Soft constraints: Polynomial classes, Applications

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Polynomial classes

- structural classes: when the constraint (hyper)-graph has good properties.
- microstructural classes: when the constraints have good properties.

Structural polynmial class: inherited by VE/BBE, problem with a tree-structured graph or more generally a partial k-tree structured graph with k bounded.

Idempotent VCSP: fuzzy CSP

The α-cut result...

Any fuzzy CSP with can be solved in $O(log(ed))$ calls to a classical CSP solver.

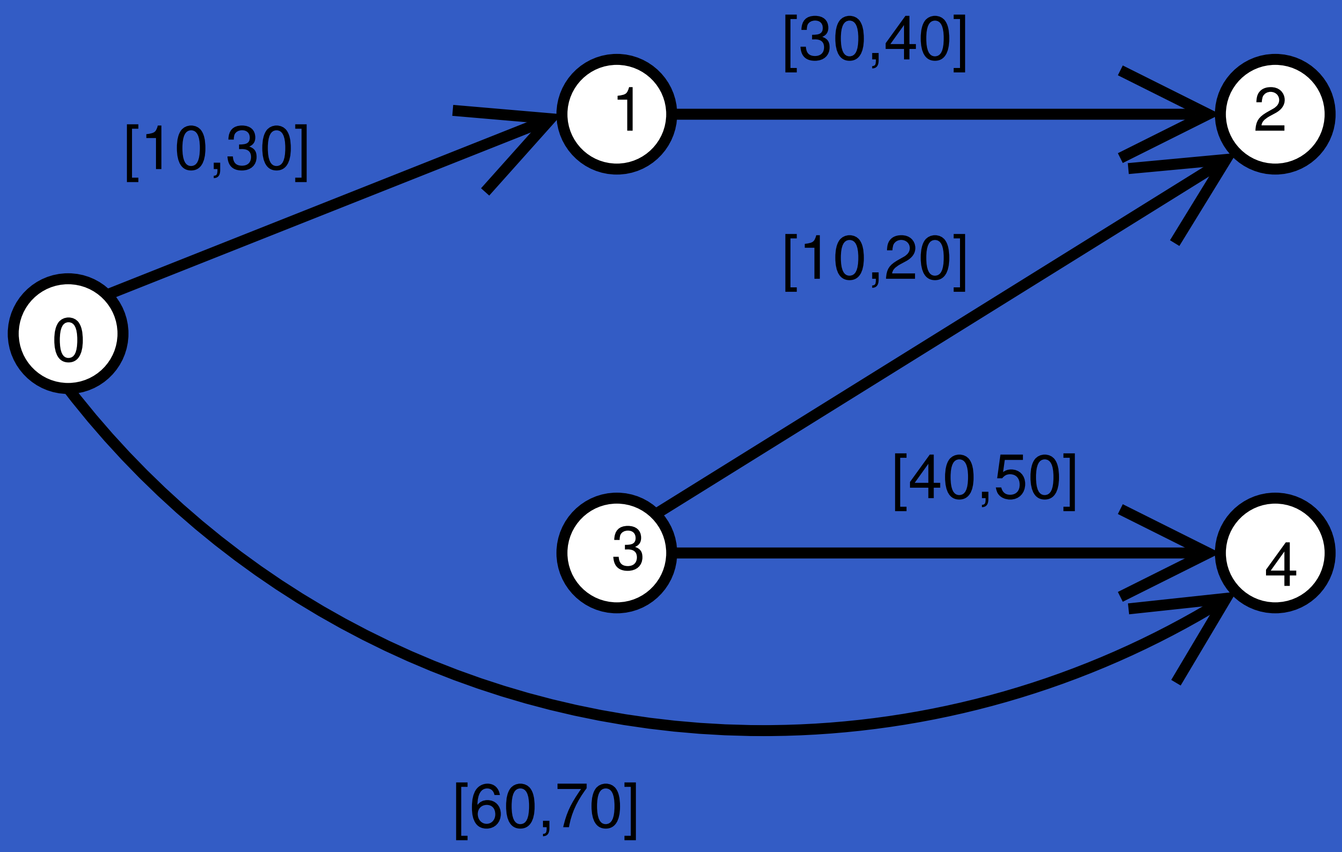
All classical CSP polynomial classes that are not affected by α-slicing are polynomial time classes for fuzzy CSP.

Temporal CSP

- each variable x_i represents a time point.
- each constraint is a set of intervals $[a, b]$.
  - T_i unary: restricts the domain to the union of the intervals
  - T_{ij} binary:restricts the distance $x_j - x_i$ to the union of the intervals

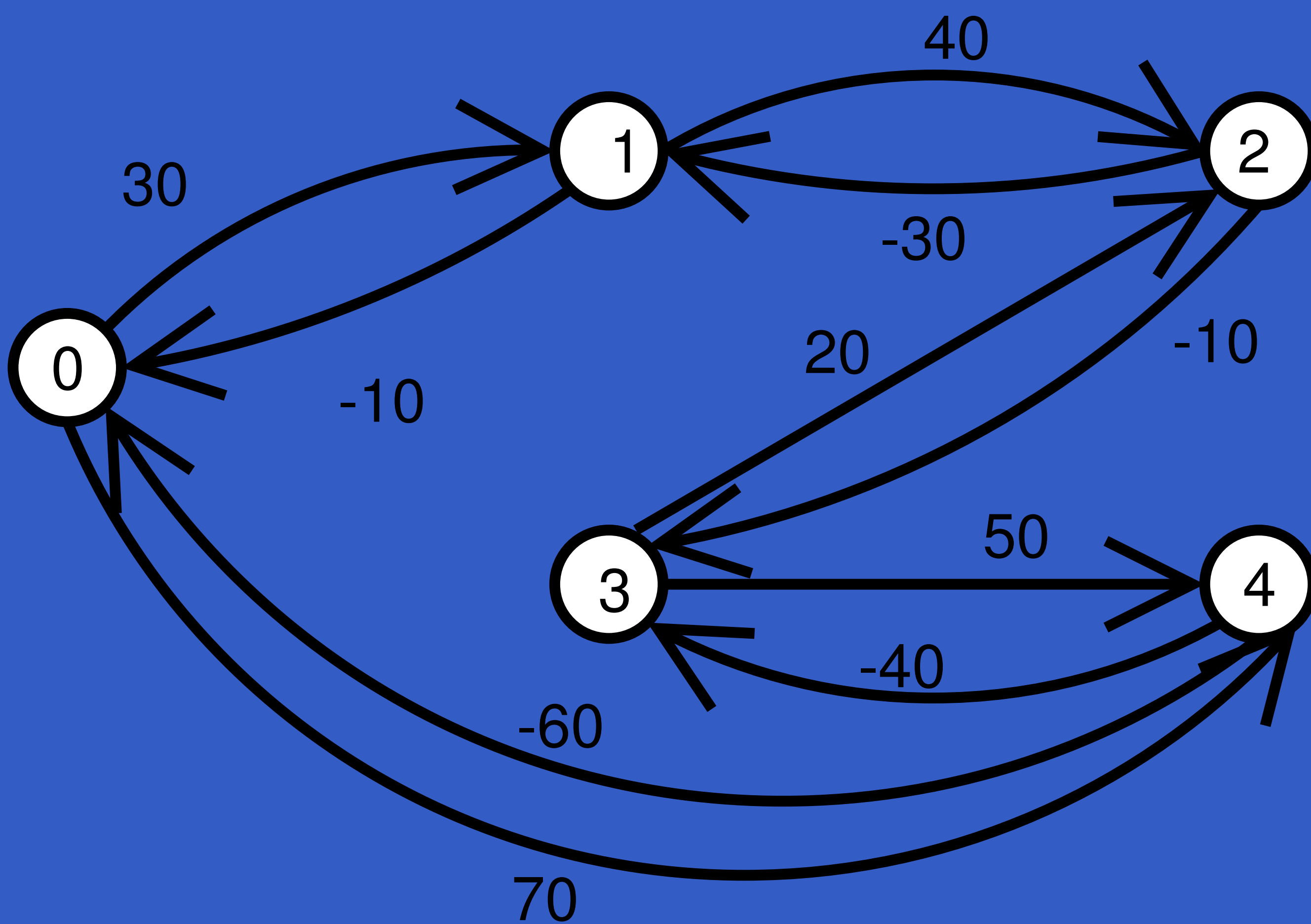
Can be represented as a directed graph with labelled vertices and edges. NP-complete. STCSP: one interval in each constraint. Polynomial time solvable.

The directed graph of a problem



The distance graph

$$l \leq x_j - x_i \leq u \Leftrightarrow (x_j - x_i \leq u) \wedge (x_i - x_j \leq -l)$$



Properties

- each path from i to j: $x_j - x_i \leq$ path length.
- $x_j - x_i \leq d_{ij}$ where d_{ij} is the shortest path from i to j.

A STCSP is consistent iff there is no negative (absorbing cycle).

Computing all pairs shortest path solves the problem completely (Floyd-Warshall, detects neg. cycles, $O(n^3)$).

Fuzzy TCSP

For temporal problems with preferences.

- each variable x_i has a continuous time domain (time point)
- each constraint is a fuzzy subset of $\mathbb{R}$.
  - T_i unary: restricts the domain to the fuzzy set. intervals
  - T_{ij} binary:restricts the distance $x_j - x_i$ to the fuzzy set.

Optimal assignment: NP-hard. Pol. class ?

Simple Fuzzy TCSP

A fuzzy TCSP is simple iff every α cut it is a simple TCSP.

$\Leftrightarrow$
Every α-cut of the sets is an interval

$\Leftrightarrow$
Every fuzzy set in the network is a semi-convex function.

$\log(nbits)$ STCSP problems to solve is enough.

Tractable languages

Imagine we have a set L of allowed soft constraints for a given c-semiring.

We will say that L is a tractable language if any soft CSP built from constraints in L is tractable (the optimal assignment cost can be computed in pol. time).

Previous result: the language of semi-convex temporal constraints is tractable in fuzzy CSP. In non idempotent structures?

Existing results on Max-CSP

- $d = 2$: Max-Sat. Precisely three tractable languages. The language of c_{xor} is NP-hard.
- Max-CSP: d may be larger than $2\ldots$

The langage of binary soft equality

$$c_{eq}(x, y) = \begin{cases} 0 & \text{x = y} \\ 1 & \texttt{otherwise} \end{cases}$$

is NP-hard.

Reduction from min. 3-terminal cut

Min. 3-terminal cut: an undirected (weighted) graph $G = (V, E)$. Three distingued vertices $\{v_1, v_2, v_3\}$. Is there a set of edges of minimum weight whose removal disconnects each pair of terminals.

One variable per vertex, 3 values. One constraint c_{eq} per edge. One unary constraint per terminal:

$$c_{v_i}(x) = \begin{cases} 0 & : \quad x = i \\ |E| + 1 & : \quad \texttt{otherwise} \end{cases}$$

Generalized interval functions

Domain D ordered.

$$c^{\rho}_{[a,b]}(x,y) = \begin{cases} 0 & : \quad (x < a) \vee (y > b) \\ \rho & : \quad \texttt{otherwise} \end{cases}$$

The langage of GI functions is tractable.

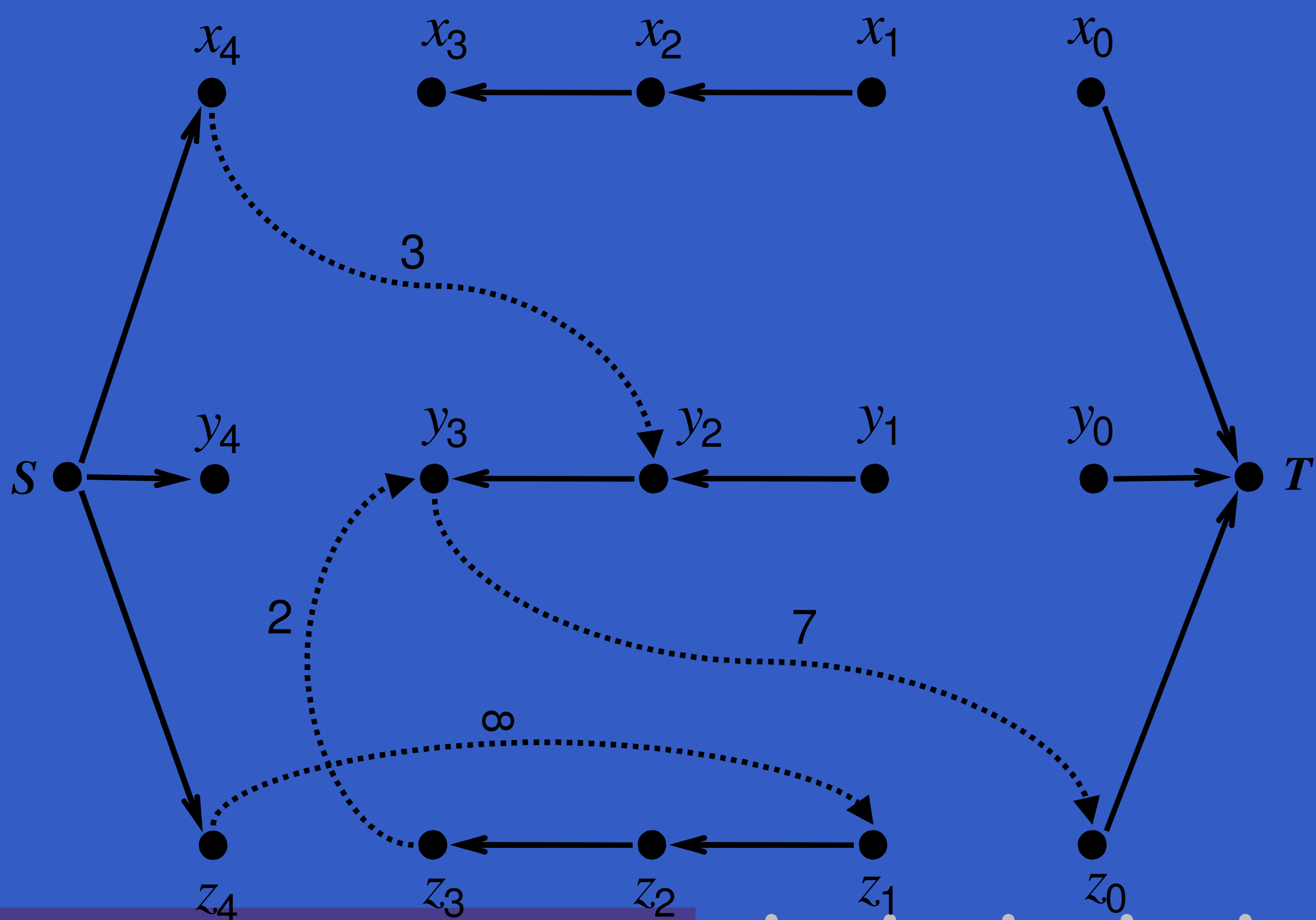
Tractability

$P = \langle X, D, C\rangle$ a maxCSP with $D_i = \{1, \ldots, M\}$.
$G = (V, E)$ with:

- $V = \{S, T\} \cup \{x_{id} \mid x_i \in X, d \in D\{1, \ldots, M\}\}$.
- for each $x_i \in X$, an edge from S to x_{iM} weight ∞
- for each $x_i \in X$, an edge from x_{i0} to T, weight ∞
- for each $x_{id} \in V, d \in [1, M-2]$, an edge from x_{id} to x_{id+1} with weight ∞.
- for each constraint $c^{\rho}_{[a,b]}(x_i, x_j)$ an edge from x_{jb} to x_{ia-1} with weight ρ (c-edges).

Example

$X = \{x, y, z\}, dom\{1, 2, 3, 4\}, C = \{c^3_{[3,4]}(y, x), c^7_{[1,3]}(z, y), c^2_{[4,3]}(y, z), c^\infty_{[2,4]}(z, z)\}$



Main results

A minimal $S - T$ cut that contains only c-edges is a proper cut $(\{\langle y_3, z_0 \rangle\}, \{\langle x_4, y_2 \rangle, \langle z_3, y_3 \rangle\})$.

For each minimal proper cut of weight Φ, there is an assignment of cost Φ and vice-versa.

Here: $\{\langle y_3, z_0 \rangle\}$ has weight 7, $\{\langle x_4, y_2 \rangle, \langle z_3, y_3 \rangle\}$ has weight 5. Both minimal.

Proof (consider Cut $\{\langle y_3, z_0\rangle\}$)

$\Rightarrow$: C_S the component connected to S.

Consider t that assigns each var. x_i to its minimum value d_i s.t. $x_{id_i} \in C_S$. $t = \langle x = 4, y = 2, z = 1\rangle$.

Note that $f < t(x_i) \Leftrightarrow x_{if} \notin C_S$.

$c^{\rho}_{[a,b]}(x_i, x_j)$ is violated by
$t \Leftrightarrow (t(x_i) \geq a) \wedge (t(x_j) \leq b) \Leftrightarrow (x_{ia-1} \notin C_S) \wedge (x_{jb} \in C_S)$.

The edge connects C_S and C_T and must be in the cut.

Consider assignment t and the edges defined by the constraints violated by t. $t = \langle x = 4, y = 2, z = 1\rangle$.

Consider a $S - T$ path and imagine all constraints on the path are satisfied. $\langle S, x_4, y_2, y_3, z_0, T\rangle$

$$(x_{i_0} > M) \vee (x_{i_1} < a_1)$$
$$(x_{i_1} > b_2) \vee (x_{i_2} < a_2) \qquad b_2 \geq a_1$$
$$\ldots$$
$$(x_{i_k} > b_{k+1}) \vee (x_{i_{k+1}} < 1) \qquad b_{k+1} \geq a_k$$

One must be violated. Violated constraints define a cut and must all appear in it. Its weight is the assignment cost.

Extends to submodular functions

A function such $\forall x, y, u, v, u \leq x, v \leq y$, we have:

$$c(u, v) + c(x, y) \leq c(u, y) + c(x, v)$$

A submodular fonction cost matrix decomposes in a sum of GI functions.

$$ax + by + c, \sqrt{x^2 + y^2}, ||x - y|^r (r \geq 1), max(x, y, 0)^r (r \geq 1)$$

This class is maximal.

Q: link with semi-convex fuzzy temporal functions submodular.

RNA secondary structure prediction

RNA is a single strand molecule composed of A,U,G,C.

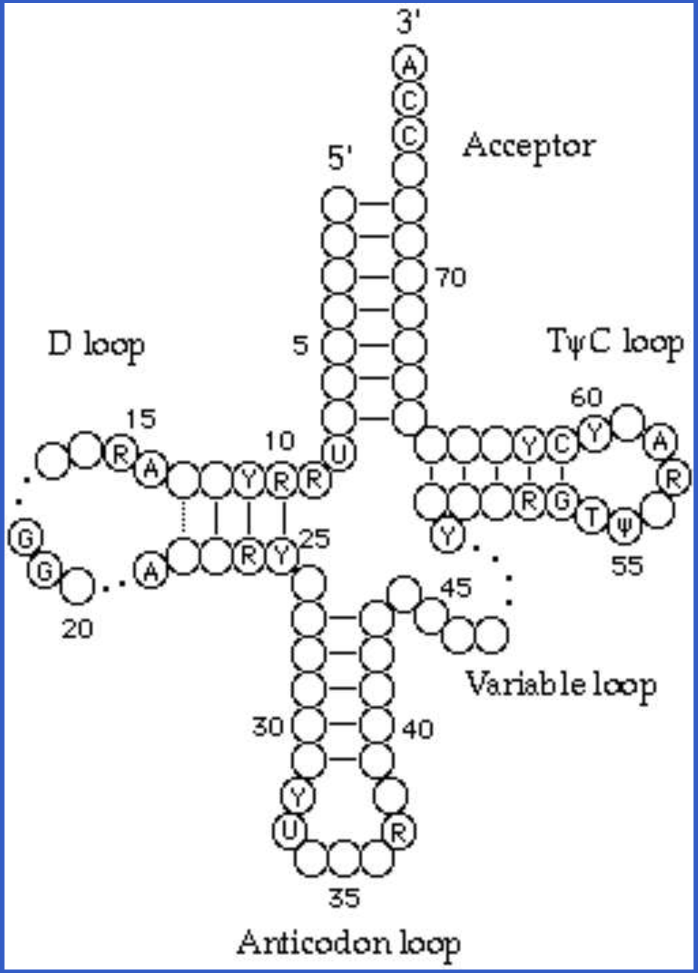
Functional RNA are structured (3d structure). Structure is related to function.

The structure is induced by base pairing: Watson-Crick (A-U,G-C) and Wobble (G-U).

Secondary structure: set of all Watson-Crick and Wobble base pairs.

Problem: determine the secondary structure of an RNA molecule from a single sequence.

A transfert RNA



RNA secondary structure prediction

Other sources of information:

- thermodynamics.
  Zuker's algorithm: DP algorithm that finds an optimal secondary structure. Pb: thermodynamics is not precise enough.

  McCaskill matrix: given an RNA sequence, computes the probability that a given base is paired to another given base (based on thermodynamics).
- biological knowledge: one may know/test that a given base is paired or not, is paired to a given other base.

A CSP model (C. Gaspin, 1995)

For a sequence of length $n = (b_1, \ldots b_n)$:

- one variable x_i per base
- domains: $d_i = \{1, \ldots, n\}$. $b_i = i$ means b_i unpaired.
- constraints: Watson-Crick/Wobble only.

  $x_i = j \Leftrightarrow x_j = i$

  No pseudo-knot: for $i < j, k < l, (j, l)$ is forbidden for x_i, x_k if $i < k < j < l$ or $k < i < l < j$.

  Many other constraints. . .

  Experimental knowledge: a base is unpaired, is paired, with a specific base...

Usually too many solutions. Need more information.

Exploiting thermodynamics

McCaskill matrix $P(i, j)$ probability that b_i is paired with b_j.

For algorithmic reasons (satisfaction problem):

- fix a threshold p.
- forbid all pairs $b_i = j$ such that $P(i, j) < p$.

Poor handling of probabilities, Choice of p...

Enforce arc consistency, then solve as a Max-CSP with unary soft constraints (maximize the number of paired bases).

Satellite scheduling

- var/dom: a set S of pictures. Each picture can be taken at different time points.
- binary constraints: only three instruments are available and each picture requires some instruments with possible transition times for reconfiguration.
- ternary constraints: the data bus bandwidth is limited.
- global constraint: the local memory is limited.

Overconstrained: instanciate a subset of S which maximizes the sum of the weights of the pictures (and satisfies all constraints).

RDS (no global constraint)

val = # of pictures, $*$ = optimality proof (within 30')

pb	n	e	FC	(cpu ")	RDS	(cpu ")
404	100	610	48	1800	49*	0.5
408	199	2032	3076	1800	3082*	14
412	300	4048	15078	1800	16102*	29
414	364	9744	21096	1800	22120*	86
503	105	403	8095	1800	9096*	2.5
505	240	2002	12088	1800	13100*	15
507	311	5421	12110	1800	15137*	55
509	348	8276	19104	1800	19125*	106

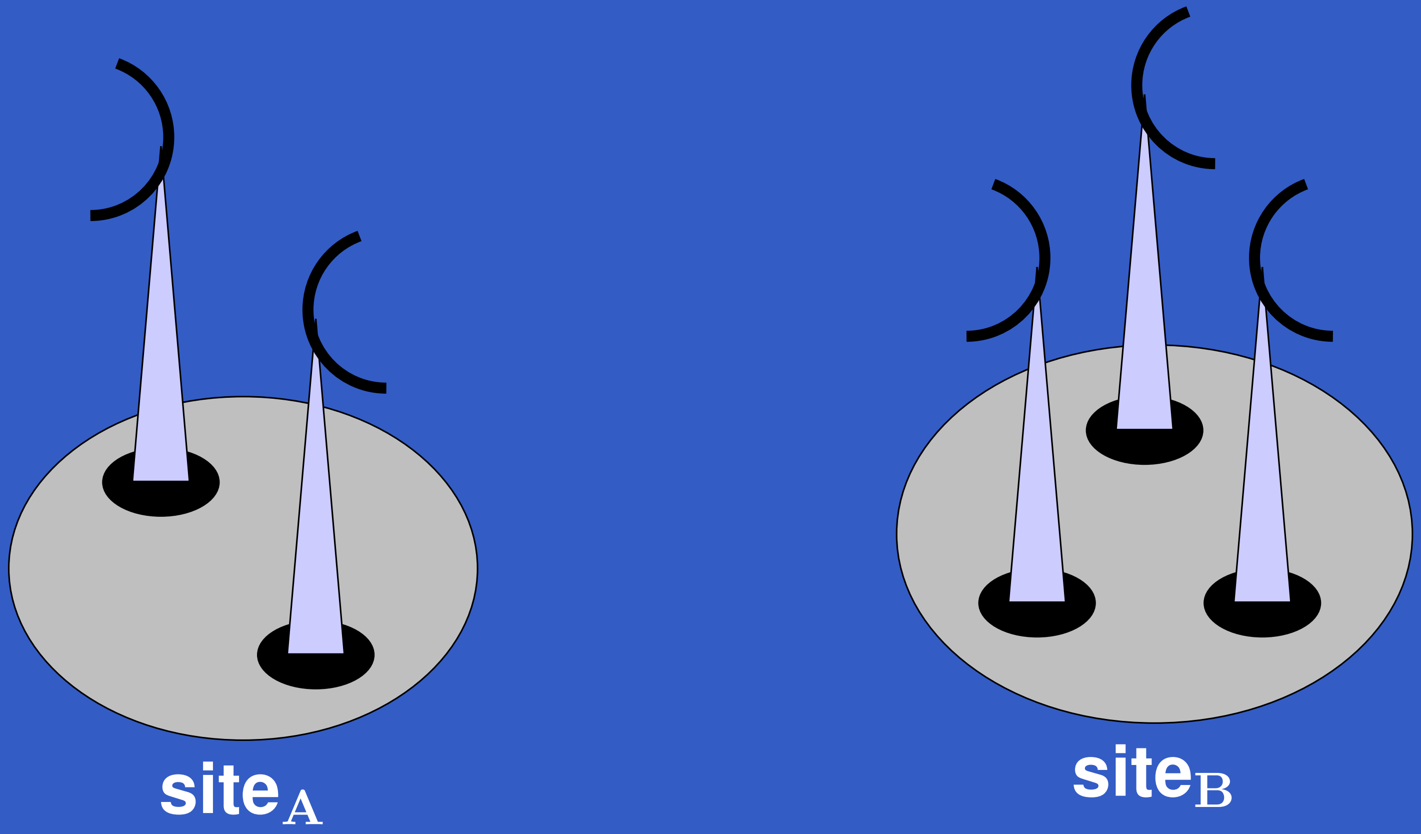
Satellite scheduling

Beyond RDS, these instances have been tackled by several approaches:

- local search: taboo search
- LP + column generation: to provide global lower bounds
- 0/1 LP: as a multidimensional Knapsack (MKP01), to provide global lower bounds

The MPK01 model is solved to apparent optimality by CPLEX 7.0 (but with float tolerance problems) on most instances. Cpu-time may reach 5.10^4 sec. on a modern Pentium machine and may violate known lower bounds.

Frequency assignment (CELAR)



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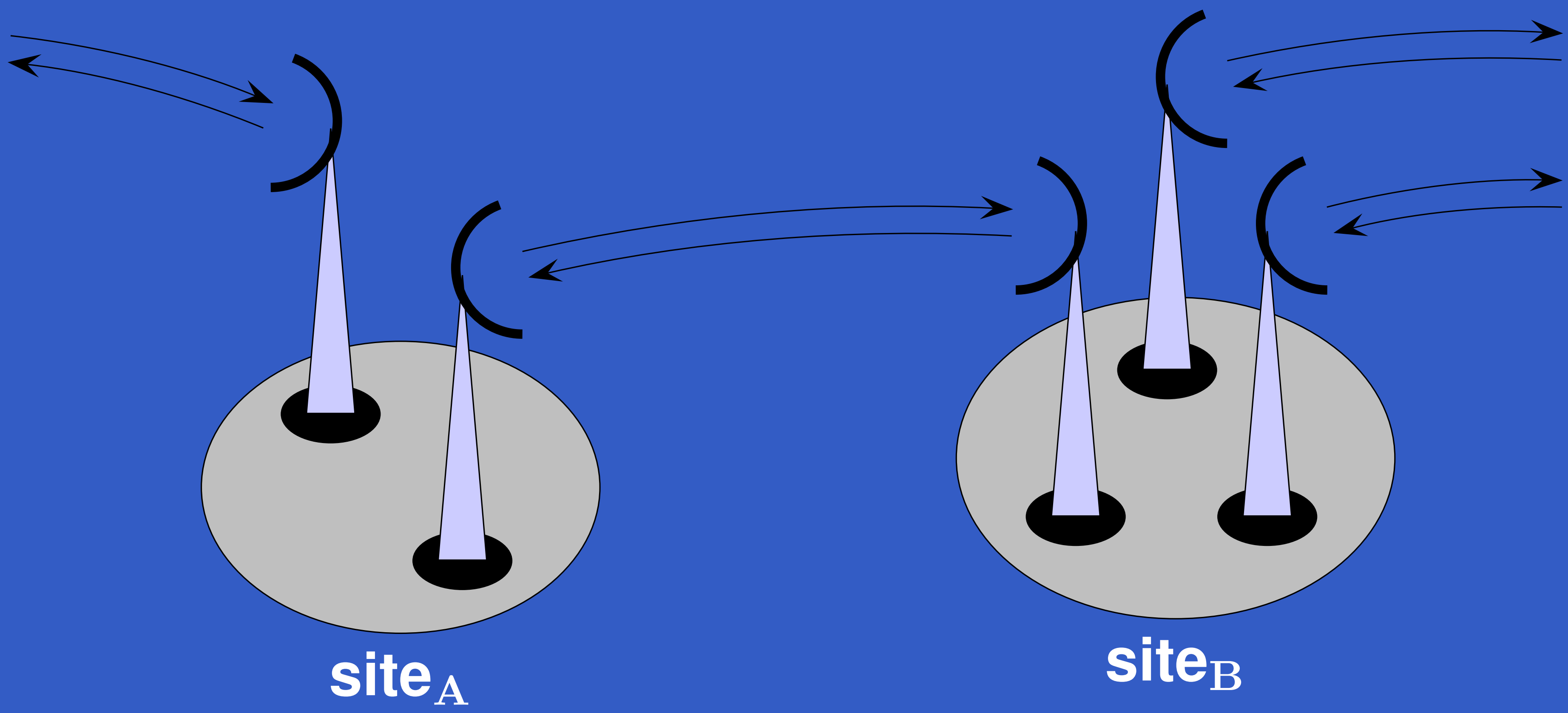

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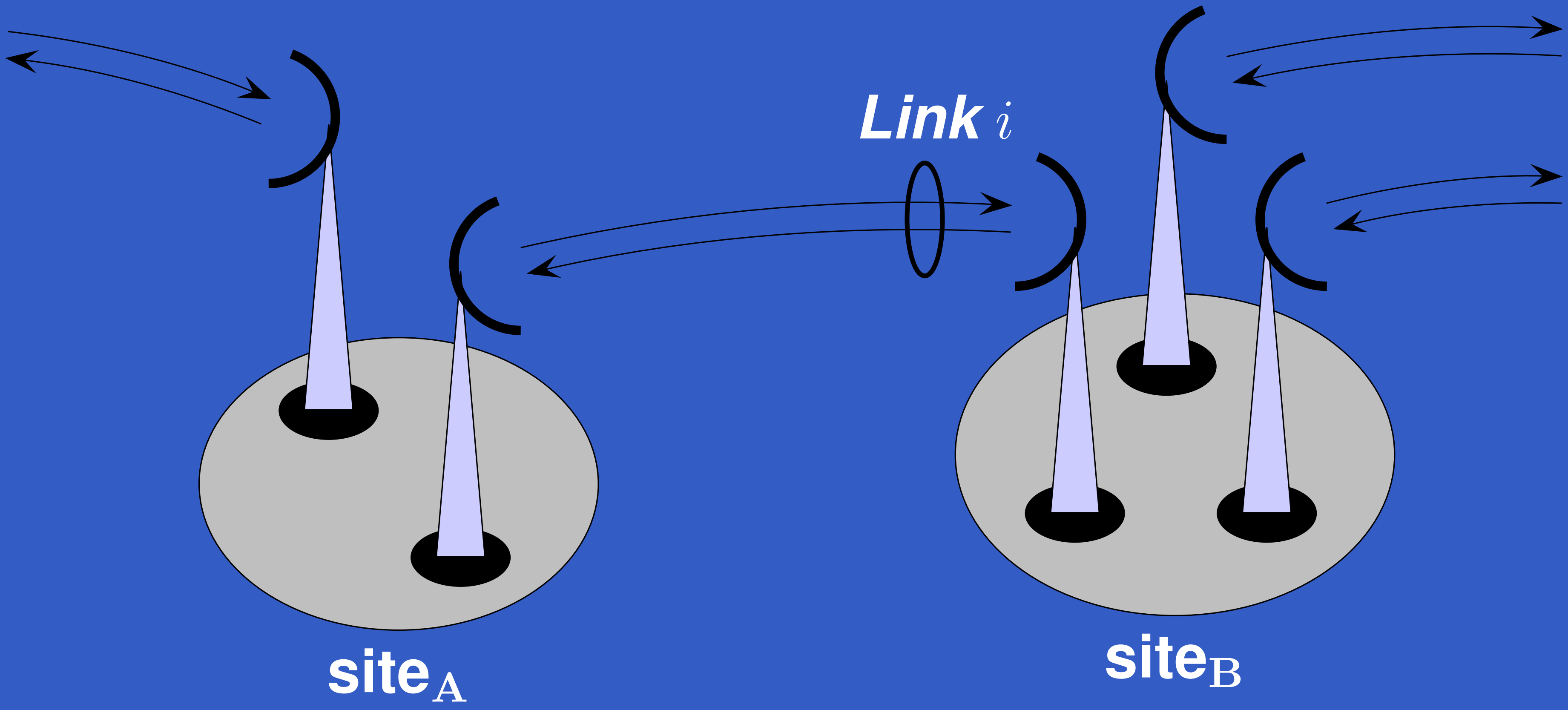

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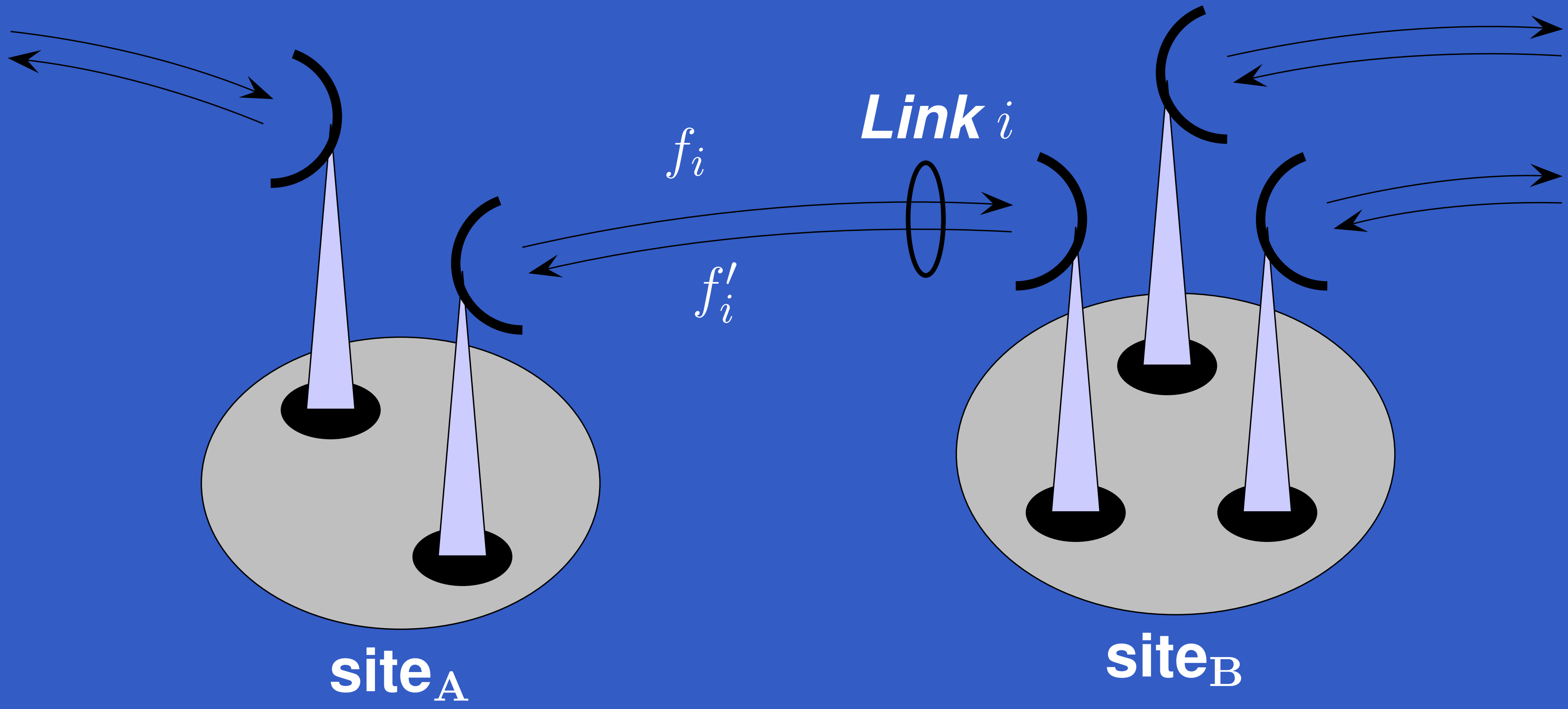

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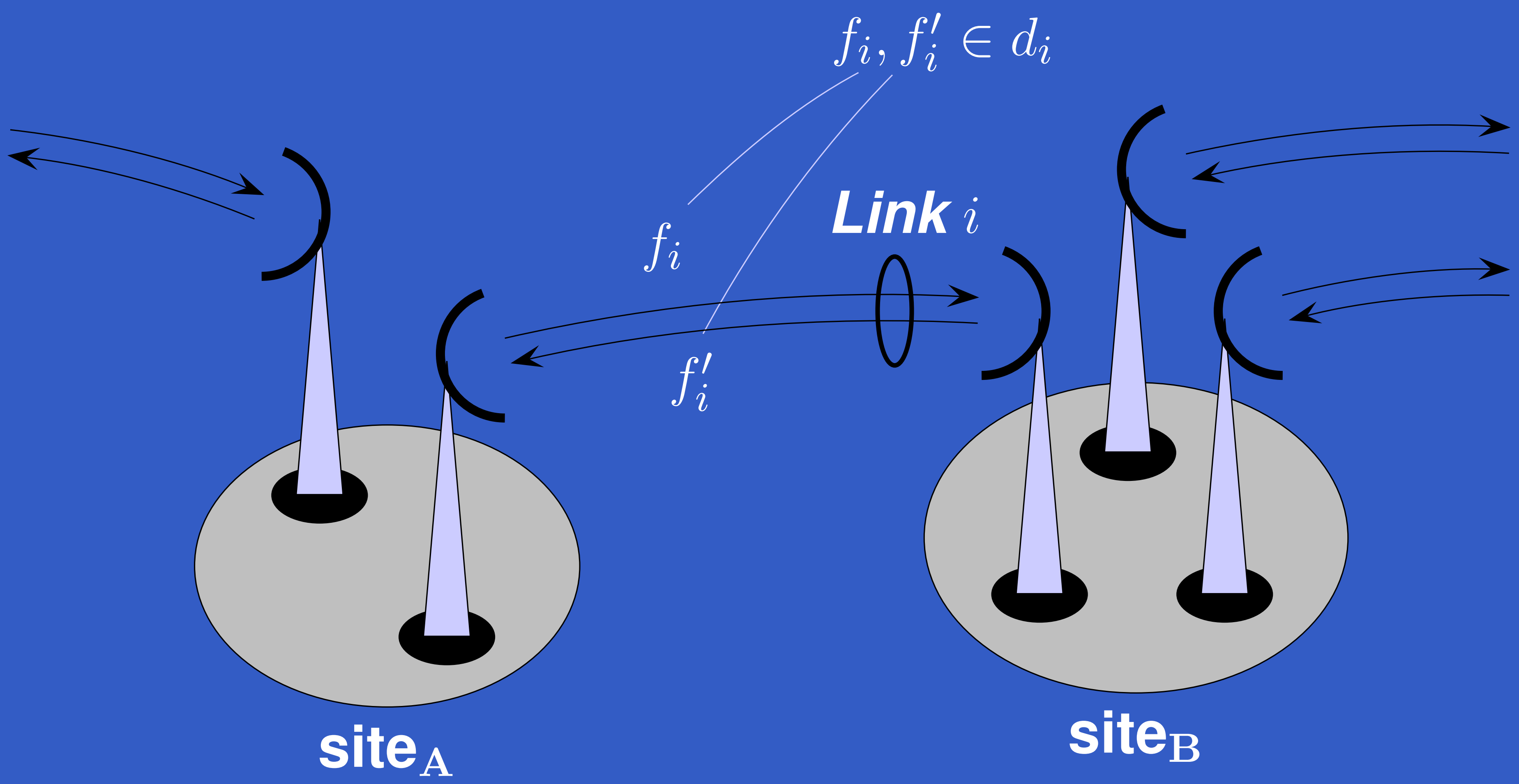

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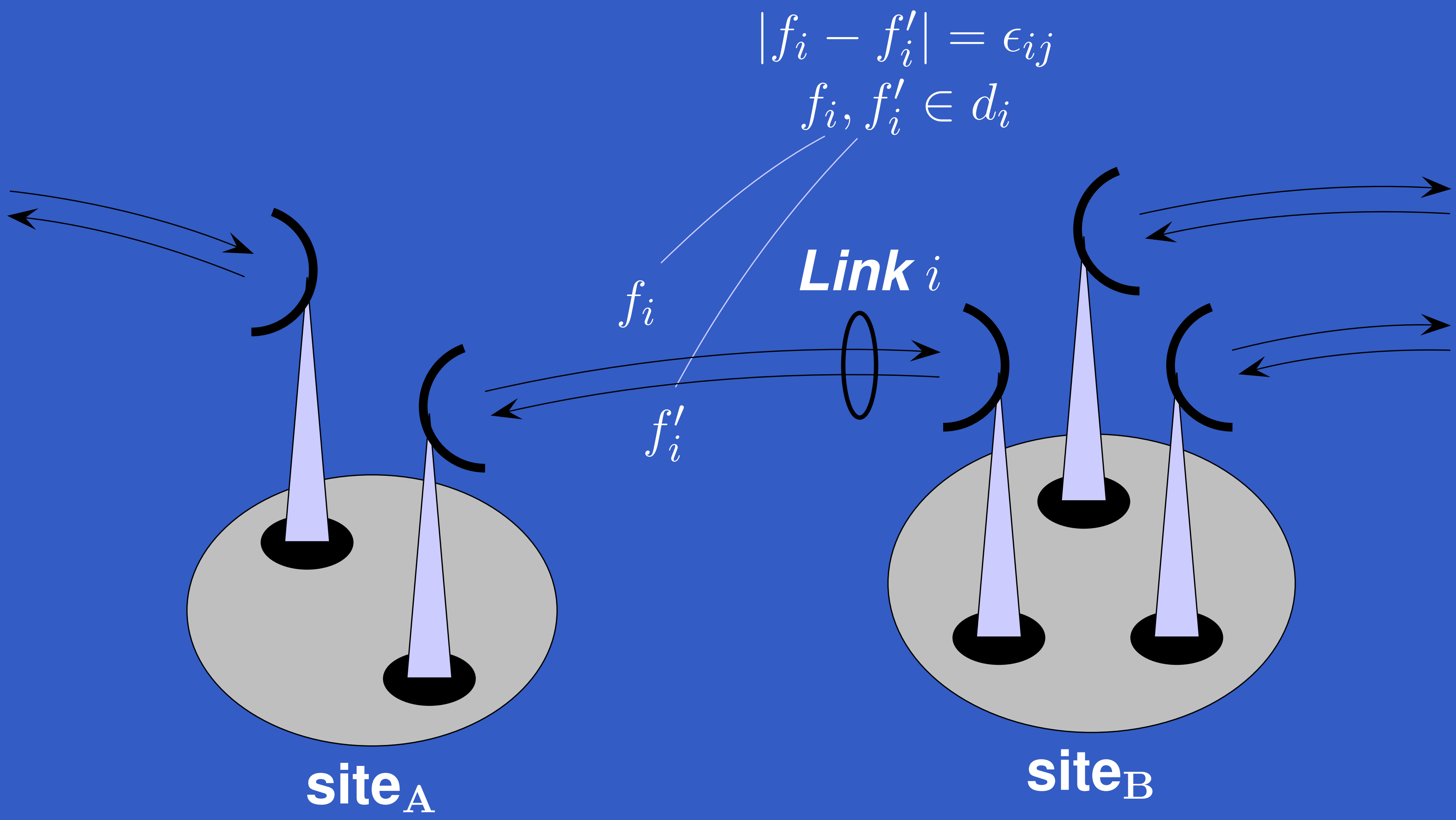

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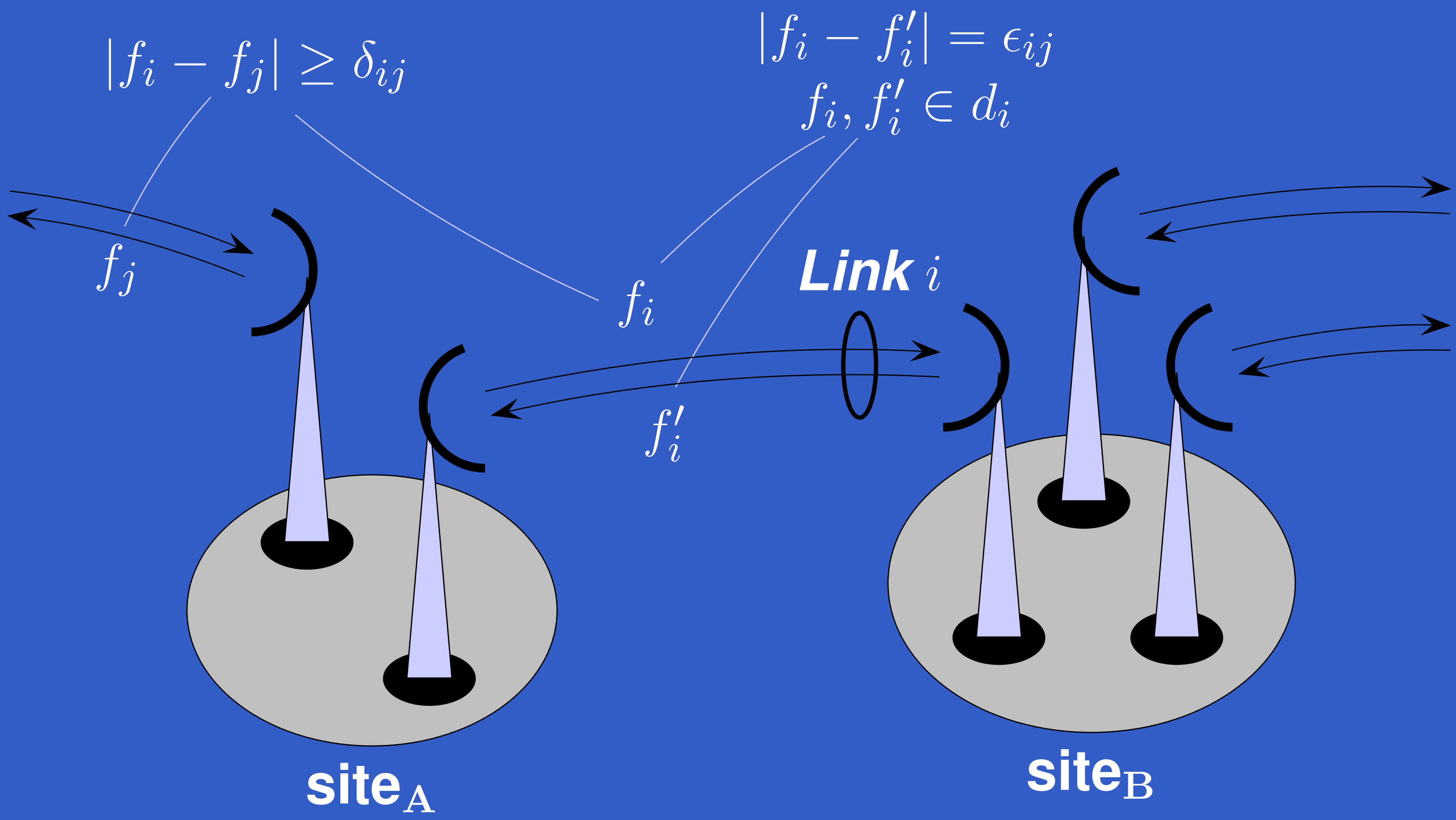


FAP - criteria

- minimize the maximum frequency used (possibilistic CSP)
- minimize the number of frequencies used (optimisation/global soft constraint)
- minimize the weighted constraint violation (Max-CSP)

Several instances available: from 200 to 916 vars, from 1200 to more than 5000 binary constraints. Domains usually have more than 30 values.

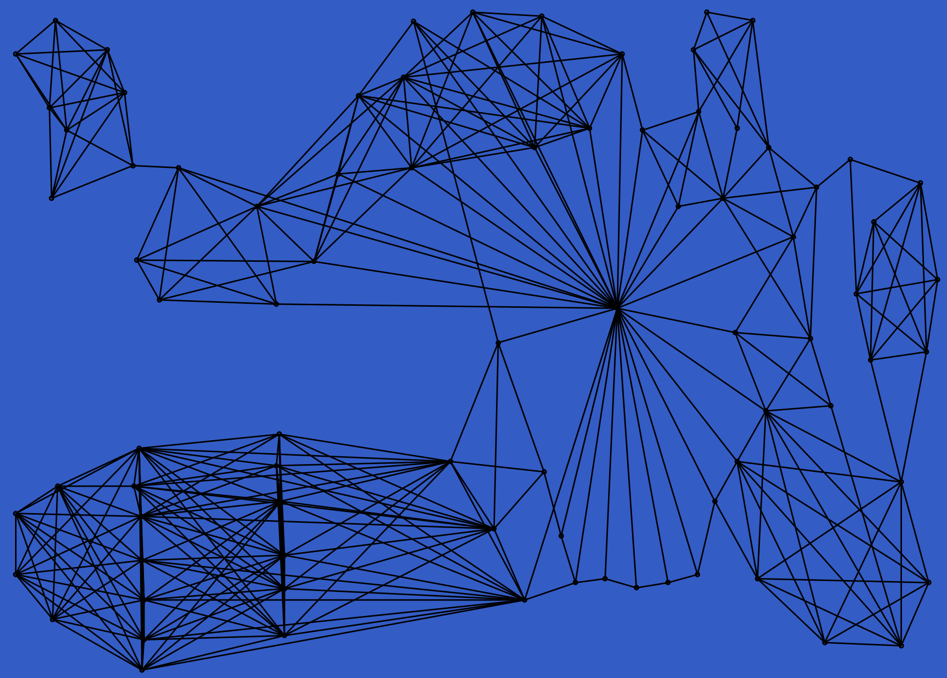
FAP: results

Tackled in the CALMA project (1994) and then by individuals. Most problems solved to optimality. . .

Max-CSP problems are very hard (even for local search). No proof of optimality after CALMA.

- 1997: graph decomposition + RDS proved optimality of Celar06 $(5.10^6$ sec., Sparc 5). PFC-MRDAC (2.610^5 sec, Sparc 2).

A nice approach: DP



- 1999: preprocessing + non serial dynamic programming + a lot more: solves most instances to optimality (Arie Koster, PhD thesis).

Conclusion

Soft constraint technology is still in its enfancy.
There is much to do:

- use existing frameworks to build more realistic models for existing problems, that may exploit recent algorithms (eg. bucket elimination, PFC-MRDAC. . .)
- improve algorithms for solving existing models in existing frameworks:
  - stronger preprocessing
  - global soft constraints
  - combination of bucket elimination, branching and local consistency or other preprocessing.

Existing implementations (I know...)

- Con'Flex: Conjunctive fuzzy CSP system with integer, symbolic and numerical constraints (`www.inra.fr/bia/T/conflex`).
- clp(FD,S): semi-ring CLP. (`pauillac.inria.fr/~georget/clp_fds/clp_fds.html`).
- LVCSP: Common-Lisp library for Valued CSP with an emphasis on strictly monotonic operators (`ftp.cert.fr/pub/lemaitre/LVCSP`).
- Choco: a claire library for CSP. Existing layers above Choco implements Weighted Max-CSP algorithms (part of LVCSP, (`www.choco-constraints.net`).
- toolbar: C library for MaxCSP and related problems (`carlit.toulouse.inra.fr/cgi-bin/awki.cgi/ToolBarIntro`).