

WEIGHTED CP & RELATED FRAMEWORKS

SOFT LOCAL CONSISTENCIES AS BOUNDS

DAGSTHUL 2025



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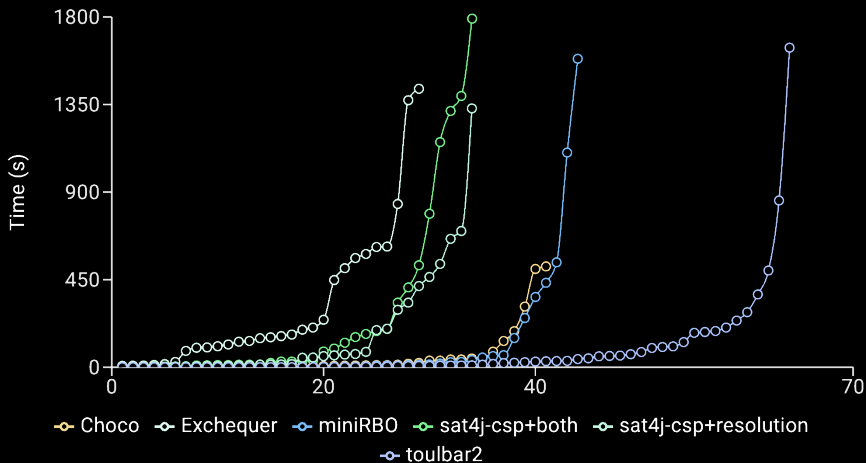
MAY 5, 2025



SOME MOTIVATION FIRST?

XCSP3 MiniCOP track

Problems solved with a proof of optimality



A Constraint Network $\langle V, \Phi \rangle$

- a sequence of discrete domain variables V
- a set Φ of e Boolean functions (or constraints)
- Each $c_S \in \Phi$ is a truth function from $D^S \rightarrow \{t, f\}$

Joint truth function

$$\Phi_{\mathcal{M}} = \bigwedge_{c_S \in \Phi} c_S$$

The Constraint Satisfaction Problem (NP-complete)

- Is it possible to have $\Phi_{\mathcal{M}} = t$?

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Languages for domains and constraints

- Constraint Networks: Boolean tables (tensors) for domains and constraints
- SAT/CNF: Boolean variables and clauses
- Constraint Programming: interval domains, global constraints,...

Tables (or tensors) for c_S

- A multidimensional table with a Boolean for every tuple in D^S
- Says if it is authorized (t) or not (f)

Pairwise difference (3 values)

$$\begin{bmatrix} f & t & t \\ t & f & t \\ t & t & f \end{bmatrix}$$

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Beyond dense tensors

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Most famous

`ALLDIFFERENTS`

Cost Function Network $\langle V, \Phi, \top \rangle$

- a sequence of discrete domain variables V
- a set Φ of e **non negative** integer cost functions
- each $c_S \in \Phi$ is a numerical function bounded by $\top$ $c_\emptyset$ lower bound

Joint cost function using $a +^\top b = \min(a + b, \top)$ saturating arithmetics

$$\Phi_{\mathcal{M}} = \sum_{c_S \in \Phi}^\top c_S$$

The Weighted Constraint Satisfaction Problem (decision NP-complete)

- What/where is the minimum of $\Phi_{\mathcal{M}}$?

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Soft difference (3 values)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example

WEIGHTEDREGULAR_{A,S}

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Example

WEIGHTEDREGULAR $_{A,S}$

EXAMPLE: MIN-CUT

Graph $G = (V, E)$ with edge weight function w

- A Boolean variable X_i per vertex $i \in V$
- A cost function per edge $e = (i, j) \in E : c_{ij} = w(i, j) \times \mathbb{1}[x_i \neq x_j]$

A simple graph

- vertices $\{1, 2, 3, 4\}$
- cut weight 1 or 1.5 (1, 3)
- edge (1, 2) hard

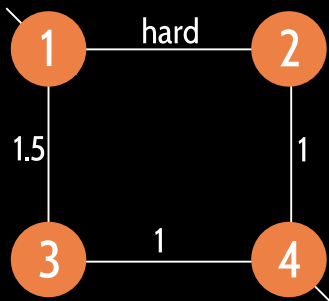
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Min-CUT on 4 variables

```
import pytoulbar2
myCFN = pytoulbar2.CFN(100,1)    # T/ub, resolution
for i in range(4):
    myCFN.AddVariable("x"+str(i+1),["l", "r"]) # returns an index
myCFN.AddFunction(["x1"],[0,100])
myCFN.AddFunction(["x4"],[100,0])
myCFN.AddFunction(["x1","x3"], [0,1.5,1.5,0])
...
sol = myCFN.Solve() # returns a triple (sol, cost, _)
```

Expressing probabilities through cost/energy distributions

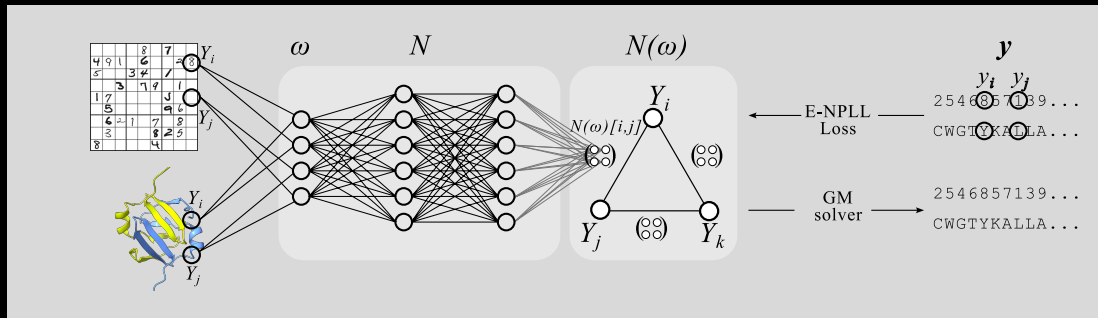
- log-linear models

$$P(t) \propto \exp(-\Phi(t))$$

- Normalization is #P-hard
- Optimisation and counting (marginals)

Markov Random Fields, Bayesian nets, Factor Graphs, Ising/Potts models [12]

- log-linear MRFs $\approx$ CFNs (cost = energy). Image/signal processing.
- CFNs can be learned from data.



This can

[5, 9, 10]

- Simultaneously learn how to decode numbers and play Sudoku
- Or learn costs from contextual data and solutions (Decision focused learning)
- Or learn how to design new proteins (CoViD neutralizers)

Uses a 0/1 encoding of finite domains: $x_{ia} = 1$ means $x_i = a$.

Pairwise models

[2]

$$\text{Minimize } \sum_{i,a} c_i(a) \cdot x_{ia} + \sum_{\substack{c_{ij} \in \Phi \\ a \in D^i, b \in D^j}} c_{ij}(a,b) \cdot x_{ia} x_{jb} \quad \text{such that}$$

$$\sum_{a \in D^i} x_{ia} = 1 \quad \forall i \in \{1, \dots, n\}$$

$$x_{ia} \in \{0, 1\} \quad \forall i \in \{1, \dots, n\}$$

nd variables, n constraints: concise but challenging

The “local polytope” [22, 14, 24, 23]

(without eq. (1))

$$\text{Minimize } \sum_{i,a} c_i(a) \cdot x_{ia} + \sum_{\substack{c_{ij} \in \Phi \\ a \in D^i, b \in D^j}} c_{ij}(a,b) \cdot p_{iajb} \quad \text{such that}$$

$$\sum_{a \in D^i} x_{ia} = 1 \quad \forall i \in \{1, \dots, n\}$$

$$\sum_{b \in D^j} p_{iajb} = x_{ia} \quad \forall c_{ij} \in \Phi, \forall a \in D^i$$

$$\sum_{a \in D^i} p_{iajb} = x_{jb} \quad \forall c_{ij} \in \Phi, \forall b \in D^j$$

$$x_{ia} \in \{0, 1\} \quad \forall i \in \{1, \dots, n\} \quad (1)$$

$nd + ed^2$ variables, $n + 2ed$ constraints: strong but expensive

UNIVERSALITY OF THE LOCAL POLYTOPE

A well-studied object

The hope was that local polytopes could be solved faster than LP

Plenty of fast **approximate** Block Coordinate Descent algorithms [20]

Local polytope universality: constructive proof [18]

Any LP can be reduced in linear time to a linear optimization over a local polytope (with infinite weights).

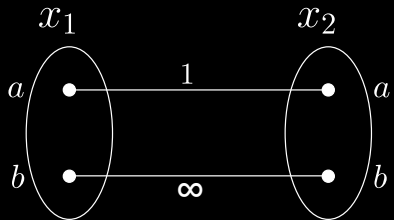
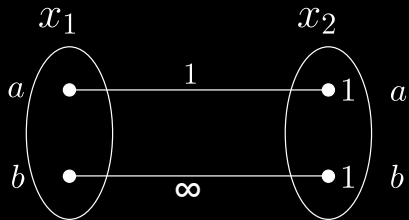
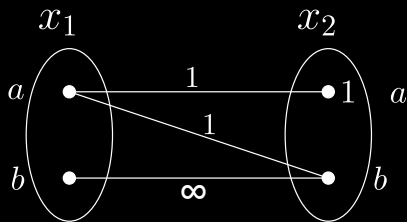
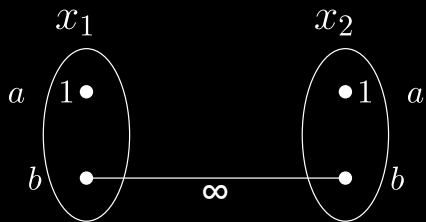
Fundamental for Tree Search efficiency

- Tight bounds
- Efficient bounds

Main difference with CP

- (G)AC: variables and constraints communicate through domains (value deletion)
- Soft (G)AC: variables and cost functions communicate through unary cost functions [21].
- This includes value deletions ($\top$)

SIMPLE EXAMPLE



$$c_{\emptyset} = 1$$

EQUIVALENCE PRESERVING TRANSFORMATIONS

Procedure MoveCost($c_S, c_{S'}, \tau, \alpha$): Move α units of cost between the tuple τ of scope S and tuples τ' that extend τ in scope S'

Data: Scopes $S \subset S'$

Data: $\tau \in \ell(S)$

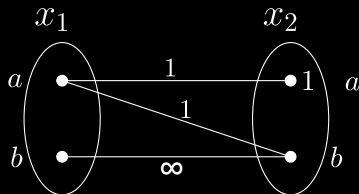
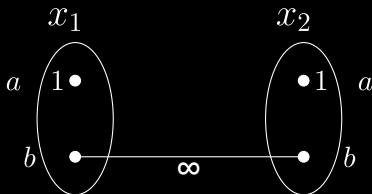
Data: cost α to move

$c_S(\tau) \leftarrow c_S(\tau) + \alpha$;

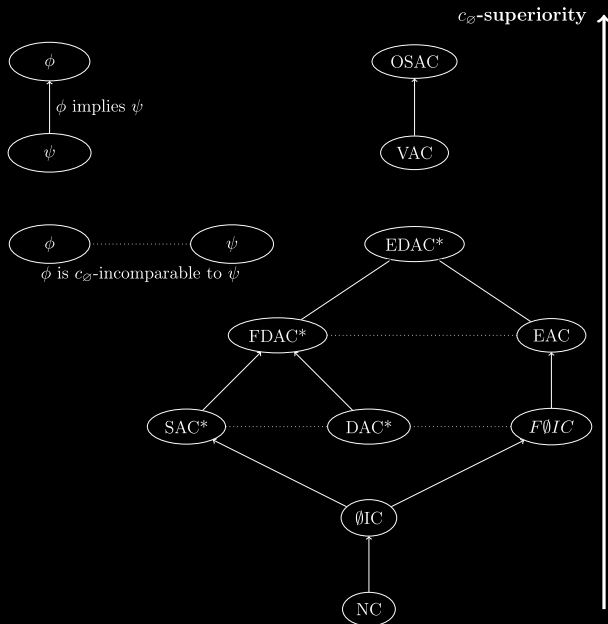
foreach $\tau' \in \ell(S') \mid \tau'_S = \tau$ **do**

$c_{S'}(\tau') \leftarrow c_{S'}(\tau') - \alpha$;

$$S = \{x_1\} \quad S' = \{x_1, x_2\} \quad \tau = (a) \quad \alpha = -1$$



FIX-POINT ALGORITHMS USING A WELL-DEFINED SET OF MOVE OPERATIONS



Algorithm	Time Complexity
NC	$O(nd)$
$\emptyset IC$	$O(ed^2)$
F $\emptyset IC$	$O(ed^2)$
SAC	$O(n^2d^2 + ed^3)$
DAC	$O(ed^2)$
FDAC	$O(end^3)$
EDAC	$O(ed^2 \max(nd, T))$
VAC_ϵ	$O(ed^2 T / \epsilon)$
OSAC	$O(e^{4.5} d^{5.5} \log M)$

Generalize usual (G)AC

- Enforce GAC when applied to constraints
- Most are polytime (some depend on T)
- Produce an equivalent problem (incremental)
- Can have several fixpoints

Definition

A CFN is VAC iff its hardened version has a non empty arc consistent closure.

Hardened: non zero-costs are forbidden ($\top$).

Enforcing VAC

AC-core guided

- Iteratively enforces AC on the hardened version to build a sequence of moves that increase $c_\emptyset$.
- Generalization of Ford Fulkerson (Augmenting DAGs [15])
- With capacity scaling (MaxSAT stratification)
- Solves submodular problems (Min-CUT)

What “Move” operations are used in enforcing Soft AC

- Moving to the 0-ary scope from a unary scope y_i
- Moving between a value a 's cost and associated pairs in c_{ij} y_{iaj}
- We should not create negative costs (lower bound)

OPTIMAL SOFT AC (YET ANOTHER LP)

$$\max \left(c_{\emptyset} + \sum_{i \in \{1, \dots, n\}} y_i \right)$$

such that:

$$c_i(a) - y_i + \sum_{j \in \{1, \dots, n\}} y_{iaj} \geq 0 \quad \forall i \in \{1, \dots, n\}, \forall a \in D_i$$

$$c_{ij}(ab) - y_{iaj} - y_{jbi} \geq 0 \quad \forall (i, j), i > j, \forall (a, b) \in D_i \times D_j$$

OSAC IS THE DUAL OF THE LOCAL POLYTOPE

Soft Arc Consistencies are

- Fast combinatorial heuristics that find good feasible dual solutions
- The reduced costs are used in the reformulated problem
- The problem remains equivalent

VAC vs. LP ON PROTEIN DESIGN PROBLEMS

CPLEX V12.4.0.0

```
Problem '3e4h.LP' read.  
Root relaxation solution time = 811.28 sec.  
...  
MIP - Integer optimal solution: Objective = 150023297067  
Solution time = 864.39 sec.
```

tb2 and VAC

(AC3 based)

```
loading CFN file: 3e4h.wcsp  
Lb after VAC: 150023297067  
Preprocessing time: 9.13 seconds.  
Optimum: 150023297067 in 129 backtracks, 129 nodes and 9.38 seconds.
```

Could this be useful for ILP?

By reversing the universal polytope construction somehow?

Let P be an LP $\{\min c^T z \mid Az = b, z \geq 0\}$

y a dual solution of P with cost $y^T b$

associated reduced costs $rc^y = c - A^T y$.

. Then the LP $\{\min rc^y \cdot z + y^T b \mid Az = b, z \geq 0\}$ is equivalent to P .

The ingredients

- An All-different constraint
- Unary cost functions on all involved variables

Reformulate this subproblem to maximally increase the lower bound $c_\emptyset$

How to reformulate?

- Find an optimal dual solution of this Linear Assignment Problem
- Reformulate using the dual solution

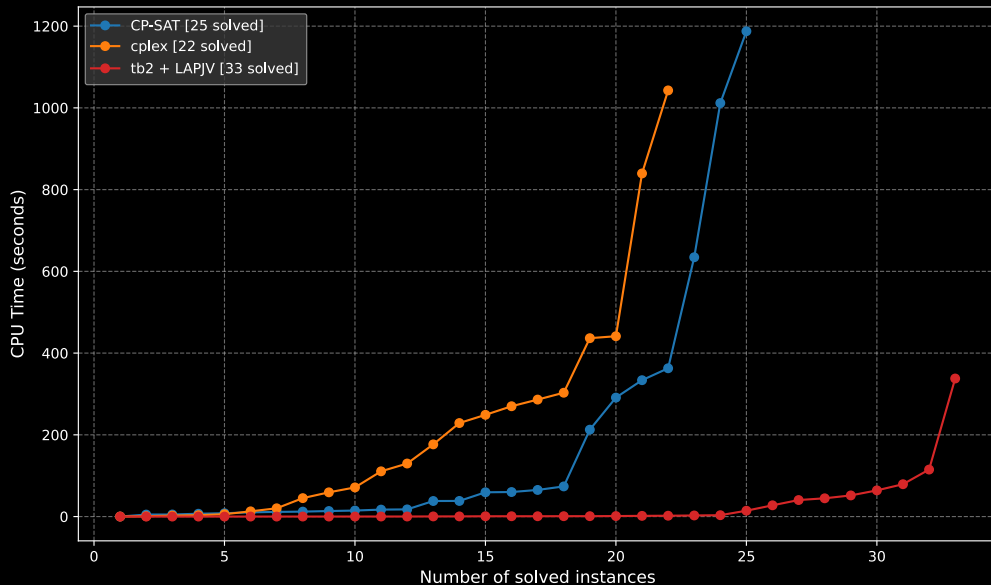
$$\min \sum_{i,j \in [1,n]} c_i[j] x_{ij} \quad s.t.$$

$$\sum_{j \in [1,n]} x_{ij} = 1 \quad \forall i \in [1, n]$$

$$\sum_{i \in [1,n]} x_{ij} = 1 \quad \forall j \in [1, n]$$

We can

- solve it using a primal-dual algorithm such as LAPJV (Jonker & Volgenant [13]) algorithm.
- Preserve primal/dual solutions during tree search for incrementality



CONCLUSION

We only skimmed through the surface

Soft Local consistencies

- On-the-fly variable elimination [16]
- Parallel Hybrid BFS [1, 3]
- Tree decomposition-aware tree-search $\div$ [11, 19, 1] (closed several RLFAP instances)
- Dominance analysis [8, 2]
- A parallel complete VNS-LDS-WCP local search algorithm [6, 17]...

Visit toulbar2 websites: github.com/toulbar2/toulbar2 and toulbar2.github.io

THANK YOU ALL FOR YOUR ATTENTION!

And to all CFN/toulbar2 contributors

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Questions?

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