Multi-Language Evaluation of Exact Solvers in Graphical Model Discrete Optimization

Barry Hurley[1] Barry O'Sullivan[1] David Allouche[2]
George Katsirelos[2] Thomas Schiex[2] Matthias Zytnicki[2]
Simon de Givry[2]

[1] Insight Centre for Data Analytics, University College Cork, Ireland
[2] MIAT, UR-875, INRA, F-31320 Castanet Tolosan, France

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Outline

1. Combinatorial optimization languages: *Cost Function Network, Markov Random Field, 0/1 Linear Programming, Max-CSP, Max-SAT, Constraint Programming*
2. Translations between formalisms
3. Graphical model evaluation
4. Exploitation: a portfolio approach
5. Conclusions

Cost Function Network (CFN)

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

Cost Function Network (CFN)

$$\left.\begin{array}{l} x \in \mathbb{N}^+ \\ y \in \mathbb{N}^+ \end{array}\right\}$$ *small domains (∝ 100 values)*

$$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$$

$$f(x,y) = \quad x \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \overset{\begin{array}{cccc} & & y & \\ 0 & 1 & 2 & 3 \end{array}}{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix}}$$

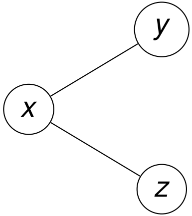
Cost Function Network (CFN)

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$
$z \in \mathbb{N}^+$
$\dots$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$
$g : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$
$\dots$

$$f(x,y) = \begin{array}{cc} & y \\ & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ x \; \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix} \end{array}$$

$$\underset{x,y,z,\cdots}{\text{Minimize}} \quad f(x,y) + g(x,z) + \cdots$$



Problem is NP-hard

Cost Function Network (CFN)

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x, y) = \quad x \begin{array}{c c} & \begin{array}{cccc} & y & & \\ 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix} \end{array}$$

$f(x, y) = max(x - y, 0)$ */* soft(x $<$ y) in scheduling*/*

Cost Function Network (CFN)

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x, y) = \quad x \begin{array}{c c} & \begin{array}{cccc} & y & & \\ 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \end{array}$$

Cost Function Network (CFN)

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \quad x \begin{array}{c c} & \begin{array}{cccc} & & y & \\ 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \end{array}$$

$f(x,y) =$
$max(2 - |x - y|, 0)$ */* soft($|x - y| \geq 2$) in telecoms (CELAR)*/*

Cost Function Network (CFN)

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \quad x \begin{array}{c c} & \begin{array}{cccc} & & y & \\ 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$f(x,y) = max(1 - |x - y|, 0)$

/ soft($x \neq y$) in overconstrained graph coloring (fix. nb. of colors)*/*

/ It defines a Max-CSP problem */*

Markov Random Field (MRF)

const $\lambda \in \mathbb{R}^+$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \color{red}{\mathbb{R}^+}$

$$f(x, y) \quad = \quad \begin{array}{ccc} & & y \\ & & \begin{array}{cc} 0 & 1 \end{array} \\ x & \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix} \end{array}$$

Markov Random Field (MRF)

const $\lambda, \alpha, \beta, \gamma, \delta \in \mathbb{R}^+$

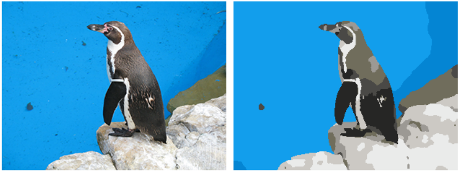
$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$$f(x, y) = \begin{array}{cc} & y \\ & \begin{array}{cc} 0 & 1 \end{array} \\ x \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix} \end{array}$$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{R}^+$
$g : \mathbb{N}^+ \mapsto \mathbb{R}^+$
$h : \mathbb{N}^+ \mapsto \mathbb{R}^+$

/ l1-norm distance to class colors */*

$$g(x) = \begin{pmatrix} \alpha & \beta \end{pmatrix}$$
$$h(y) = \begin{pmatrix} \gamma & \delta \end{pmatrix}$$



320 x 240 = 76,800 pixels/variables, 8 labels [Kappes et al., 2015]

From Bayesian Network to additive MRF

$x, y, z \in \{0, 1, 2\}$

/ Mendelian law in genetics [Sánchez et al., 2008] */*

$\mathbb{P}(z|x, y) =$

$$
\begin{array}{cc}
 & \begin{array}{ccc} & y & \\ 0 & 1 & 2 \end{array} \\
x \begin{array}{c} 0 \\ 1 \\ 2 \end{array} &
\left(\begin{array}{ccc}
(1 \quad 0 \quad 0) & (0.5 \quad 0.5 \quad 0) & (0 \quad 1 \quad 0) \\
(0.5 \quad 0.5 \quad 0) & (0.25 \quad 0.5 \quad 0.25) & (0 \quad 0.5 \quad 0.5) \\
(0 \quad 1 \quad 0) & (0 \quad 0.5 \quad 0.5) & (0 \quad 0 \quad 1)
\end{array} \right)
\end{array}
$$

$f(x, y, z) = -\log(\mathbb{P}(z|x, y))$

Cost Function Network

const $k \in \mathbb{N}^+ \cup \infty$
/ problem upper bound */*

$$f(x,y) = \begin{array}{cc} & \begin{array}{cc} & y \\ & \begin{array}{cc} 0 & 1 \end{array} \end{array} \\ x \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix} \end{array}$$

$x \in \{0,1\}$
$y \in \{0,1\}$

$$g(x) = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$f : \{0,1\} \times \{0,1\} \mapsto [0,k]$
$g : \{0,1\} \mapsto [0,k]$
$h : \{0,1\} \mapsto [0,k]$

$$h(y) = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$f(x,y) = max(k(1-x-y), 0)$ */* hard($x \vee y$) */*
$g(x) = x$ */* minimum vertex covering*/*
$h(y) = y$

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \{0, 1\}$
$y \in \{0, 1\}$

$f : \{0, 1\} \times \{0, 1\} \mapsto [0, k]$
$g : \{0, 1\} \mapsto [0, k]$
$h : \{0, 1\} \mapsto [0, k]$

$$f(x, y) = \begin{array}{cc} & \begin{array}{cc} & y \\ 0 & 1 \end{array} \\ x \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix} \end{array}$$

$$g(x) = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$h(y) = \begin{pmatrix} 0 & \color{red}{1} \end{pmatrix}$$

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \{0, 1\}$
$y \in \{0, 1\}$

$f : \{0, 1\} \times \{0, 1\} \mapsto [0, k]$
$g : \{0, 1\} \mapsto [0, k]$
$h : \{0, 1\} \mapsto [0, k]$

$$f(x, y) = x \begin{array}{c} \\ \\ 0 \\ 1 \end{array} \begin{array}{c} y \\ \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} k & 1 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$g(x) = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$h(y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \{0, 1\}$
$y \in \{0, 1\}$

$f : \{0, 1\} \times \{0, 1\} \mapsto [0, k]$
$g : \{0, 1\} \mapsto [0, k]$
$h : \{0, 1\} \mapsto [0, k]$

$$f(x, y) = x \begin{array}{cc} & y \\ & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$g(x) = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$h(y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \{0, 1\}$
$y \in \{0, 1\}$

$f : \{0, 1\} \times \{0, 1\} \mapsto [0, k]$
$g : \{0, 1\} \mapsto [0, k]$
$h : \{0, 1\} \mapsto [0, k]$

$$f(x, y) = x \begin{array}{cc} & y \\ & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$g(x) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$h(y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$f_\emptyset() = 1$$ */* problem lb */*

const $k \in \mathbb{N}^+ \cup \infty$

$$f(x,y) = \begin{array}{cc} & \begin{array}{cc} & y \\ 0 & 1 \end{array} \\ x \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$x \in \{0,1\}$
$y \in \{0,1\}$

$$g(x) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$f : \{0,1\} \times \{0,1\} \mapsto [0,k]$
$g : \{0,1\} \mapsto [0,k]$
$h : \{0,1\} \mapsto [0,k]$

$$h(y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$f_\emptyset() = 1 \qquad \textit{/* problem lb */}$$

Maximize $f_\emptyset$ is NP-hard using integer costs,

but it is polynomial using rationals (OSAC [Cooper et al., 2010])

Soft Arc Consistency by Equivalence Problem Transformations

const $k \in \mathbb{N}^+ \cup \infty$

$$x \in \{0, 1\}$$
$$y \in \{0, 1\}$$

$$f : \{0, 1\} \times \{0, 1\} \mapsto [0, k]$$
$$g : \{0, 1\} \mapsto [0, k]$$
$$h : \{0, 1\} \mapsto [0, k]$$

$$f(x, y) = \begin{array}{cc} & y \\ & \begin{array}{cc} 0 & 1 \end{array} \\ x \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$g(x) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$h(y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$f_\emptyset() = 1 \qquad \textit{/* problem lb */}$$

CFN solver toulbar2 exploits weaker (incremental) lower bounds (EDAC & VAC [Cooper et al., 2010])

const $k \in \mathbb{N}^+ \cup \infty$

$$f(x,y) = \begin{array}{cc} & \begin{array}{cc} & y \\ 0 & 1 \end{array} \\ x \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$x \in \{0,1\}$
$y \in \{0,1\}$

$$g(x) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$f : \{0,1\} \times \{0,1\} \mapsto [0,k]$
$g : \{0,1\} \mapsto [0,k]$
$h : \{0,1\} \mapsto [0,k]$

$$h(y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$f_\emptyset() = 1 \qquad \textit{/* problem lb */}$$

CFN solver toulbar2 exploits weaker (incremental) lower bounds
(EDAC & VAC [Cooper et al., 2010])
+ dominance rules [de Givry et al., 2013]
+ hybrid best-first search [Allouche et al., 2015]

Translation from CFN to 0/1 Linear Programming (01LP)

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \begin{array}{cc} & y \\ & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ x \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

Direct encoding:

Translation from CFN to 0/1 Linear Programming (01LP)

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \quad x \begin{array}{c|c} & \begin{array}{ccc} & y & \\ y_0 & y_1 & y_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

Direct encoding:

Subject to: $\sum_{d=0}^{2} x_d = 1, \sum_{d=0}^{2} y_d = 1, \ldots$
$x_d, y_d, \ldots \in \{0, 1\}$ /* domain values */

Translation from CFN to 0/1 Linear Programming (01LP)

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \begin{array}{cc} & y \\ & \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ x \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

Direct encoding:

Minimize $\quad 2p_{x=0,y=0} + p_{x=1,y=0} + p_{x=2,y=1} + 2p_{x=2,y=2} + \cdots$

Subject to: $\sum_{d=0}^{2} x_d = 1, \sum_{d=0}^{2} y_d = 1, \ldots$
$x_d, y_d, \ldots \in \{0,1\}$ */* domain values */*
$p_{x=u,y=v}, \ldots \in [0,1]$ */* finite positive costs */*

Translation from CFN to 0/1 Linear Programming (01LP)

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \quad x \begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \!\!\! \begin{array}{c} y \\ \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

Direct encoding:

Minimize $\quad 2p_{x=0,y=0} + p_{x=1,y=0} + p_{x=2,y=1} + 2p_{x=2,y=2} + \cdots$

Subject to: $\sum_{d=0}^{2} x_d = 1, \sum_{d=0}^{2} y_d = 1, \ldots$
$x_d, y_d, \ldots \in \{0,1\}$ */* domain values */*
$p_{x=u,y=v}, \ldots \in [0,1]$ */* finite positive costs */*
$(1 - x_u) + (1 - y_v) + p_{x=u,y=v} \geq 1, \ldots$ */* channeling */*

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x, y) = \quad x \begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \overset{\displaystyle y}{\begin{array}{c} \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}}$$

Direct encoding:

Minimize $\quad 2p_{x=0,y=0} + p_{x=1,y=0} + p_{x=2,y=1} + 2p_{x=2,y=2} + \cdots$

Subject to: $\sum_{d=0}^{2} x_d = 1, \sum_{d=0}^{2} y_d = 1, \ldots$
$x_d, y_d, \ldots \in \{0, 1\}$ */* domain values */*
$p_{x=u,y=v}, \ldots \in [0, 1]$ */* finite positive costs */*
$(1 - x_u) + (1 - y_v) + p_{x=u,y=v} \geq 1, \ldots$ */* channeling */*
$(1 - x_1) + (1 - y_1) \geq 1, \ldots$ */* forbidden assignments */*

Translation from CFN to 0/1 Linear Programming (01LP)

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x, y) = \quad x \; \begin{array}{c|c} & \begin{array}{ccc} & y & \\ y_0 & y_1 & y_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

Tuple encoding:

Minimize $\quad 2p_{x=0,y=0} + p_{x=1,y=0} + p_{x=2,y=1} + 2p_{x=2,y=2} + \cdots$

Subject to: $\sum_{d=0}^{2} x_d = 1, \sum_{d=0}^{2} y_d = 1, \ldots$

$x_d, y_d, \ldots \in \{0, 1\}$ /* domain values */

$p_{x=u,y=v}, \ldots \in [0, 1]$ /* finite positive or zero costs */

$\sum_{v=0}^{2} p_{x=u,y=v} = x_u, \sum_{u=0}^{2} p_{x=u,y=v} = y_v, \ldots$ /* channeling */

Translation from CFN to 0/1 Linear Programming (01LP)

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \begin{array}{cc} & \begin{array}{ccc} & y & \\ y_0 & y_1 & y_2 \end{array} \\ x \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

Tuple encoding: local polytope [Schlesinger, 1976]

Minimize $\quad 2p_{x=0,y=0} + p_{x=1,y=0} + p_{x=2,y=1} + 2p_{x=2,y=2} + \cdots$

Subject to: $\sum_{d=0}^{2} x_d = 1, \sum_{d=0}^{2} y_d = 1, \ldots$
$x_d, y_d, \ldots \in \{0,1\}$ */* domain values */*
$p_{x=u,y=v}, \ldots \in [0,1]$ */* finite positive or zero costs */*
$\sum_{v=0}^{2} p_{x=u,y=v} = x_u, \sum_{u=0}^{2} p_{x=u,y=v} = y_v, \ldots$ */* channeling */*

Translation from CFN to 0/1 Linear Programming (01LP)

const $k \in \mathbb{N}^+ \cup \infty$

$x \in \mathbb{N}^+$
$y \in \mathbb{N}^+$

$f : \mathbb{N}^+ \times \mathbb{N}^+ \mapsto \mathbb{N}^+$

$$f(x,y) = \begin{array}{cc} & y \\ & \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ x \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

Tuple encoding: local polytope [Schlesinger, 1976]
$=$ dual(OSAC) [Werner, 2007]

Minimize $2p_{x=0,y=0} + p_{x=1,y=0} + p_{x=2,y=1} + 2p_{x=2,y=2} + \cdots$

Subject to: $\sum_{d=0}^{2} x_d = 1, \sum_{d=0}^{2} y_d = 1, \ldots$
$x_d, y_d, \ldots \in \{0,1\}$ */* domain values */*
$p_{x=u,y=v}, \ldots \in [0,1]$ */* finite positive or zero costs */*
$\sum_{v=0}^{2} p_{x=u,y=v} = x_u, \sum_{u=0}^{2} p_{x=u,y=v} = y_v, \ldots$ */* channeling */*

Translations between CFN and Constraint Programming

- CFN to CP: use extra cost variables and Table constraints.

$$f(x,y) = x\; \begin{array}{c|ccc} & & y & \\ & 0 & 1 & 2 \\ \hline 0 & & & \\ 1 & & & \\ 2 & & & \end{array}$$

$$f(x,y) = \begin{array}{cc} & \begin{array}{ccc} & y & \\ 0 & 1 & 2 \end{array} \\ x\;\begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

var $c_f \in [0, k]$

min $c_f + \cdots$

Table$([c_f, x, y], [(2, 0, 0), (0, 0, 1),$
$(0, 0, 2), (1, 1, 0), (0, 1, 2),$
$(0, 2, 0), (1, 2, 1), (2, 2, 2)])$

- CP to CFN: Extract cost functions from the objective variable and decompose global constraints into small arity cost functions.

Translations between CFN and Constraint Programming

- CFN to CP: use extra cost variables and Table constraints.

$$f(x,y) = \begin{array}{cc} & \begin{array}{ccc} & y & \\ 0 & 1 & 2 \end{array} \\ x \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \left(\begin{array}{ccc} 2 & 0 & 0 \\ 1 & k & 0 \\ 0 & 1 & 2 \end{array}\right) \end{array}$$

var $c_f \in [0, k]$

$\min c_f + \cdots$

Table($[c_f, x, y], [(2,0,0), (0,0,1), (0,0,2), (1,1,0), (0,1,2), (0,2,0), (1,2,1), (2,2,2)]$)

Costs limited to 32-bit representation

- CP to CFN: Extract cost functions from the objective variable and decompose global constraints into small arity cost functions.
  No large domains (10^3), limited size of cost functions (10^6)

Benchmarks

- MRF: Probabilistic Inference Challenge 2011 (uai format)
- CVPR: Computer Vision and Pattern Recognition OpenGM2 (uai)
- CFN: MaxCSP 2008 Competition and CFLib (wcsp format)
- WPMS: Weighted Partial MaxSAT Evaluation 2013 (wcnf format)
- CP: MiniZinc Challenge 2012 & 2013 (minizinc format)

Number of instances and their total compressed (gzipped) size:

Benchmark	Nb.	UAI	WCSP	LP(direct)	LP(tuple)	WCNF(direct)	WCNF(tuple)	MINIZINC
MRF	319	187MB	475MB	2.4G	2.0GB	518MB	2.9GB	473MB
CVPR	1461	430MB	557MB	9.8GB	11GB	3.0GB	15GB	N/A
CFN	281	43MB	122MB	300MB	3.5GB	389MB	5.7GB	69MB
MaxCSP	503	13MB	24MB	311MB	660MB	73MB	999MB	29MB
WPMS	427	N/A	387MB	433MB	N/A	717MB	N/A	631MB
CP	35	7.5MB	597MB	499MB	1.2GB	378MB	1.9GB	21KB
Total	**3026**	0.68G	2.2G	14G	18G	5G	27G	1.2G

Experimental Results (1-hour CPU time limit and 8 GB of RAM)

	DAOOPT	TOULBAR2	CPLEXdirect	CPLEXtuple	MAXHSdirect	MAXHStuple	GECODE
Total	1832 (534.34)	**2433 (107.26)**	1273 (123.70)	1862 (57.35)	1417 (191.45)	1567 (143.45)	202 (219.37)
Nb. of 1st position	0	**16 [1]**	7 [3]	3 [5]	9 [2]	0	4 [4]
Nb. of best solution	2209 [2]	**2562 [1]**	1355 [5]	1300 [6]	1626 [4]	1706 [3]	229[7]
Nb. of single best sol.	57 [4]	88 [2]	43 [5]	**95 [1]**	80 [3]	1 [7]	13 [6]
Zscore (time)	135.37 [6]	**57.84 [1]**	102.97 [3]	104.89 [4]	90.73 [2]	122.88 [5]	136.58 [7]
Zscore (cost)	63.00 [3]	**26.25 [1]**	59.24 [2]	69.92 [4]	80.55 [5]	108.76 [7]	100.55 [6]
Borda-score	89.40 [5]	**182.50 [1]**	129.60 [2]	102.78 [4]	114.37 [3]	59.54 [7]	60.64 [6]
Borda-score (norm)	2.08 [5]	**4.24 [1]**	3.01 [2]	2.86 [3]	2.66 [4]	1.65 [7]	1.84 [6]

Zscore$(x) = \frac{x-\mu}{\sigma}$, Borda-score: sum of mean Borda-scores per category (solution quality then time, see MiniZinc Challenge)

Exact Solvers:

- MRF: DAOOPT 1.1.2 [Otten et al., 2012] (uai format)
- CFN: TOULBAR2 0.9.8 [Cooper et al., 2010] (wcsp, uai, wcnf, qpbo formats)
- 01LP: CPLEX 12.6.0 (lp format)
- WPMS: MAXHS 2.51 [Davies and Bacchus, 2013] (wcnf format)
- CP: GECODE 4.4.0 (minizinc format)

Experimental Results (1-hour CPU time limit and 8 GB of RAM)

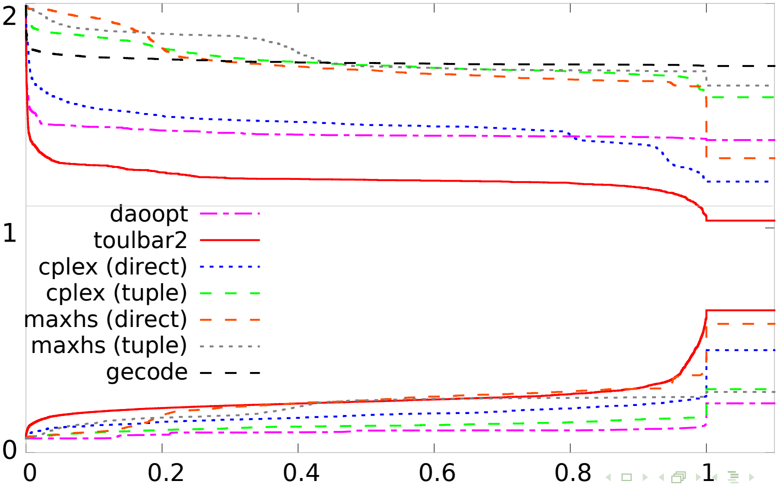
Problem/*s*/*d*/*a*	DAOOPT	TOULBAR2	CPLEX*direct*	CPLEX*tuple*	MAXHS*direct*	MAXHS*tuple*	GECODE
MRF/319/503/5 (UAI)	151 (584.39)	**226 (93.80)**	156 (111.88)	210 (82.18)	118 (172.27)	72 (98.68)	1 (1509.93)
DBN/108/2/2	60 (626.79)	**81 (192.42)**	65 (124.66)	69 (155.12)	38 (366.15)	2 (1748.65)	0 (-)
Grid/21/2/2	5 (1223.67)	0 (-)	**15 (120.90)**	1 (3354.21)	8 (557.01)	0 (-)	0 (-)
ImageAlignment/10/93/2	10 (754.96)	**10 (5.27)**	0 (-)	9 (88.41)	0 (-)	0 (-)	0 (-)
Linkage/22/7/5	17 (576.94)	14 (364.73)	16 (365.09)	**22 (21.99)**	20 (52.62)	20 (124.04)	0 (-)
ObjectDetection/37/21/2	0 (-)	0 (-)	0 (-)	0 (-)	0 (-)	0 (-)	0 (-)
ProteinFolding/21/503/2	0 (-)	**21 (20.24)**	10 (169.28)	9 (176.17)	2 (268.51)	0 (-)	0 (-)
Segmentation/100/21/2	59 (460.33)	**100 (0.29)**	50 (0.06)	100 (3.35)	50 (7.38)	50 (22.54)	1 (1509.93)
CVPR/1461/20/3 (HDF5)	1274 (481.02)	**1340 (22.81)**	382 (179.96)	1332 (8.70)	483 (355.71)	1038 (58.83)	N/A
ChineseChars/100/2/2	0 (-)	0 (-)	0 (-)	0 (-)	0 (-)	0 (-)	N/A
ColorSeg/21/12/2	0 (-)	**15 (1340.56)**	0 (-)	5 (190.33)	0 (-)	0 (-)	N/A
GeomSurf/600/7/3	555 (509.01)	**600 (0.96)**	382 (179.96)	600 (2.89)	387 (183.53)	321 (63.15)	N/A
InPainting/4/4/2	0 (-)	**2 (325.72)**	0 (-)	1 (339.90)	0 (-)	0 (-)	N/A
Matching/4/20/2	4 (319.24)	**4 (3.20)**	0 (-)	3 (765.25)	0 (-)	0 (-)	N/A
MatchingStereo/2/20/2	0 (-)	0 (-)	0 (-)	0 (-)	0 (-)	0 (-)	N/A
ObjectSeg/5/8/2	0 (-)	4 (2292.28)	0 (-)	**5 (1057.88)**	0 (-)	0 (-)	N/A

Experimental Results (1-hour CPU time limit and 8 GB of RAM)

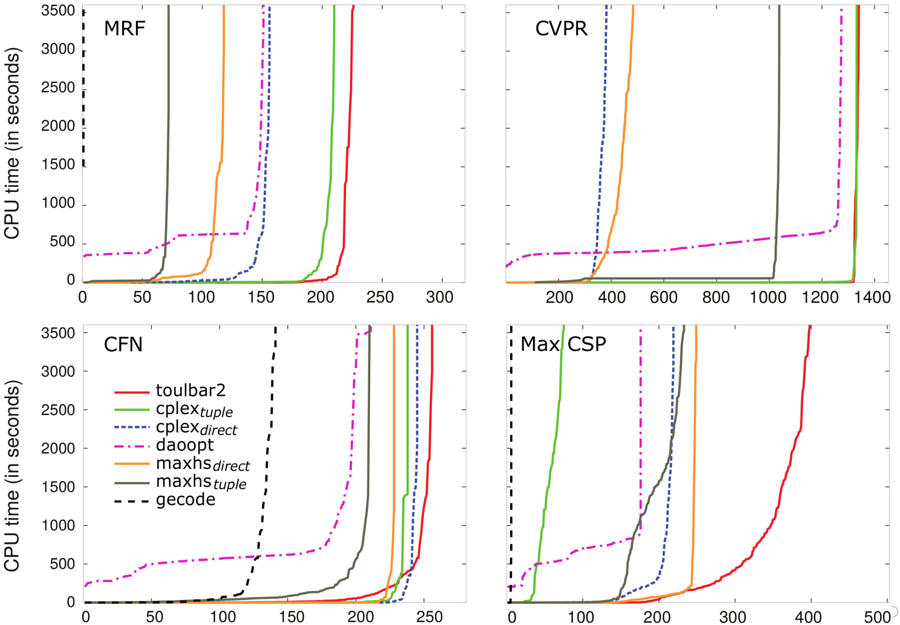
Problem/*s*/*d*/*a*	DAOOPT	TOULBAR2	CPLEX direct	CPLEX tuple	MAXHS direct	MAXHS tuple	GECODE
CFN/281/300/3 (WCSP)	211 (768.93)	**256 (109.84)**	245 (33.85)	238 (34.00)	228 (14.91)	210 (121.20)	141 (224.77)
Auction/170/2/2	169 (663.04)	170 (93.10)	170 (0.03)	170 (0.14)	**170 (0.03)**	170 (121.16)	113 (231.55)
CELAR/16/44/2	4 (598.72)	**14 (279.00)**	0 (-)	3 (560.44)	0 (-)	0 (-)	0 (-)
Pedigree/10/28/3	4 (373.43)	**10 (10.58)**	5 (44.28)	9 (57.27)	10 (190.49)	6 (99.28)	0 (-)
ProteinDesign/10/198/2	4 (597.46)	**9 (13.40)**	0 (-)	7 (298.88)	0 (-)	4 (477.72)	0 (-)
SPOT5/20/4/3	6 (309.04)	4 (40.44)	**16 (22.99)**	12 (294.94)	6 (200.82)	5 (5.40)	0 (-)
Warehouse/55/300/2	24 (1752.42)	49 (163.23)	**54 (142.57)**	37 (6.46)	42 (6.78)	25 (92.83)	28 (197.39)
MaxCSP/503/50/2 (XCSP)	176 (603.56)	**398 (386.08)**	219 (152.73)	75 (876.84)	249 (76.21)	233 (538.93)	6 (115.39)
BlackHole/37/50/2	10 (222.19)	10 (0.08)	**30 (141.91)**	10 (2.22)	10 (0.30)	10 (2.78)	0 (-)
Coloring/22/6/2	17 (319.29)	17 (11.39)	**17 (7.14)**	16 (72.33)	14 (17.67)	14 (50.80)	4 (171.61)
Composed/80/10/2	26 (543.73)	**80 (0.13)**	80 (4.48)	37 (1667.07)	80 (79.81)	73 (1383.72)	0 (-)
EHI/200/7/2	0 (-)	**179 (773.86)**	0 (-)	0 (-)	1 (3078.96)	0 (-)	0 (-)
Geometric/100/20/2	92 (755.46)	**95 (134.57)**	65 (419.39)	0 (-)	89 (31.52)	84 (138.98)	0 (-)
Langford/4/29/2	2 (272.24)	**2 (0.12)**	2 (38.79)	1 (0.03)	2 (0.32)	2 (2.19)	2 (2.97)
[illegible]	29	15	25	11	**53**	50	[illegible]

Problem/*s*/*d*/*a*	DAOOPT	TOULBAR2	CPLEX direct	CPLEX tuple	MAXHS direct	MAXHS tuple	GECODE
WPMS/427/2/580 (WCNF)	11 (536.35)	197 (110.33)	269 (109.76)	N/A	**321 (168.67)**	N/A	28 (243.39)
Haplotyping/100/2/580	N/A	1 (784.32)	18 (679.90)	N/A	**44 (674.01)**	N/A	0 (-)
MIPLib/12/2/93	2 (365.31)	3 (102.39)	3 (49.85)	N/A	**3 (9.47)**	N/A	3 (28.61)
MaxClique/62/2/2	9 (574.36)	33 (209.07)	38 (229.33)	N/A	**40 (362.26)**	N/A	24 (280.38)
PackupWeighted/99/2/177	N/A	53 (167.82)	**99 (0.72)**	N/A	99 (7.14)	N/A	0 (-)
PlanningWithPref/29/2/372	N/A	7 (515.22)	11 (751.65)	N/A	**28 (65.82)**	N/A	1 (0.03)
TimeTabling/25/2/36	N/A	0 (-)	0 (-)	N/A	**7 (1020.73)**	N/A	0 (-)
Upgradeability/100/2/77	N/A	100 (12.43)	**100 (0.84)**	N/A	100 (2.73)	N/A	N/A
CP/35/*163/4* (MINIZINC)	9 (387.13)	16 (354.57)	2 (0.99)	7 (584.10)	18 (145.94)	14 (400.03)	**26 (138.55)**
AMaze/6/17/4	0 (-)	3 (279.71)	0 (-)	4 (998.46)	**6 (12.00)**	5 (161.25)	4 (176.91)
FastFood/6/*5/2*	1 (200.32)	1 (0.00)	1 (0.00)	1 (0.00)	1 (0.00)	1 (0.00)	**6 (14.22)**
Golomb/6/*163/3*	0 (-)	3 (44.97)	0 (-)	0 (-)	3 (117.34)	1 (78.01)	**6 (111.17)**
OnCallRostering/5/*89/4*	1 (253.25)	2 (27.27)	1 (1.98)	2 (47.44)	**3 (162.22)**	3 (362.19)	2 (75.13)
ParityLearning/7/20/4	7 (432.94)	7 (663.51)	0 (-)	0 (-)	5 (343.24)	4 (907.40)	**7 (248.10)**

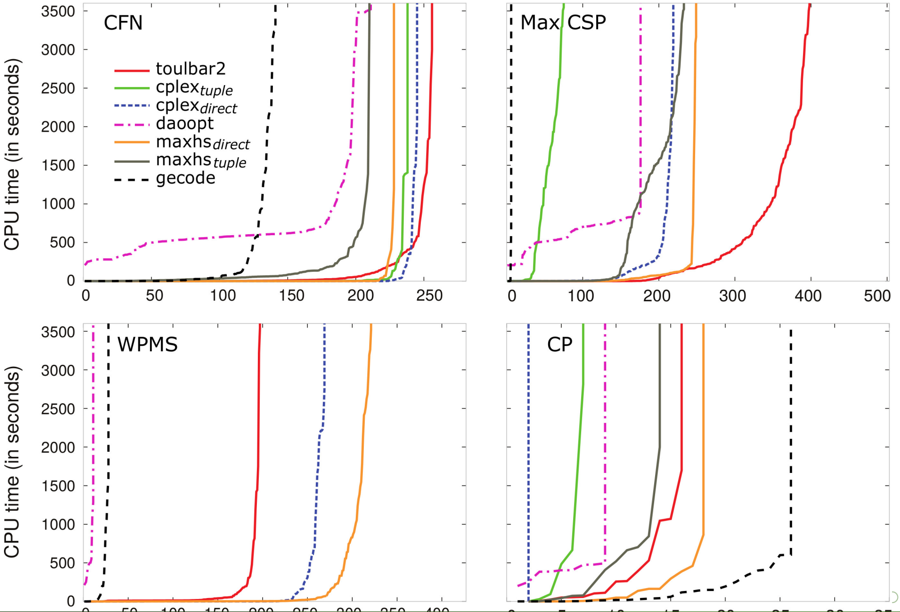
Normalized lower and upper bounds on a set of 1208 hardest instances ($\geq$ 5 sec. of CPU time for TOULBAR2)



Complete Results



Complete Results



Exploitation: a Portfolio Approach

Number of problems solved over 2,564 instances (exclud. WPMS and CP)

Solver	Solved time (sec.) Mean	Std. dev.	Num. solved	Num. best	Misclass. pen. solved	total time
VBS(6)	93.0	385.1	2,321			
M5P regression	91.5	376.1	2,298			
J48 classification	84.7	368.1	2,294			
Random Forest	74.6	327.6	2,279			
k-means clustering	66.9	301.4	2,259			
TOULBAR2	105.2	408.3	2,220	1,863	224	28,000.1
CPLEX_{tuple}	55.4	316.6	1,852	27	3	10,345.3
DAOOPT	535.1	340.1	1,812	3	0	3,236.8
MAXHS_{tuple}	140.0	414.5	1,551	3	1	8.4
MAXHS_{direct}	199.0	565.4	1,078	208	4	9,261.4
CPLEX_{direct}	127.7	433.4	1,002	217	36	14,381.9

Important features are:

- Cost function arity
- Domain size
- Initial upper bound (*k*)

TOULBAR2+CPLEX win at UAI 2014 Inference Competition (MAP entry)

Conclusions

- Collection of benchmarks in uai, wcnf, wcsp, lp, mzn formats
  `http://genoweb.toulouse.inra.fr/~degivry/evalgm`
- MRF portfolio (uai format)
  `https://github.com/9thbit/uai-proteus`
- TOULBAR2 exact solver for cost function networks
  `http://www.inra.fr/mia/T/toulbar2`
  `http://numberjack.ucc.ie` (python interface, mzn reader)

Try different languages!

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