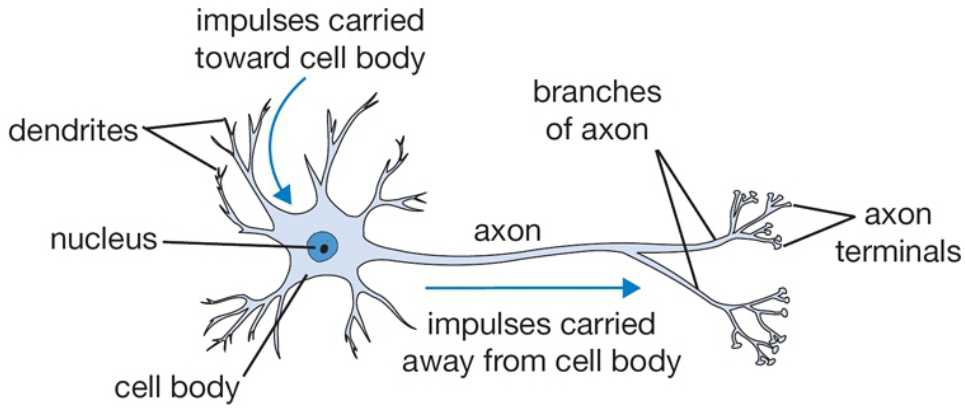

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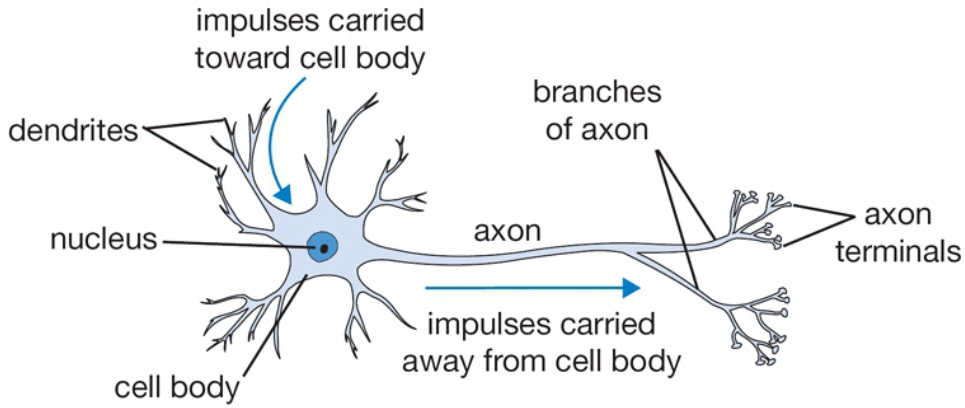

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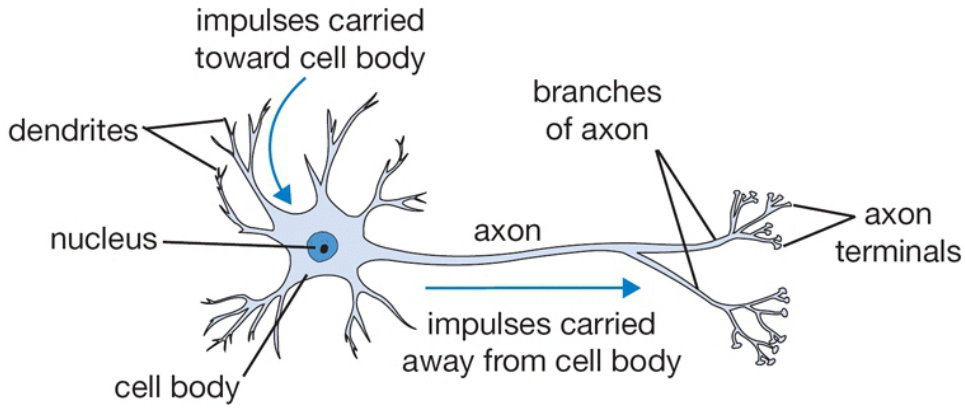

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- In brief, the activation function makes a transformation of a weighted linear combination of the inputs.

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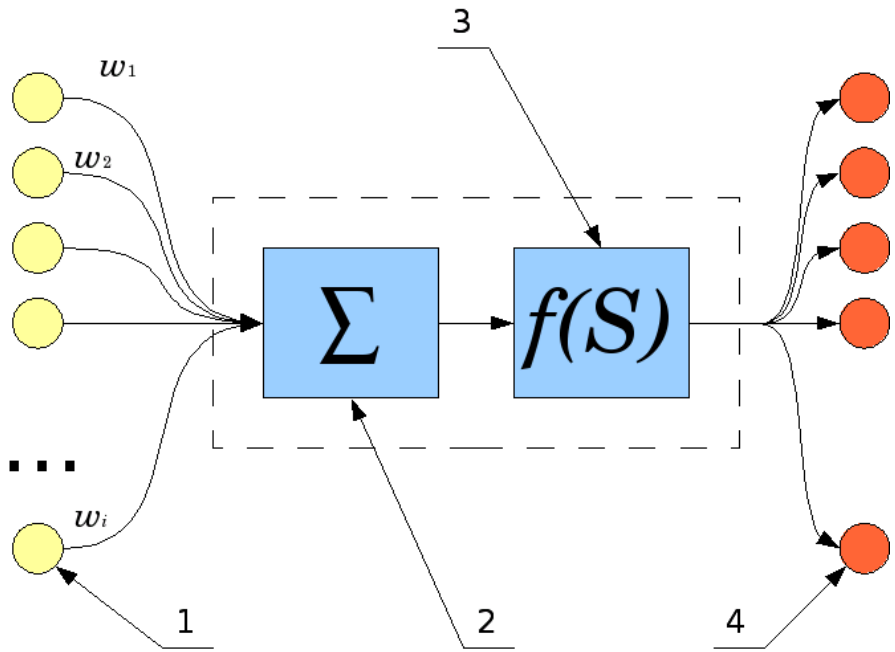

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- We restrict this course to the elementary (feedforward and not recurrent) static structure of the network for supervised learning.

Multilayer perceptron

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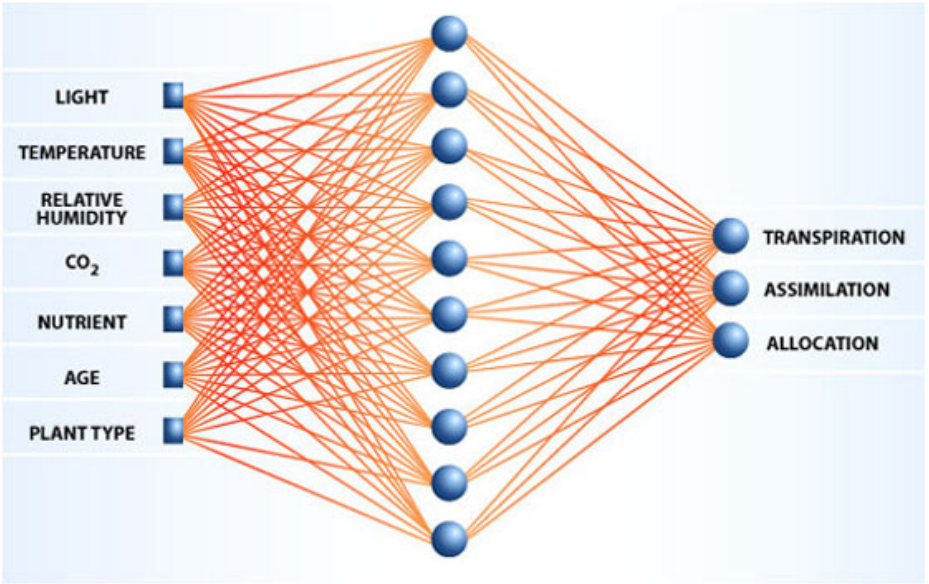

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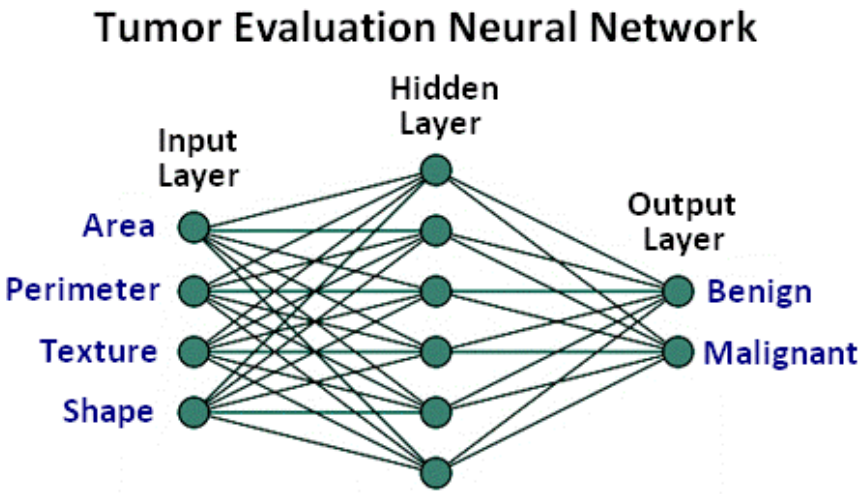

Neural networks

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Transfer function

A multilayer perceptron does a transformation of the input variables in the form:

$$Y = \varphi\left(X_1 \dots X_p; \alpha\right),$$

where α is the vector with elements α_{jkl}: parameter for entry j of neuron k of layer l. Note that the entry layer ($l = 0$) is not parameterised.

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Theorem (of "universal approximation")

A not overwhelming structure with one hidden layer is enough to account for most classical problems in statistics modelling or learning. Stems from the approximation of any **regular** *function with arbitrary accuracy in a finite domain of variable space by a neural networks with a finite number of neurons in a unique hidden layer and a linear output neuron.*

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- Parameters can be estimated using a least square criterion by minimising the quadratic loss $Q(\alpha, \beta) = \sum_i Q^{(i)} = \sum_i \left[y^{(i)} - \varphi(x^{(i)}; \alpha, \beta) \right]^2$.

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- Several classical algorithms exist and are generally based on an evaluation of the gradient by retro-propagation.

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\frac{\partial Q^{(i)}}{\partial \beta_k} &= -2(y^{(i)} - \varphi(x^{(i)}))({}^\top \beta z^{(i)}) z_k^{(i)} = \delta_i z_k^{(i)} \\
\frac{\partial Q^{(i)}}{\partial \alpha_{kj}} &= -2(y^{(i)} - \varphi(x^{(i)}))({}^\top \beta z^{(i)}) \beta_k f'({}^\top \alpha_k x^{(i)}) x_p^{(i)} = s_{ki} x_p^{(i)},
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- They are evaluated in 2 steps: "before" with current weight values, input values are applied to get evaluations $\hat{\varphi}(x^{(i)})$ and "after" to determine δ_i which are back-propagated to compute s_{ki} and hence access gradient evaluations.

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- Refinements include second order expansion, stochastic modifications for not being trapped in a local minimum, adding an inertia term to avoid oscillations...Be aware of the algorithm you use and be critical

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- a **decay** (penalty term of the form $Q(\theta) + \gamma \parallel \theta \parallel$) can avoid any question about the number of neurons by choosing it large and then restricting active parts of the activation functions by choosing γ only by CV.

Let's take a break before the practical sesson

Only 2 sessions left: regression trees and MCMC !