



STATSEQ: Gene Network Inference Meeting 2013  
Paris, 28-29 March

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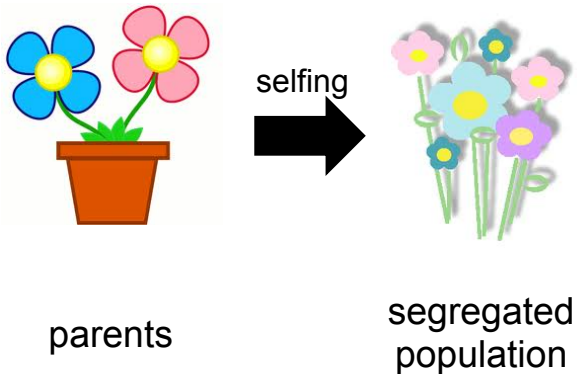
# An Effective Framework for Reconstructing Gene Regulatory Networks From Genetical Genomics Data



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TECHNISCHER SYSTEME  
MAGDEBURG



- **Wanted**
  - **understanding of gene interactions** via gene regulatory networks
  - optimization of phenotype via **optimization of gene interactions**



# Motivation

- **Wanted**
  - understanding of **gene interactions** via gene regulatory networks
  - optimization of phenotype via **optimization of gene interactions**
- **Need**
  - infer interaction structure (GRN) from data
    - **systems genetics approach: use segregated population**
    - ...

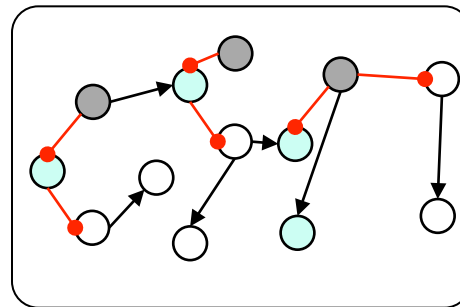
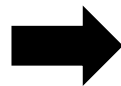


parents

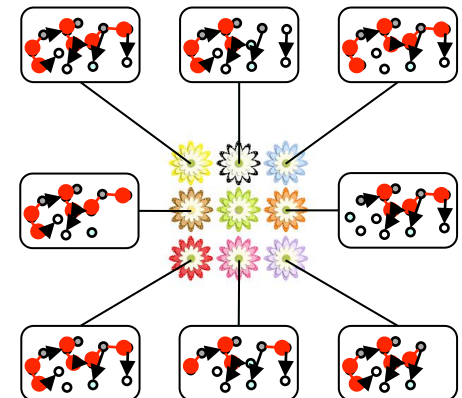
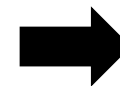
selfing



segregated population



**How to obtain?**



# Motivation

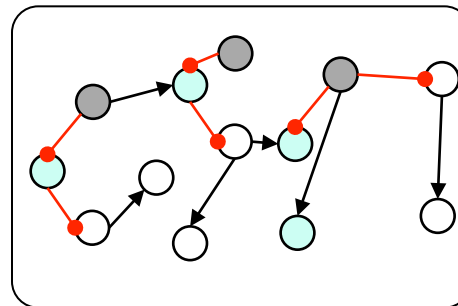
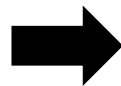
- Many (complex) methods
- What about simple correlation?



parents



segregated  
population



**How to obtain?**

## Challenge

$$n_{\text{sample}} / n_{\text{genes}} \ll 1$$

# Outline



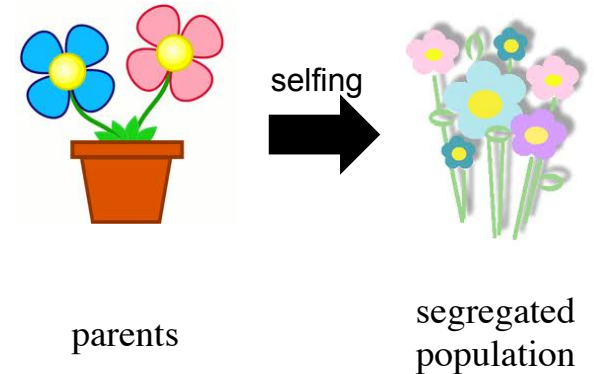
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- Motivation
- Method
- Application
- Conclusion

1. Data preprocessing
2. Reconstruct interactions between genes
  - raw perturbation graph  $G_1$  (no edge weights nor signs)
  - raw perturbation graph  $G_2$  (edge weights & signs)
  - identify eQTLs
  - select ONE candidate gene
3. Prune false positive interactions via transitive reduction
  - remove redundant candidate genes/interactions

# 1. Data Preprocessing

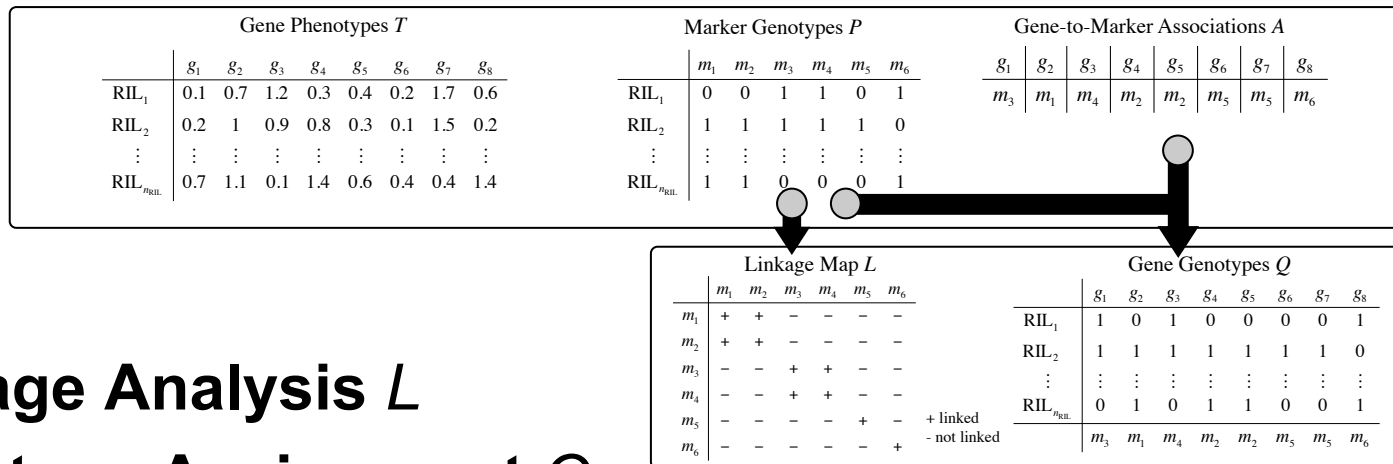
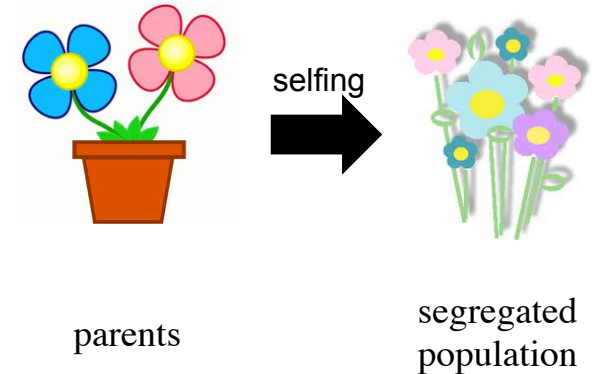
- data set from study population:
  - **phenotype** of genes  $T$
  - **genotype** of marker  $P$
  - **gene-to-marker** association  $A$



| Gene Phenotypes $T$            |       |       |       |       |       |       |       |       | Marker Genotypes $P$           |       |       |       |       |       | Gene-to-Marker Associations $A$ |       |       |       |       |       |       |       |       |  |
|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|--------------------------------|-------|-------|-------|-------|-------|---------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|--|
|                                | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ |                                | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$                           | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ |  |
| RIL <sub>1</sub>               | 0.1   | 0.7   | 1.2   | 0.3   | 0.4   | 0.2   | 1.7   | 0.6   | RIL <sub>1</sub>               | 0     | 0     | 1     | 1     | 0     | 1                               | $m_3$ | $m_1$ | $m_4$ | $m_2$ | $m_2$ | $m_5$ | $m_5$ | $m_6$ |  |
| RIL <sub>2</sub>               | 0.2   | 1     | 0.9   | 0.8   | 0.3   | 0.1   | 1.5   | 0.2   | RIL <sub>2</sub>               | 1     | 1     | 1     | 1     | 1     | 0                               |       |       |       |       |       |       |       |       |  |
| ⋮                              | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮                              | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮                               |       |       |       |       |       |       |       |       |  |
| RIL <sub>n<sub>RIL</sub></sub> | 0.7   | 1.1   | 0.1   | 1.4   | 0.6   | 0.4   | 0.4   | 1.4   | RIL <sub>n<sub>RIL</sub></sub> | 1     | 1     | 0     | 0     | 0     | 1                               |       |       |       |       |       |       |       |       |  |

# 1. Data Preprocessing

- data set from study population:
  - **phenotype** of genes  $T$
  - **genotype** of marker  $P$
  - **gene-to-marker** association  $A$



- **Linkage Analysis  $L$**
- **Genotype Assignment  $Q$**



# 1. Data Preprocessing

## Linkage Analysis

- identify genetic linkage of markers via **genotype-genotype correlation**  $r^{P_i P_j}$  and threshold  $d_{min}$
- if  $r^{P_i P_j} \geq d_{min}$  then  $m_j \in \mu_i$ , with  $\mu_i$  set of markers linked to marker  $m_i$

### Marker Genotypes $P$

|                                     | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ |
|-------------------------------------|-------|-------|-------|-------|-------|-------|
| RIL <sub>1</sub>                    | 0     | 0     | 1     | 1     | 0     | 1     |
| RIL <sub>2</sub>                    | 1     | 1     | 1     | 1     | 1     | 0     |
| ⋮                                   | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |
| RIL <sub><math>n_{RIL}</math></sub> | 1     | 1     | 0     | 0     | 0     | 1     |



### Linkage Map $L$

|       | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ |
|-------|-------|-------|-------|-------|-------|-------|
| $m_1$ | +     | +     | -     | -     | -     | -     |
| $m_2$ | +     | +     | -     | -     | -     | -     |
| $m_3$ | -     | -     | +     | +     | -     | -     |
| $m_4$ | -     | -     | +     | +     | -     | -     |
| $m_5$ | -     | -     | -     | -     | +     | -     |
| $m_6$ | -     | -     | -     | -     | -     | +     |

# 1. Data Preprocessing



## Genotype Assignment

- $n_m$  genotyped markers,  $n_g$  phenotyped genes for  $n_{\text{RIL}}$  RILs
- gene-to-marker association

# 1. Data Preprocessing

## Genotype Assignment

- $n_m$  genotyped markers,  $n_g$  phenotyped genes for  $n_{RIL}$  RILs
- gene-to-marker association
- assign genotype to genes from associated marker genotypes

Marker Genotypes  $P$

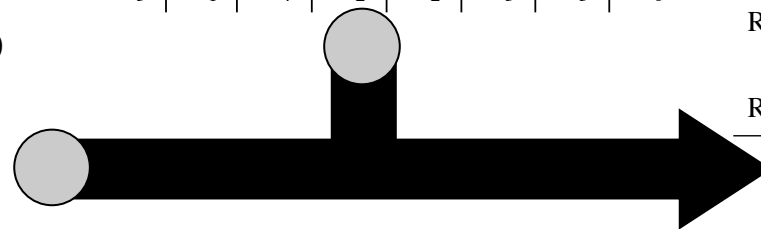
|                                     | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ |
|-------------------------------------|-------|-------|-------|-------|-------|-------|
| RIL <sub>1</sub>                    | 0     | 0     | 1     | 1     | 0     | 1     |
| RIL <sub>2</sub>                    | 1     | 1     | 1     | 1     | 1     | 0     |
| ⋮                                   | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |
| RIL <sub><math>n_{RIL}</math></sub> | 1     | 1     | 0     | 0     | 0     | 1     |

Gene-to-Marker Associations  $A$

| $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $m_3$ | $m_1$ | $m_4$ | $m_2$ | $m_2$ | $m_5$ | $m_5$ | $m_6$ |

Gene Genotypes  $Q$

|                                     | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ |
|-------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| RIL <sub>1</sub>                    | 1     | 0     | 1     | 0     | 0     | 0     | 0     | 1     |
| RIL <sub>2</sub>                    | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 0     |
| ⋮                                   | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |
| RIL <sub><math>n_{RIL}</math></sub> | 0     | 1     | 0     | 1     | 1     | 0     | 0     | 1     |



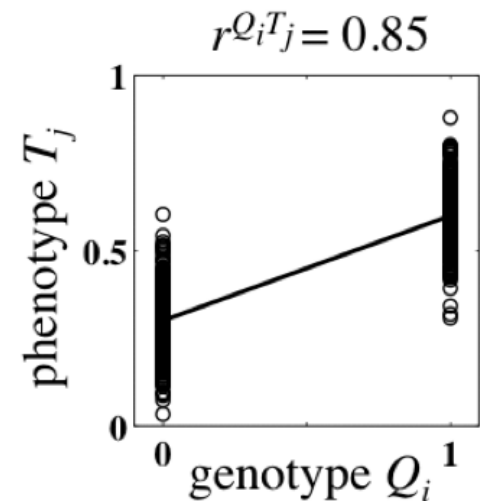
## 2. Reconstruction of Perturbation Graphs



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### Raw Perturbation Graph G1

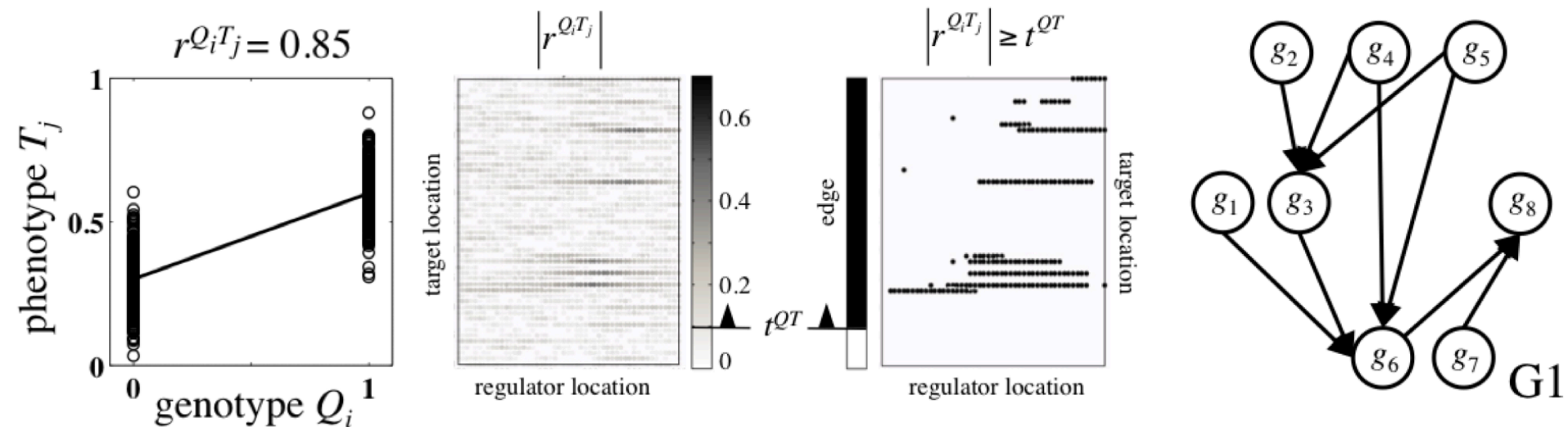
- directed **edge detection** based on **genotype-phenotype correlation** between genes and threshold  $t^{QT}$ :  $|r^{Q_i T_j}| \geq t^{QT}$



# 2. Reconstruction of Perturbation Graphs

## Raw Perturbation Graph G1

- directed **edge detection** based on **genotype-phenotype correlation** between genes and threshold  $t^{QT}$ :  $|r^{Q_i T_j}| \geq t^{QT}$



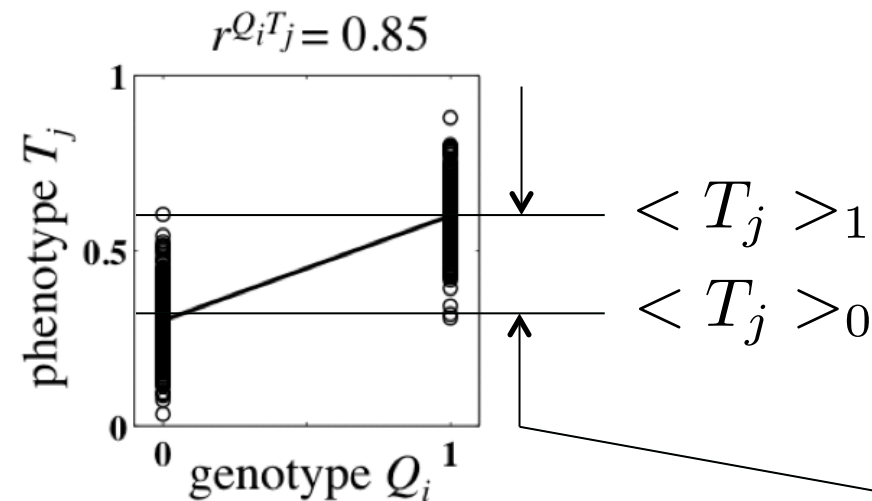
## 2. Reconstruction of Perturbation Graphs



MAX-PLANCK-GESELLSCHAFT

### Raw Perturbation Graph G1

- directed **edge detection** based on **genotype-phenotype correlation** between genes and threshold  $t^{QT}$ :  $|r^{Q_i T_j}| \geq t^{QT}$



- $r^{Q_i T_j}$  is a z-score of deviations 
$$r^{Q_i T_j} = \frac{\langle T_j \rangle_1 - \langle T_j \rangle_0}{s_{T_j} / 2}$$

## 2. Reconstruction of Perturbation Graphs



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### Raw Perturbation Graph G2

- **sign detection from pheno-phenotype correlation**  $r^{T_i T_j}$

### Gene Phenotypes $T$

|  | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ |
|--|-------|-------|-------|-------|-------|-------|-------|-------|
| RIL <sub>1</sub>                           | 0.1   | 0.7   | 1.2   | 0.3   | 0.4   | 0.2   | 1.7   | 0.6   |
| RIL <sub>2</sub>                           | 0.2   | 1     | 0.9   | 0.8   | 0.3   | 0.1   | 1.5   | 0.2   |
| ⋮  | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |
| RIL <sub><math>n_{\text{RIL}}</math></sub> | 0.7   | 1.1   | 0.1   | 1.4   | 0.6   | 0.4   | 0.4   | 1.4   |

# 2. Reconstruction of Perturbation Graphs



MAX-PLANCK-GESELLSCHAFT

## Raw Perturbation Graph G2

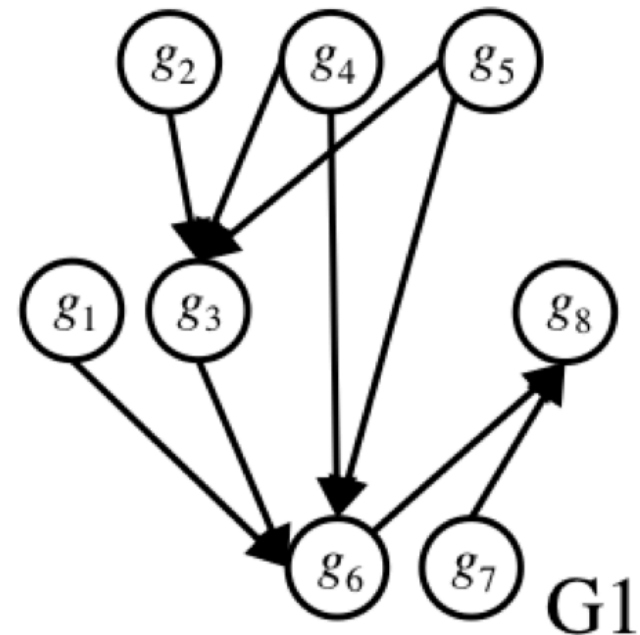
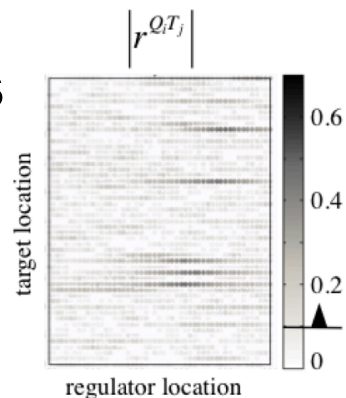
- **sign detection from pheno-phenotype correlation  $r^{T_i T_j}$**

### Gene Phenotypes $T$

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| ⋮                              | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |
| RIL <sub>n<sub>RIL</sub></sub> | 0.7   | 1.1   | 0.1   | 1.4   | 0.6   | 0.4   | 0.4   | 1.4   |

- **assign edge weights**

$$w_{ij} = \left( \left| r^{Q_i T_j} \right| + \left| r^{T_i T_j} \right| \right) / 2$$



G1



# 2. Reconstruction of Perturbation Graphs



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## Raw Perturbation Graph G2

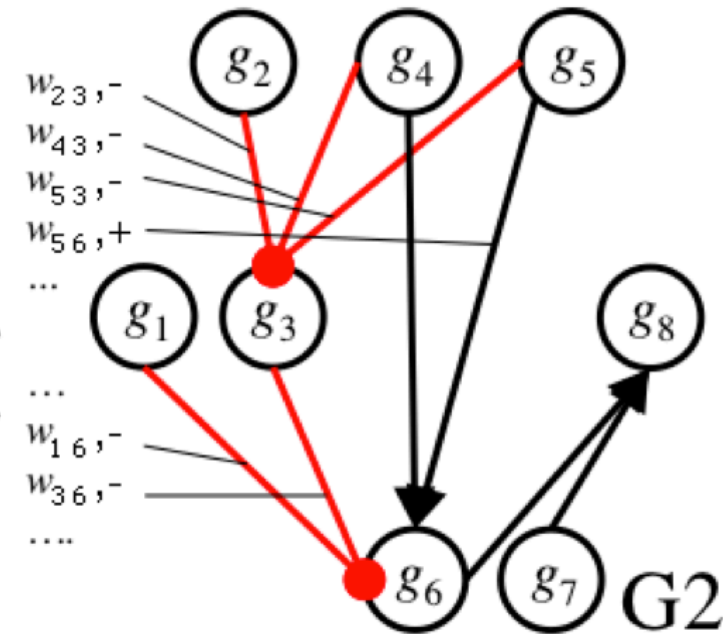
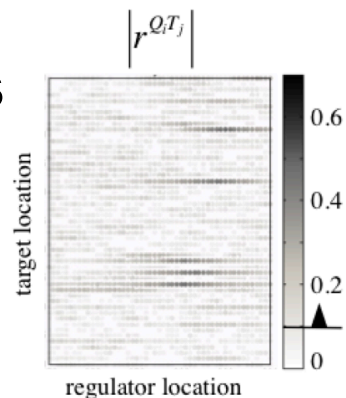
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- **assign edge weights**

$$w_{ij} = \left( \left| r^{Q_i T_j} \right| + \left| r^{T_i T_j} \right| \right) / 2$$



# 2. Reconstruction of Perturbation Graphs

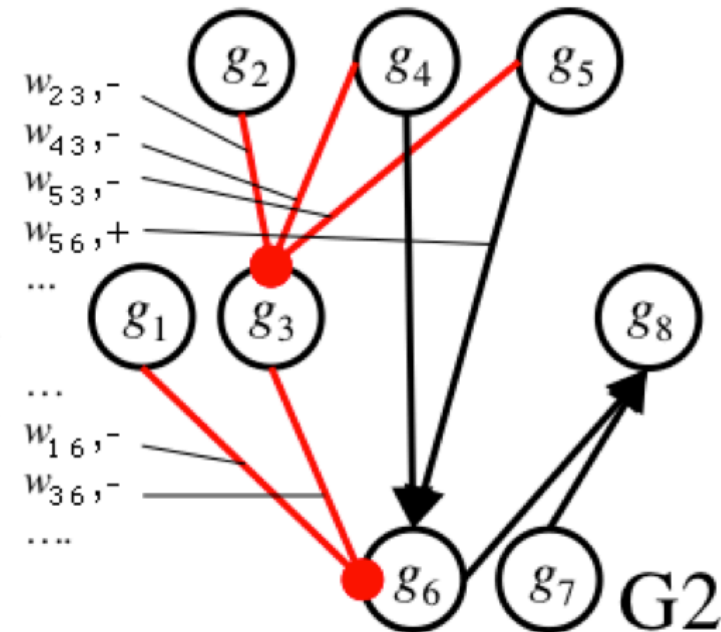
## Raw Perturbation Graph G2

= **weighted signed digraph**

eQTL Graph (G3)  
→ digraph with eQTLs

- **candidate regulator selection**, identify one regulator-target edge from each eQTL

Final Perturbation Graph (G4)  
→ weighted signed digraph



# 2. Reconstruction of Perturbation Graphs

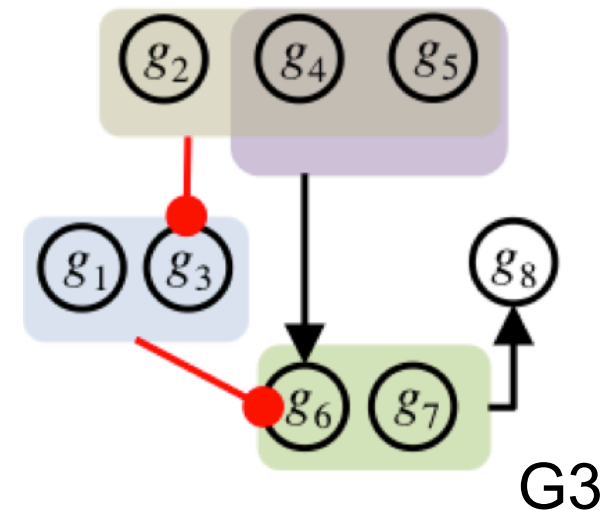


## eQTL Perturbation Graph G3

- eQTLs are derived from candidate regulators and marker linkage map
- e.g.,  $\{g_2, g_4, g_5\}$  form an eQTL for target  $g_3$ , due to linkage of their associated markers  $m_1$  and  $m_2$ .

Edge weights of regulator gene  $\rightarrow$  target gene

|       | eQTL for $g_6$ |             | eQTL for $g_3 / g_6$ |             |       | eQTL for $g_8$ |       |
|-------|----------------|-------------|----------------------|-------------|-------|----------------|-------|
|       | $g_1$          | $g_3$       | $g_2$                | $g_4$       | $g_5$ | $g_6$          | $g_7$ |
| $g_3$ | -              | -           | 0.64                 | <b>0.93</b> | 0.58  | -              | -     |
| $g_6$ | 0.41           | <b>0.92</b> | -                    | <b>0.87</b> | 0.67  | -              | -     |
| $g_8$ | -              | -           | -                    | -           | -     | <b>0.91</b>    | 0.79  |



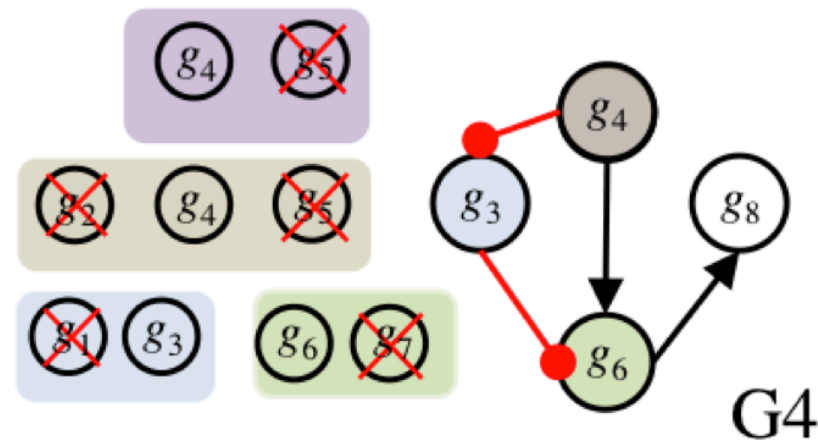
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|-------|----------------|-------------|----------------------|-------------|-------|----------------|-------|
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| $g_3$ | -              | -           | 0.64                 | <b>0.93</b> | 0.58  | -              | -     |
| $g_6$ | 0.41           | <b>0.92</b> | -                    | <b>0.87</b> | 0.67  | -              | -     |
| $g_8$ | -              | -           | -                    | -           | -     | <b>0.91</b>    | 0.79  |

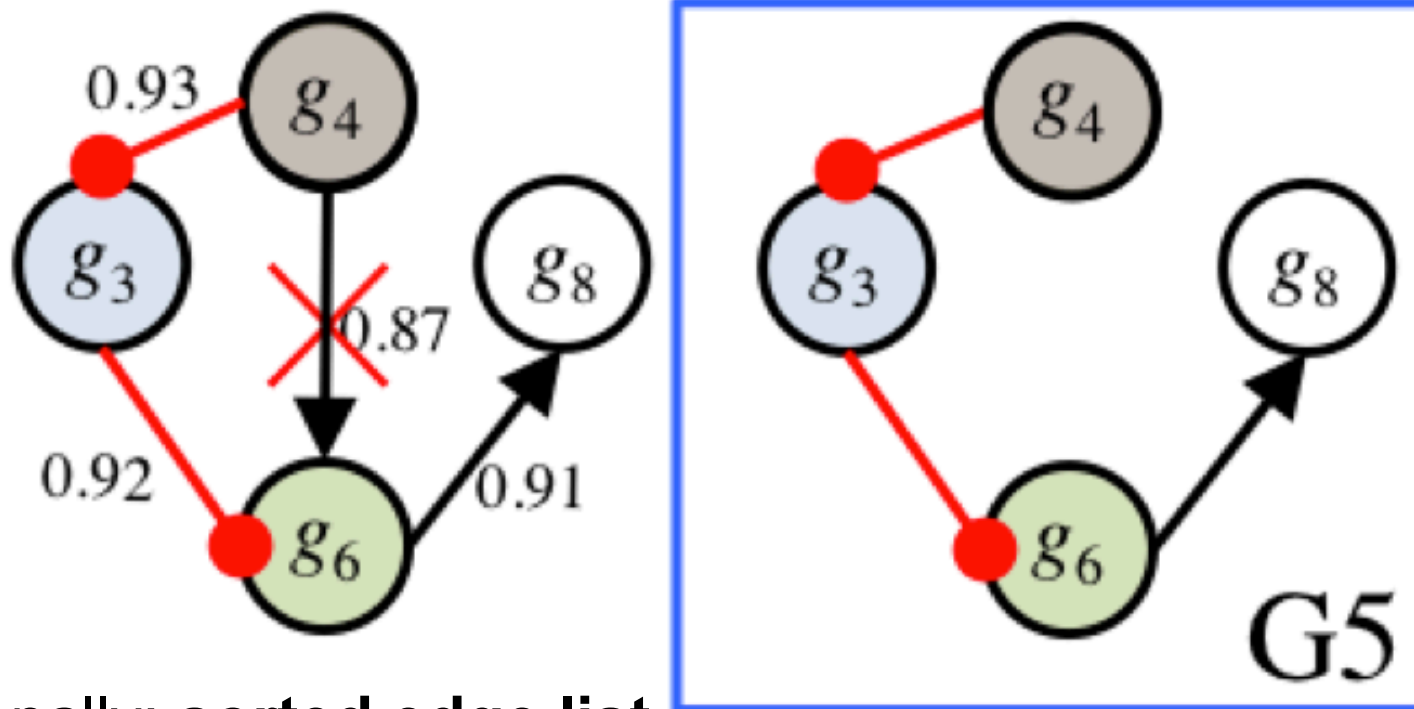


## Final Perturbation Graph G4

- from each eQTL, pick the regulator gene with highest edge weight

# 3. Transitive Reduction

- Input: final perturbation graph G4
- **remove indirect path effects** via transitive reduction using TRANSWESD → final graph G5



- optionally: **sorted edge list**

# Application DREAM 5/3A



- 100, 300, 999 RILs
- 1000 genes on 20 chromosomes

| DREAM5         | G2          | G4          | G5                 | G5*         | best performer DREAM5/3A |
|----------------|-------------|-------------|--------------------|-------------|--------------------------|
| 100/aupr auroc | 0.140 0.802 | 0.186 0.806 | <b>0.191 0.807</b> | 0.166 0.807 | 0.061 0.703              |
| 300/aupr auroc | 0.215 0.883 | 0.342 0.887 | <b>0.346 0.887</b> | 0.250 0.887 | 0.148 0.786              |
| 999/aupr auroc | 0.243 0.924 | 0.447 0.930 | <b>0.458 0.930</b> | 0.294 0.928 | 0.234 0.859              |
| 100/TP/FP      | 1138/35651  | 828/7829    | <b>765/3891</b>    |             |                          |
| 300/TP/FP      | 1682/34153  | 1328/4450   | <b>1271/3504</b>   |             |                          |
| 999/TP/FP      | 2371/51644  | 1844/3368   | <b>1734/2860</b>   |             |                          |
| 100/score      | 193.77      | 231.61      | <b>236.22</b>      | 214.89      | 81.87                    |
| 300/score      | 170.54      | 237.72      | <b>239.03</b>      | 189.42      | 89.40                    |
| 999/score      | 172.67      | 250.25      | <b>251.81</b>      | 193.49      | 140.56                   |

- G2 unpruned PG
- G4 final PG (pruned eQTLs)
- G5 final graph after TRANSWESD
- G5\* averaged over  $t^{QT}$  on [0.05...0.6]

[Flassig et al., *Bioinformatics*, 2013]

# Application DREAM 5/3A

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- 1000 genes on 20 chromosomes

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|----------------|-------------|-------------|--------------------|-------------|--------------------------|
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| 300/aupr auroc | 0.215 0.883 | 0.342 0.887 | <b>0.346 0.887</b> | 0.250 0.887 | 0.148 0.786              |
| 999/aupr auroc | 0.243 0.924 | 0.447 0.930 | <b>0.458 0.930</b> | 0.294 0.928 | 0.234 0.859              |
| 100/TP/FP      | 1138/35651  | 828/7829    | <b>765/3891</b>    |             |                          |
| 300/TP/FP      | 1682/34153  | 1328/4450   | <b>1271/3504</b>   |             |                          |
| 999/TP/FP      | 2371/51644  | 1844/3368   | <b>1734/2860</b>   |             |                          |
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- much more effective in terms of AUPR and AUROC at all sample sizes
- especially at small sample sizes good performance wrt. best performer DREAM5/3A

[Flassig *et al.*, *Bioinformatics*, 2013]

# Application *S. cerevisiae*



- data from 112 segregants obtained from a yeast cross (Brem and Kruglyak, PNAS, 2005)
- only 1573 of all 2956 markers were associated to at least one of the 5736 expression-profiled genes
- much less data than in DREAM5/3A
- compare to DREAM5/4.4 submissions

|                   | G2                   | G4                    | G5                    |
|-------------------|----------------------|-----------------------|-----------------------|
| aupr/paupr/rank   | 0.0274 / 5.7e-11 / 4 | 0.0293 / 2.34e-14 / 3 | 0.0293 / 1.89e-14 / 3 |
| auroc/pauroc/rank | 0.5396 / 6.7e-28 / 1 | 0.5407 / 6.14e-30 / 1 | 0.5407 / 6.4e-30 / 1  |

- good performance for 112 samples vs. 536 microarrays with well defined perturbations

[Flassig *et al.*, *Bioinformatics*, 2013]



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## References

- Flassig RJ, Heise S, Sundmacher K, Klamt S (2013) An effective framework for reconstructing gene regulatory networks from genetical genomics data. *Bioinformatics* 29 (2): 246-254
- Klamt S, Flassig R, Sundmacher K (2010) TRANSWESD: inferring cellular networks with transitive reduction. *Bioinformatics* 26, 2160-2168
- Brem RB and Kruglyak L. (2005) The landscape of genetic complexity across 5,700 gene expression traits in yeast. *PNAS* **102**(5), 1572-1577