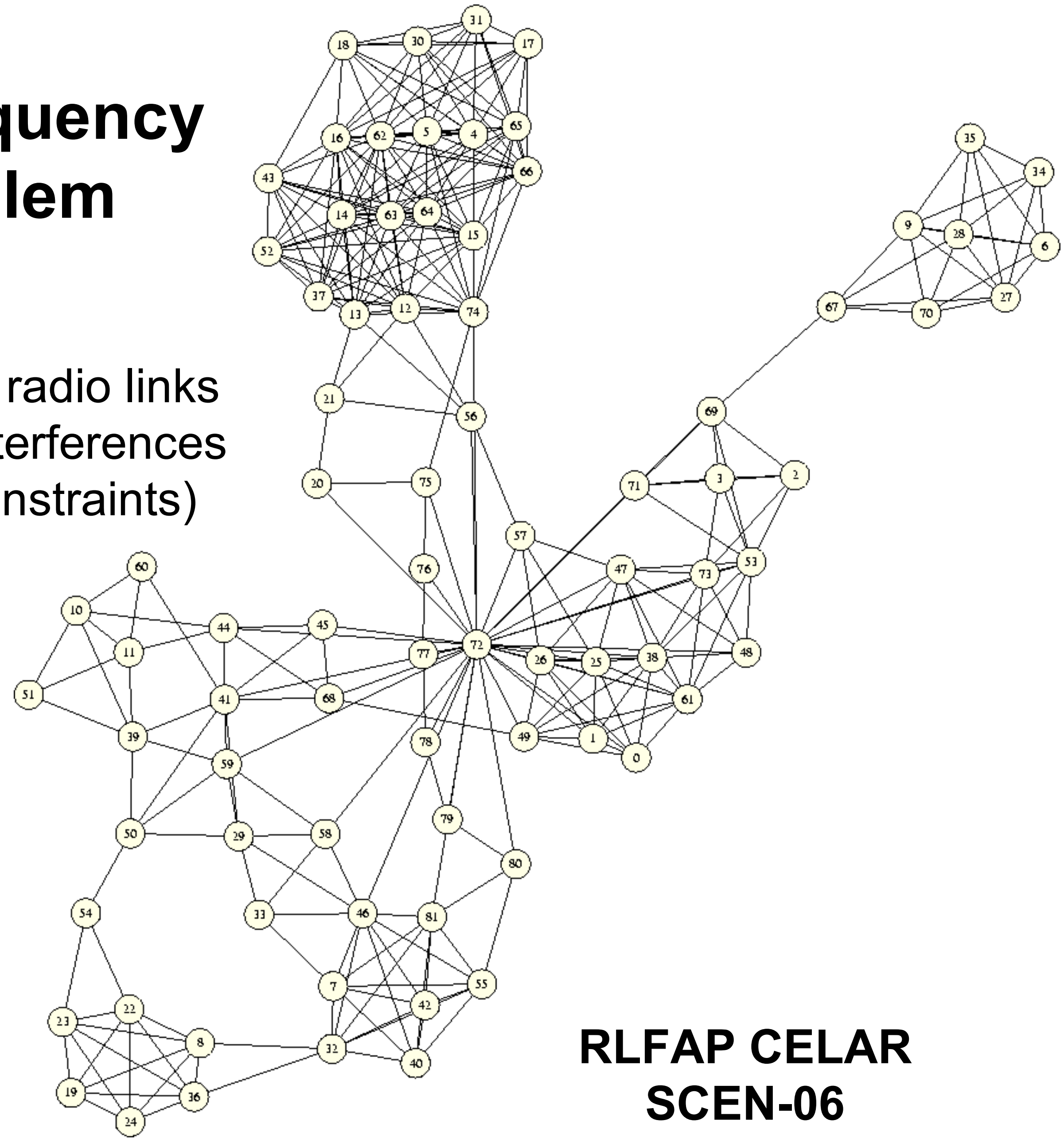
Exploiting Tree Decomposition and Soft Local Consistency in Weighted CSP

Simon de Givry, Thomas Schiex, INRA
Gérard Verfaillie, ONERA
Toulouse, France

Radio Link Frequency Allocation Problem

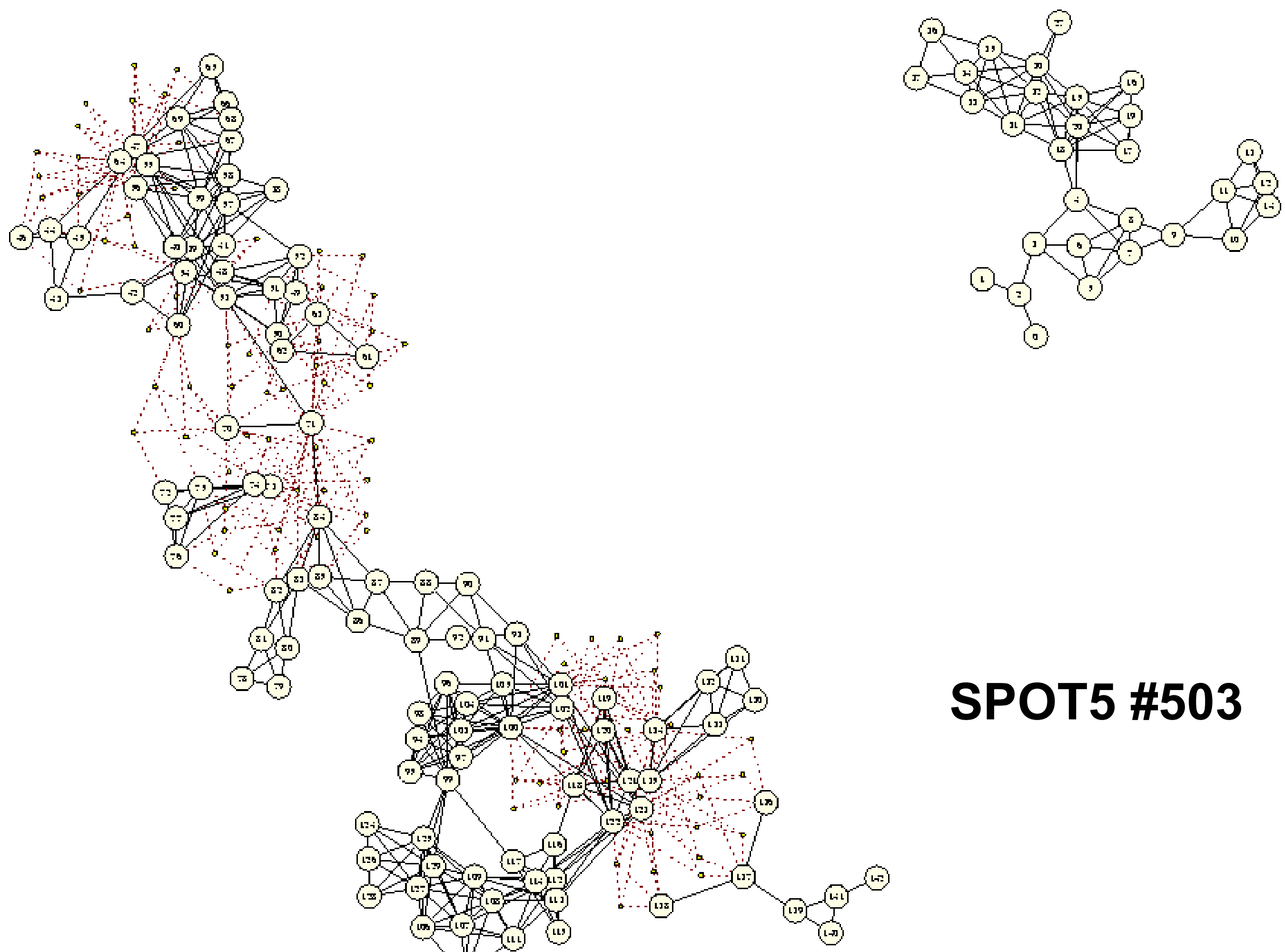
Allocate frequencies to radio links such that the sum of interferences (soft binary distance constraints) is minimized

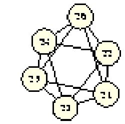


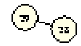
RLFAP CELAR SCEN-06

Earth Observation Satellite Management Problem

Select images among a set of candidate images such that physical constraints (hard binary and ternary constraints) are satisfied and the sum of weights associated to selected images (soft unary constraints) is maximized





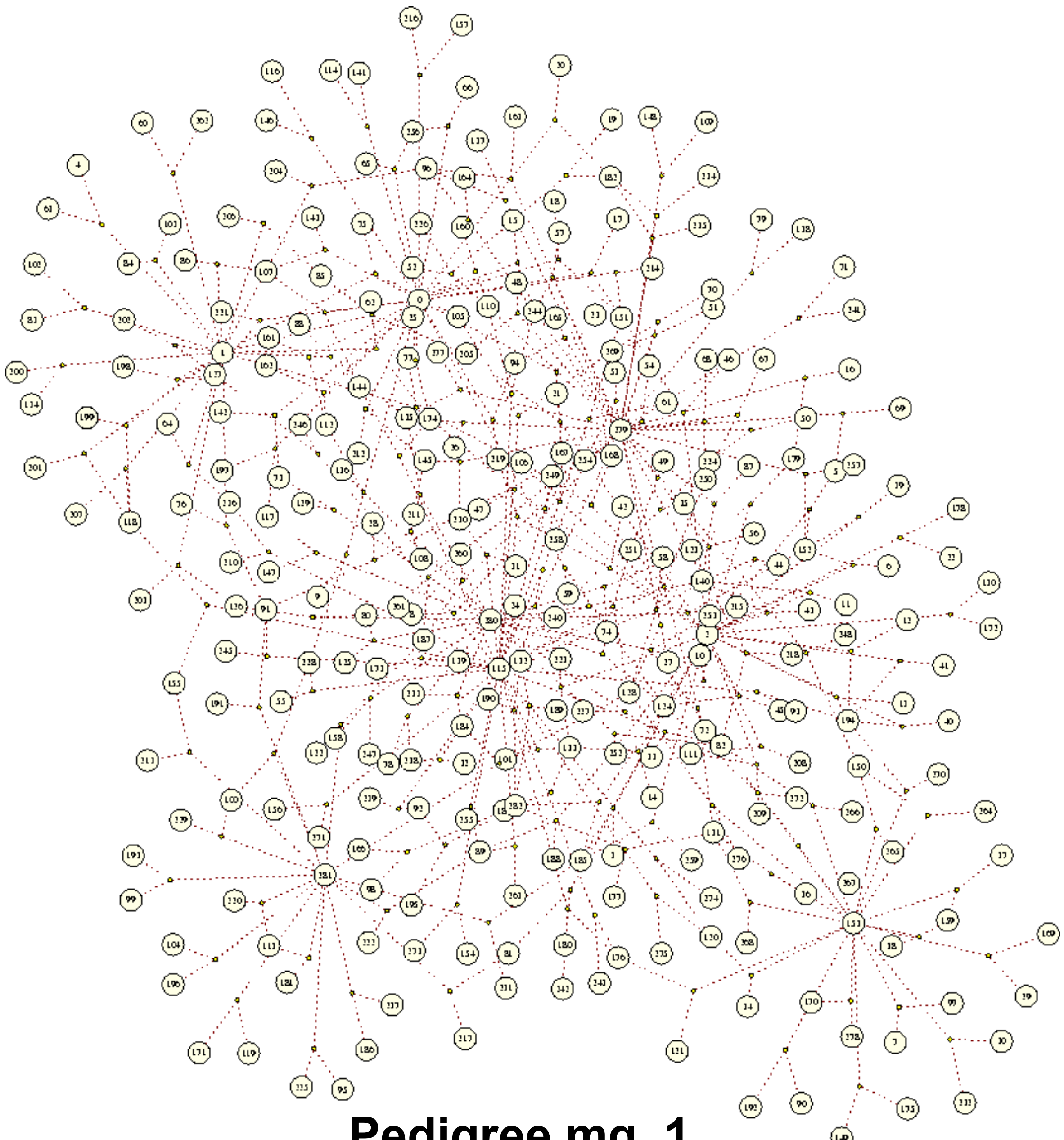


SPOT5 #503

Mendelian error detection in complex pedigree

Find a complete genotype of maximum a posteriori probability (MPE)

Conditional probability tables (soft unary, binary and ternary constraints)



Pedigree mq_1

Motivation of this work

Exploitation of the structure

present in many real problems

temporal, spatial, causal ...

Framework: weighted binary CSP

- (X,D,W)
  - $X=\{x_1,\ldots, x_n\}$ n **variables**
  - $D=\{D_1,\ldots, D_n\}$ n **finite domains** of maximum size d
  - $W=\{W_1,\ldots,W_e\}$ e **cost functions**
    - W_{ij}, W_i, $W_\varnothing$ with **scopes** $\{x_i,x_j\}$, $\{x_i\}$, $\varnothing$
    - $W_{ij} : D_i \times D_j \to [\,0, k\,]$
    - k is associated with completely **forbidden assignments**

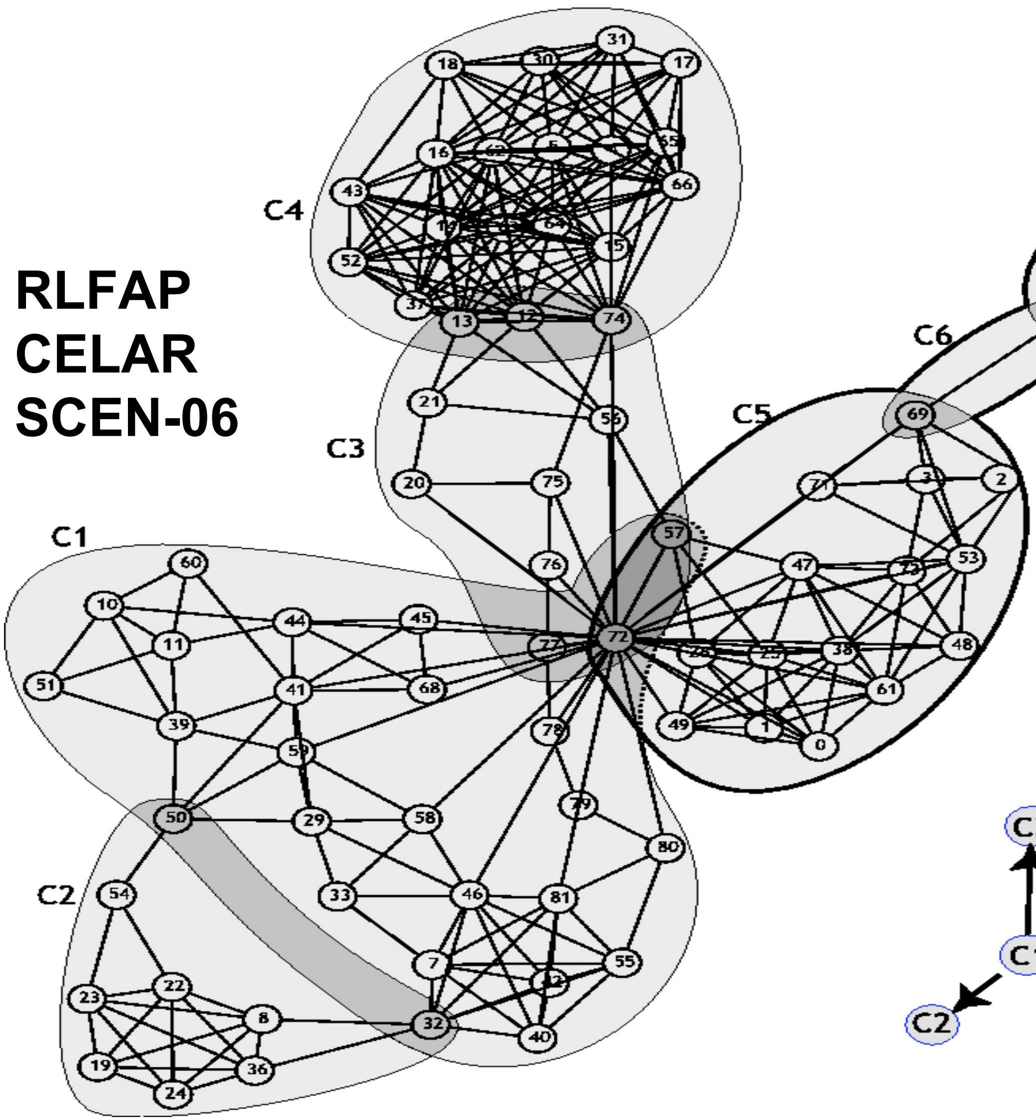
- Find a **complete assignment minimizing**

$$W_\varnothing + \sum_i W_i\,(a_i) + \sum_{ij} W_{ij}\,(a_i,a_j)$$

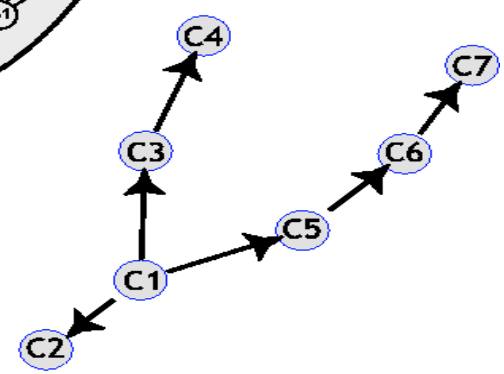
- **NP-hard** problem

Tree decomposition of a constraint graph

RLFAP
CELAR
SCEN-06

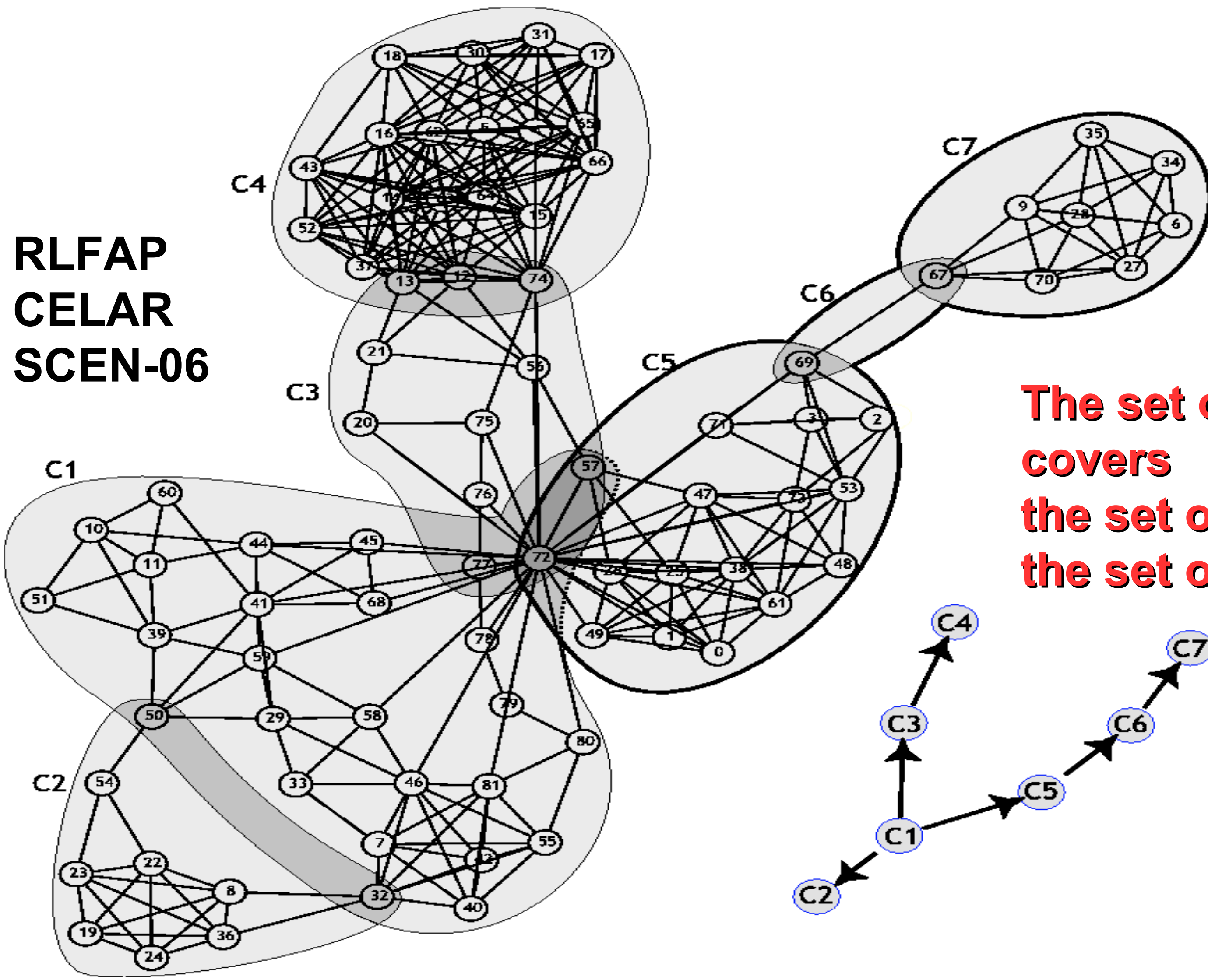


Tree of clusters



Tree decomposition of a constraint graph

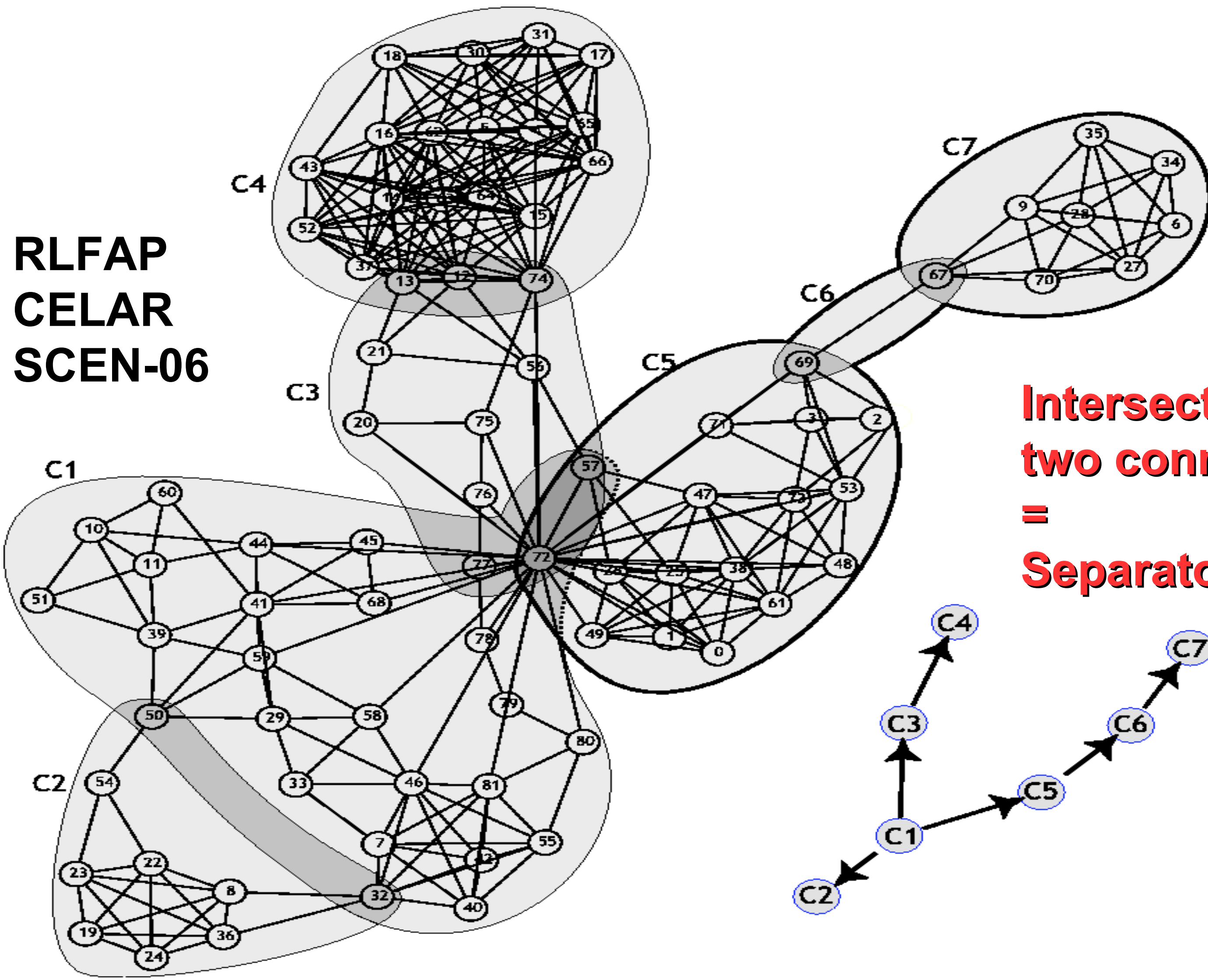
RLFAP
CELAR
SCEN-06



The set of clusters covers the set of variables and the set of constraints

Tree decomposition of a constraint graph

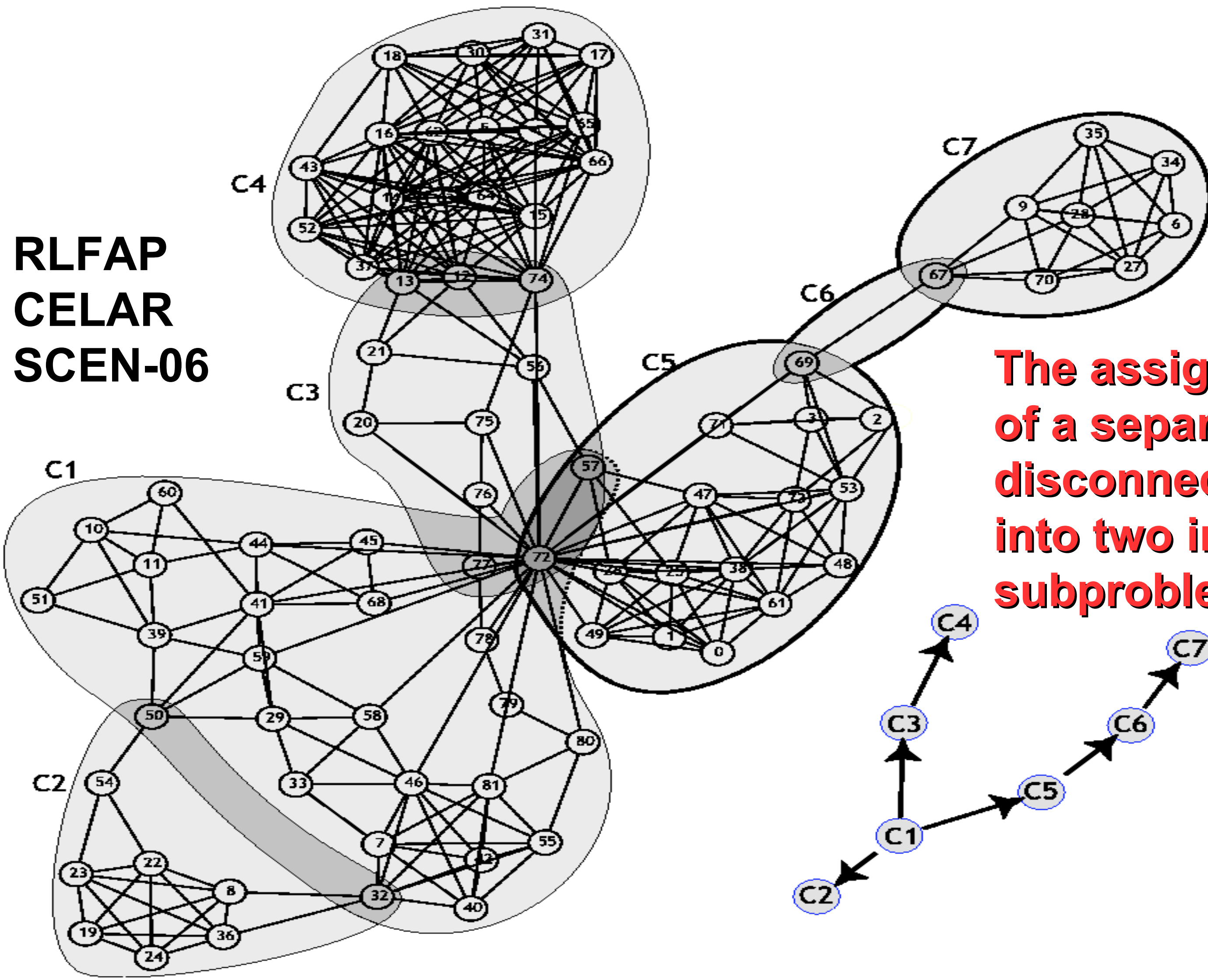
RLFAP
CELAR
SCEN-06



Intersection between two connected clusters
=
Separator

Tree decomposition of a constraint graph

RLFAP
CELAR
SCEN-06



The assignment of a separator disconnects the problem into two independent subproblems

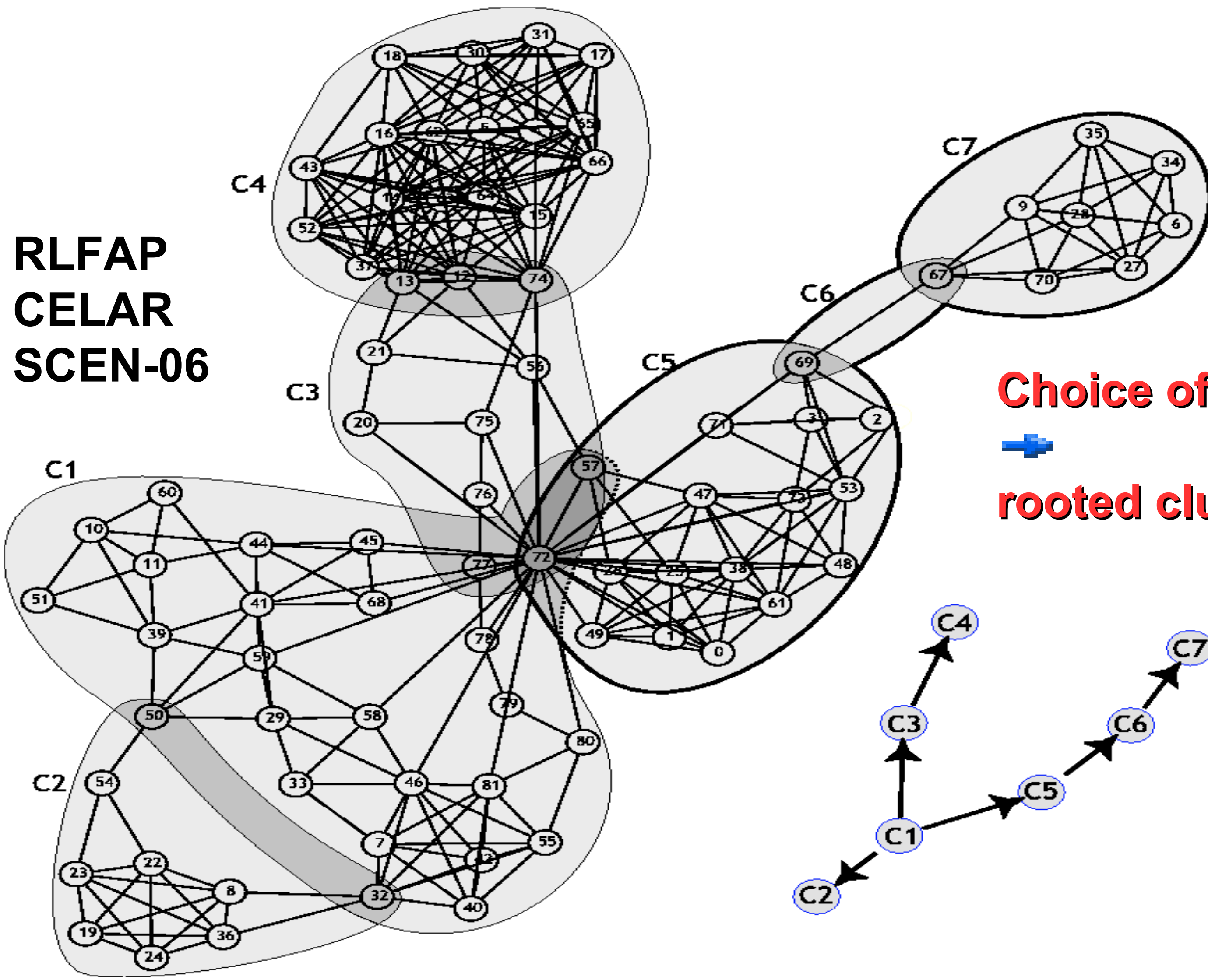
Tree decomposition of a constraint graph

RLFAP
CELAR
SCEN-06

Choice of a root cluster

➔

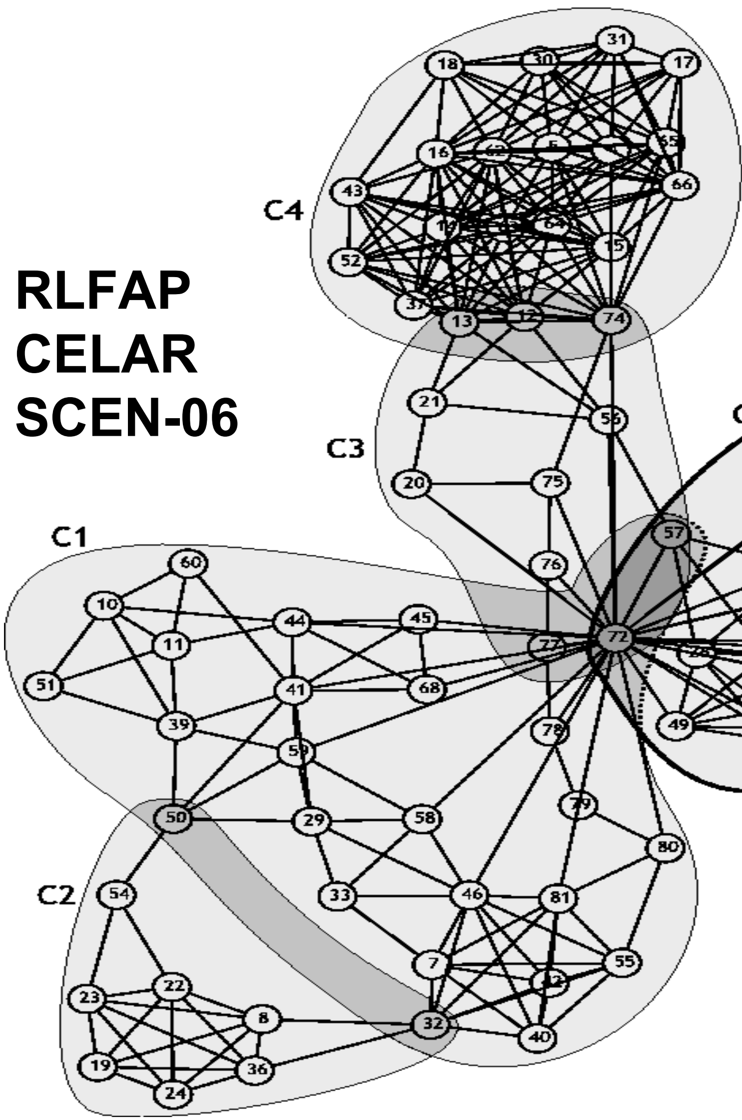
rooted cluster tree



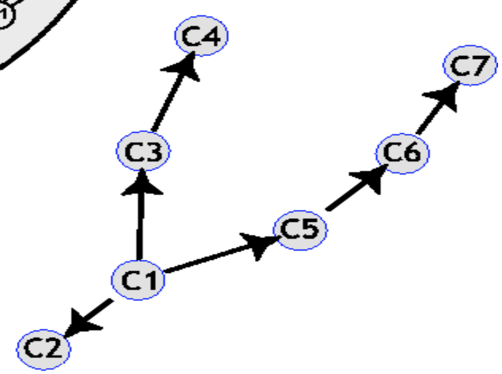
Exploitation of the tree decomposition

w = largest cluster size
s = largest separator size

RLFAP
CELAR
SCEN-06



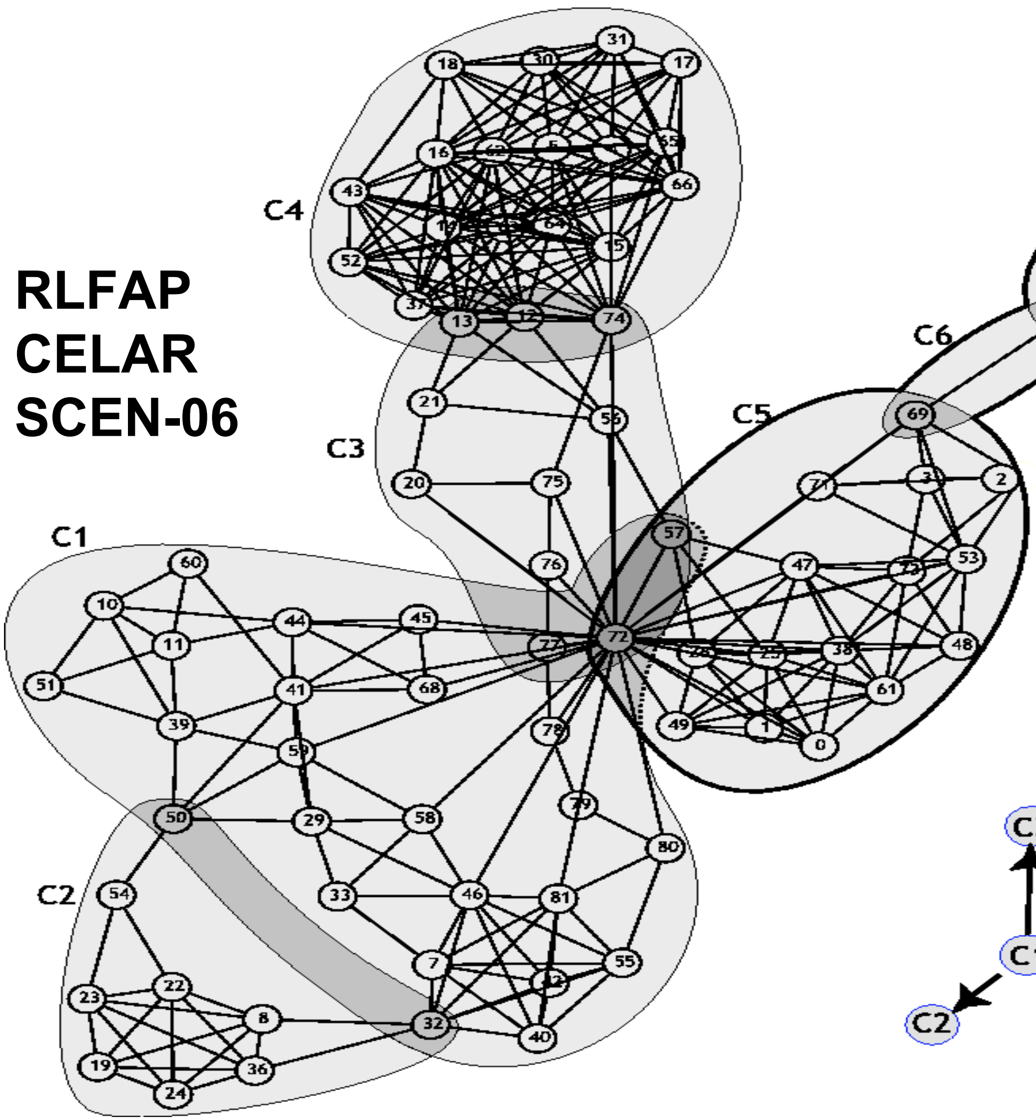
Dynamic programming
➔
Cluster tree elimination
Time: $O(d^w)$, Space: $O(d^s)$



Exploitation of the tree decomposition

h = rooted cluster tree height

RLFAP
CELAR
SCEN-06

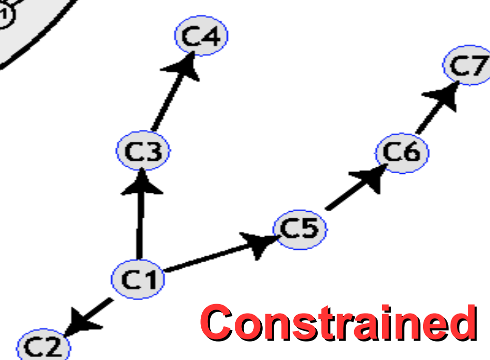


Tree search

→

Pseudo-tree search (PTS)

Time: $O(d^h)$, Space: $O(nd)$

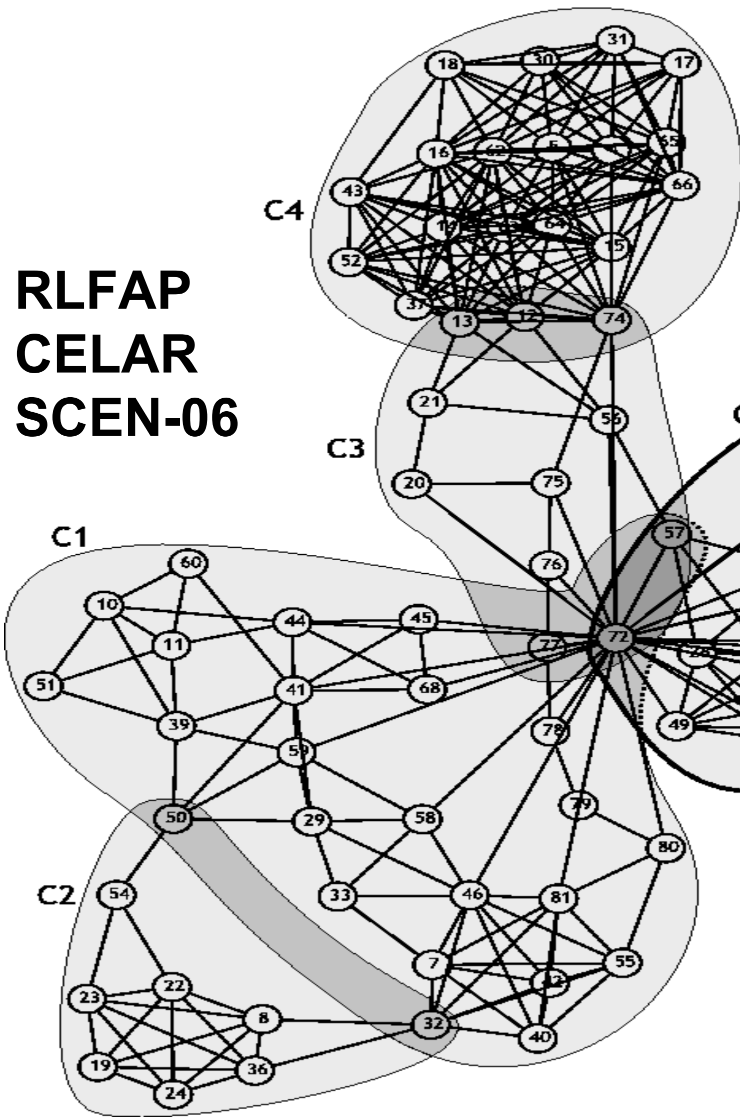


Constrained variable ordering
Graph-based backjumping

Exploitation of the tree decomposition

w = largest cluster size
s = largest separator size

RLFAP
CELAR
SCEN-06

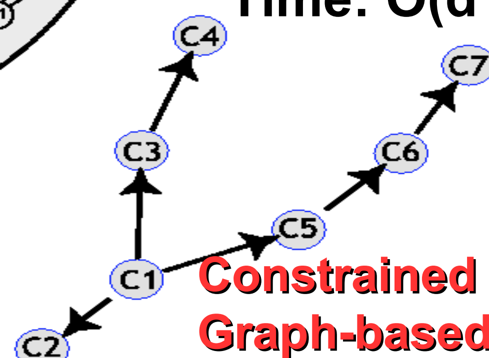


Tree search

→

Backtrack Bounded by Tree Decomposition (BTD)

Time: $O(d^w)$, Space: $O(d^s)$

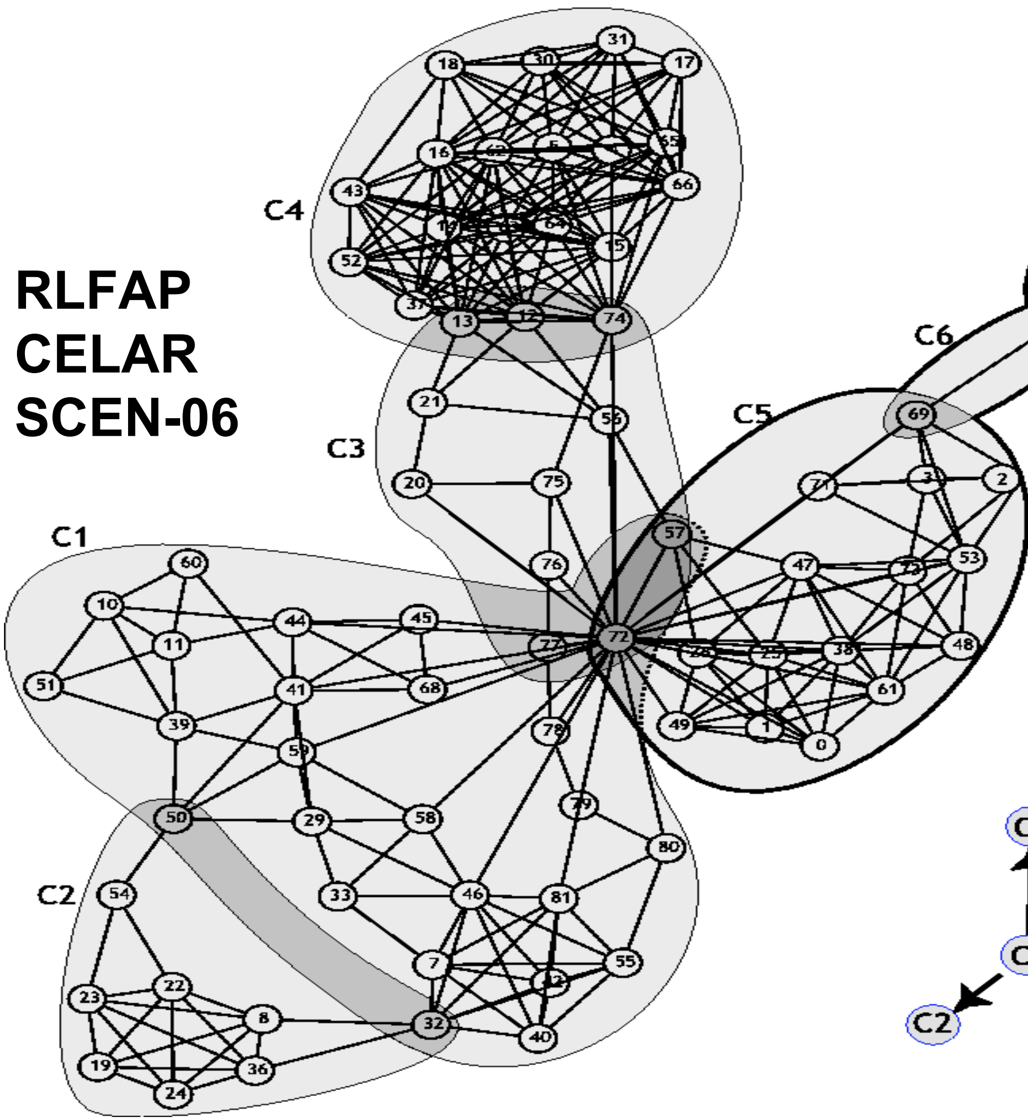


Constrained variable ordering
Graph-based backjumping
Graph-based learning

Exploitation of the tree decomposition

w = largest cluster size
s = largest separator size
k = number of valuations

RLFAP
CELAR
SCEN-06

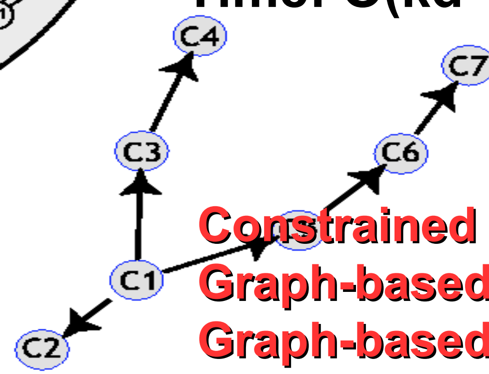


Tree search

➡

Backtrack Bounded by Tree Decomposition (BTD+)

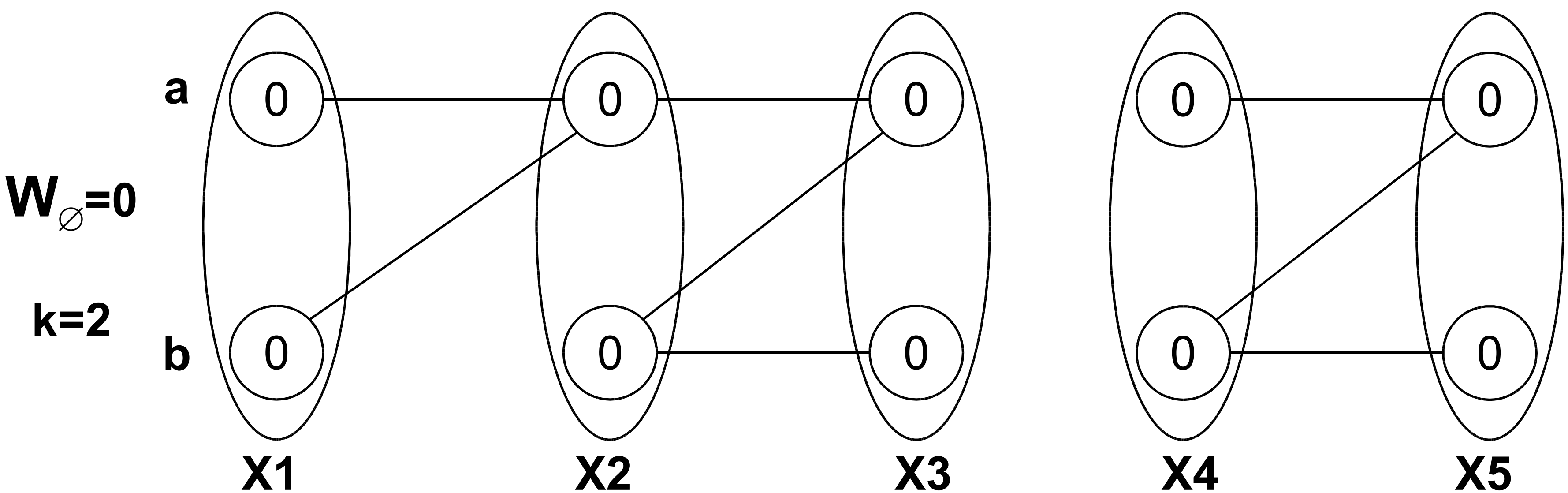
Time: $O(kd^w)$, Space: $O(d^s)$



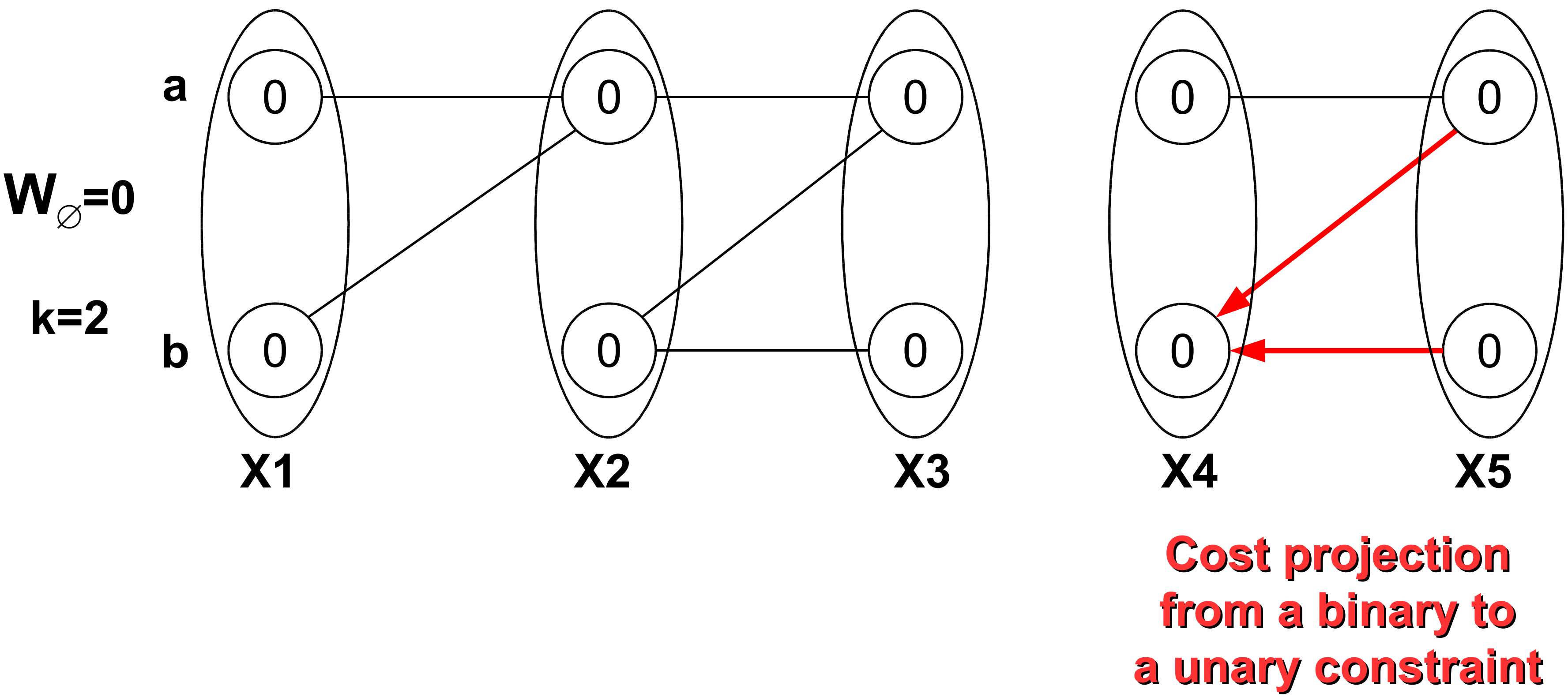
Constrained variable ordering
Graph-based backjumping
Graph-based learning
Initial upper-bounds

Soft local consistency

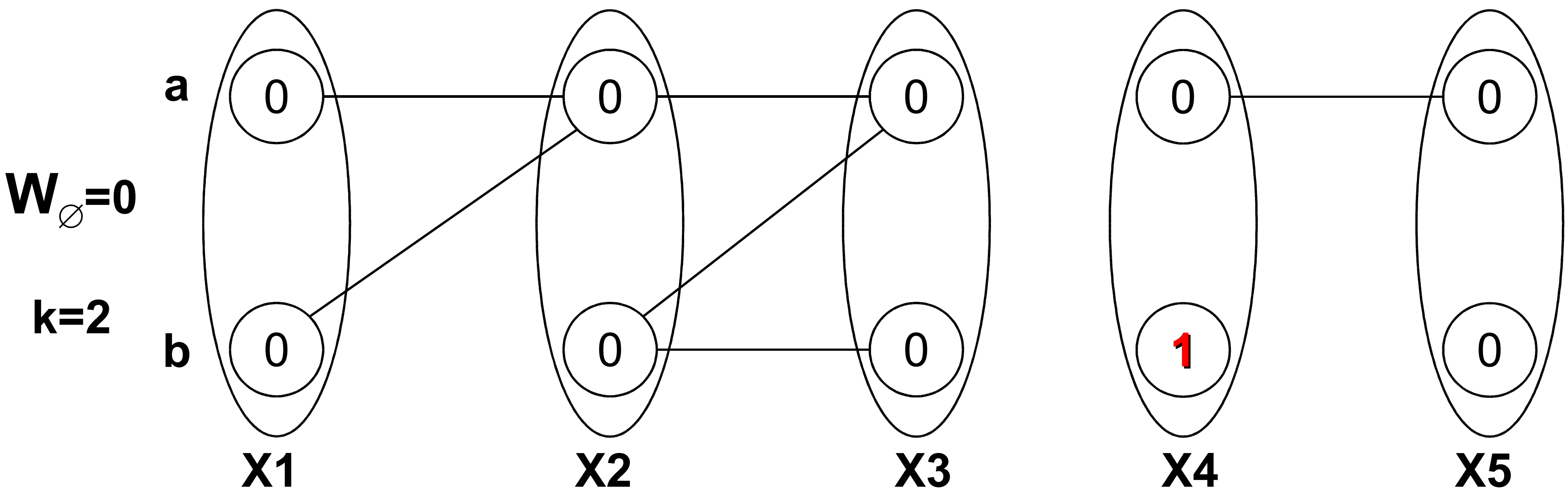
- **Goal**: transforming a problem into an **equivalent problem** with a **more explicit lower bound**
- **Means**: enforcing a **soft local consistency property** by **moving costs** from **binary** constraints to **unary** constraints and to the **zero-arity** constraint $W_\varnothing$ (problem lower bound)



Soft local consistency



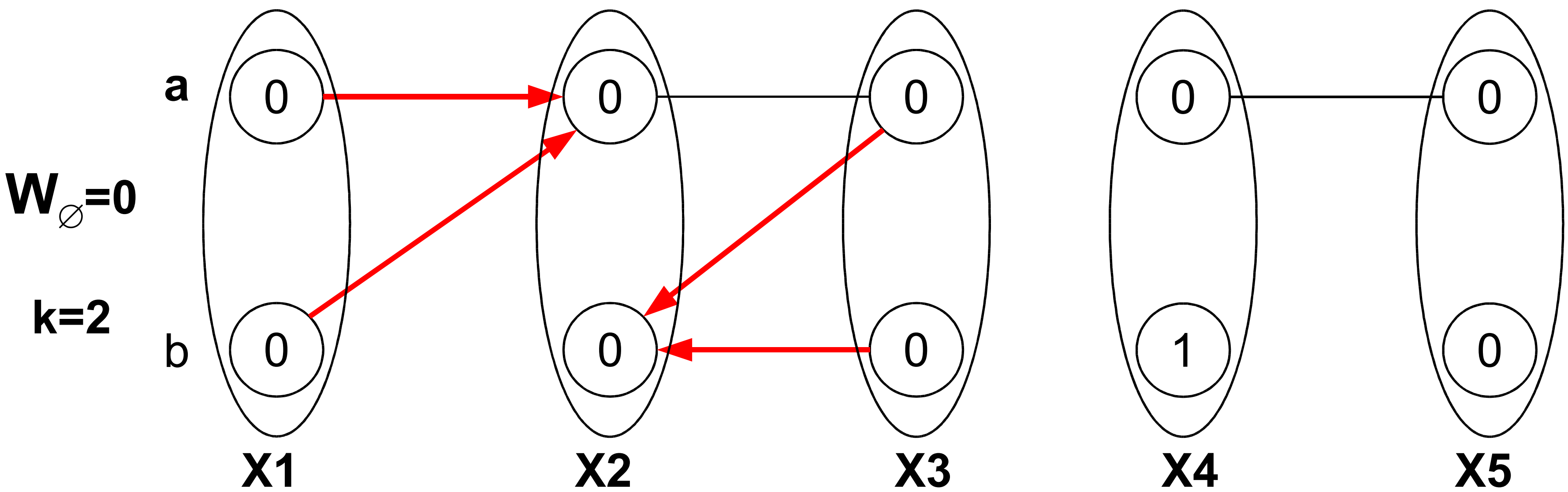
Soft local consistency



Cost projection from a binary to a unary constraint

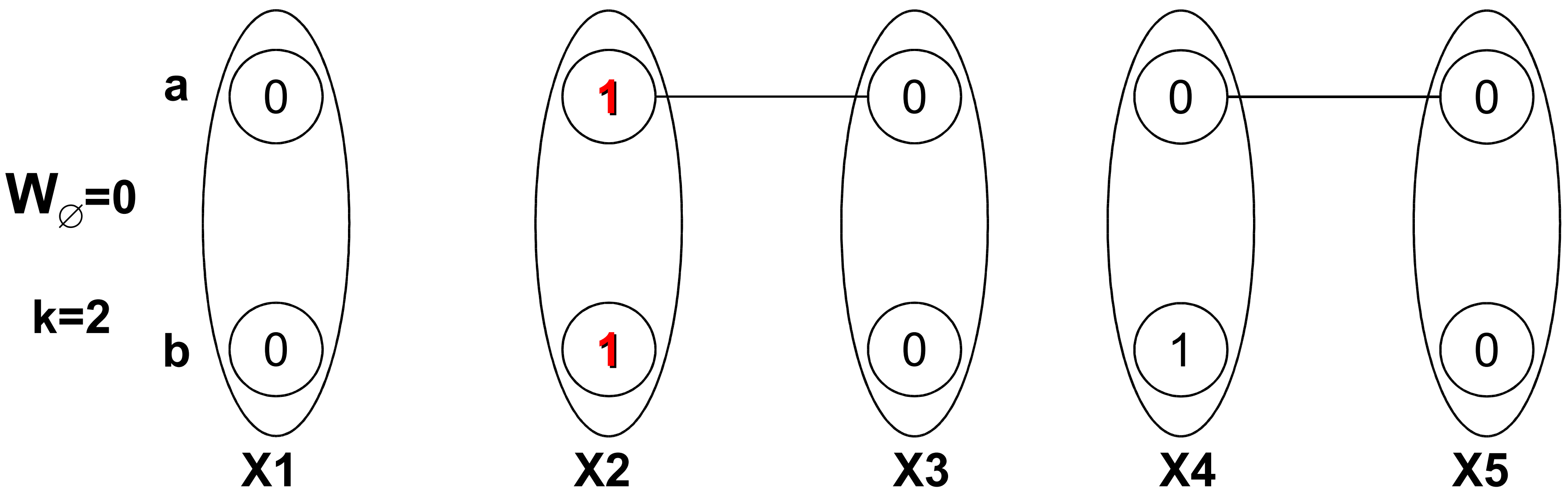
No idempotency property

Soft local consistency



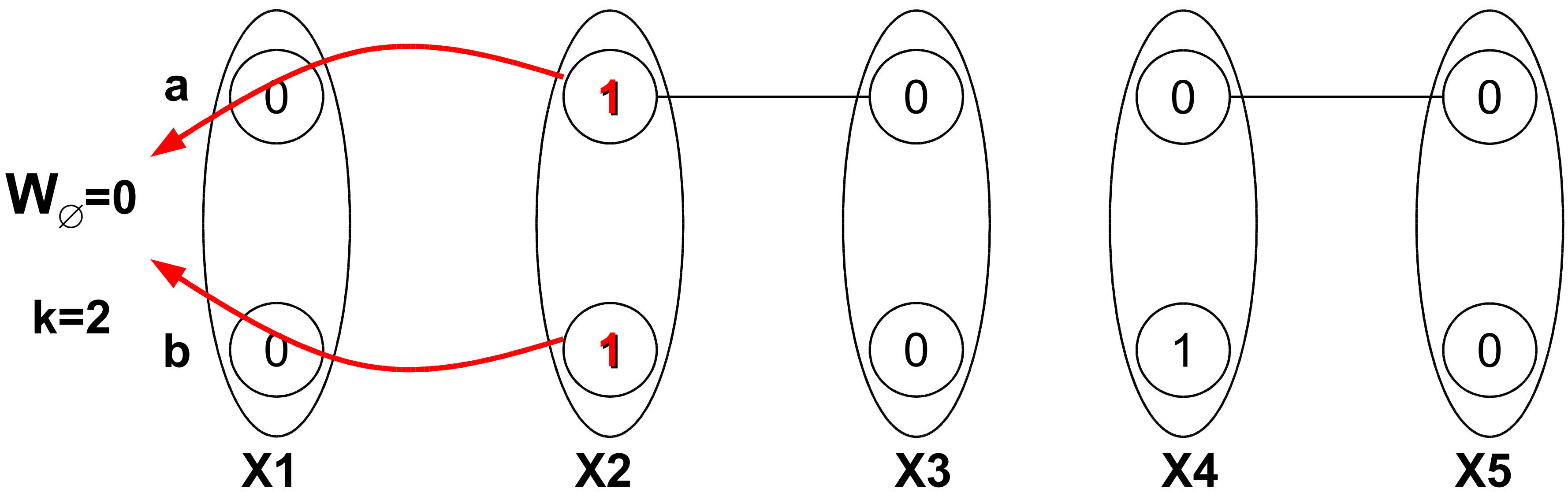
Two cost projections
from two binary constraints
to a unary constraint

Soft local consistency



Two cost projections from two binary constraints to a unary constraint

Soft local consistency

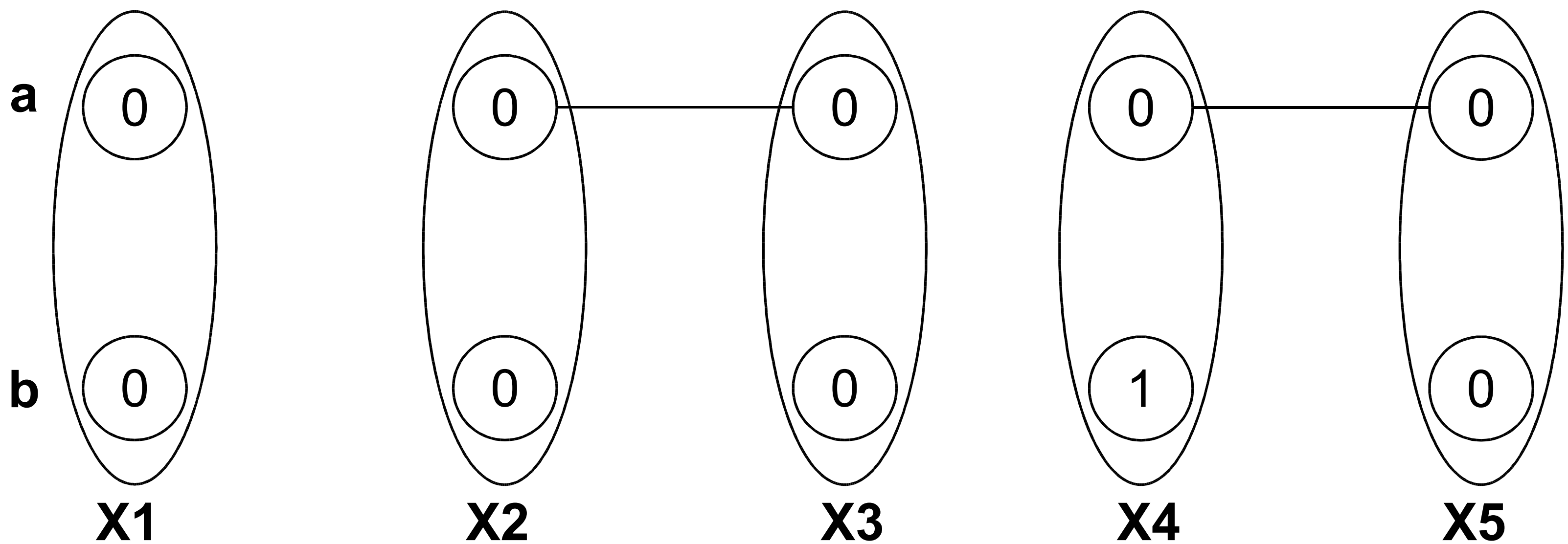


Cost projection from a unary to the zero-arity constraint

Soft local consistency

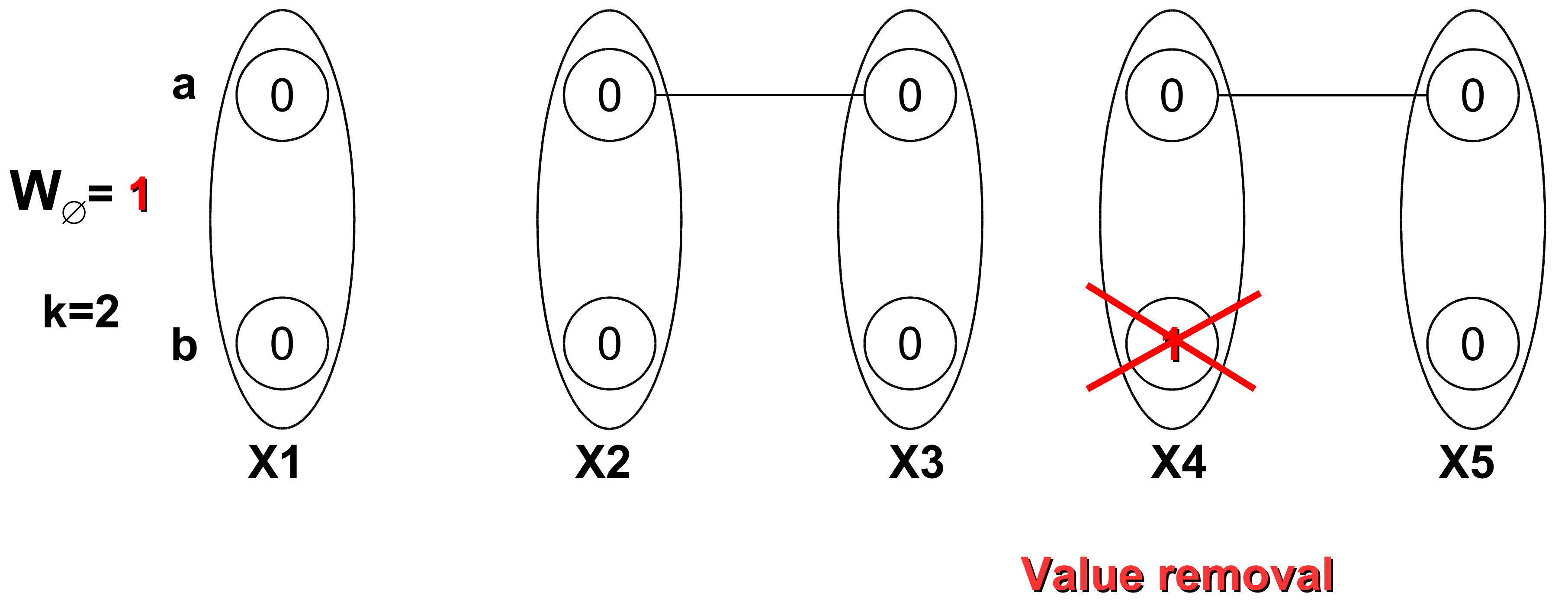
$W_{\varnothing} = 1$

k=2

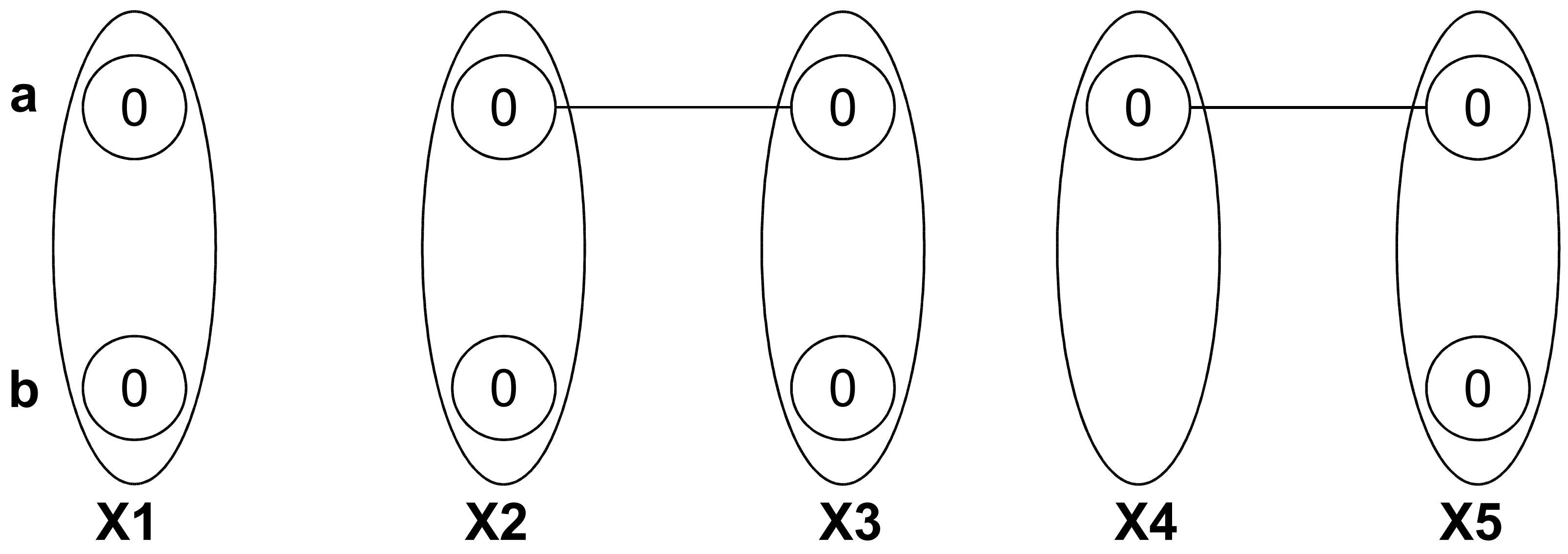


Cost projection from a unary to the zero-arity constraint

Soft local consistency



Soft local consistency

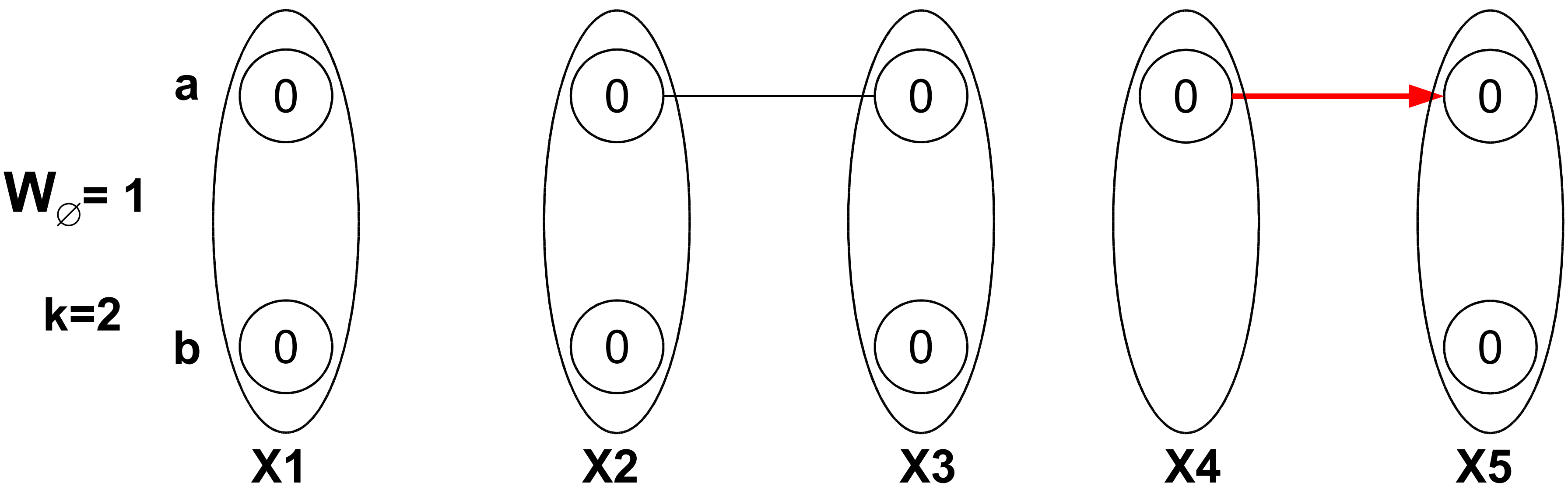


$W_{\varnothing} = 1$

k=2

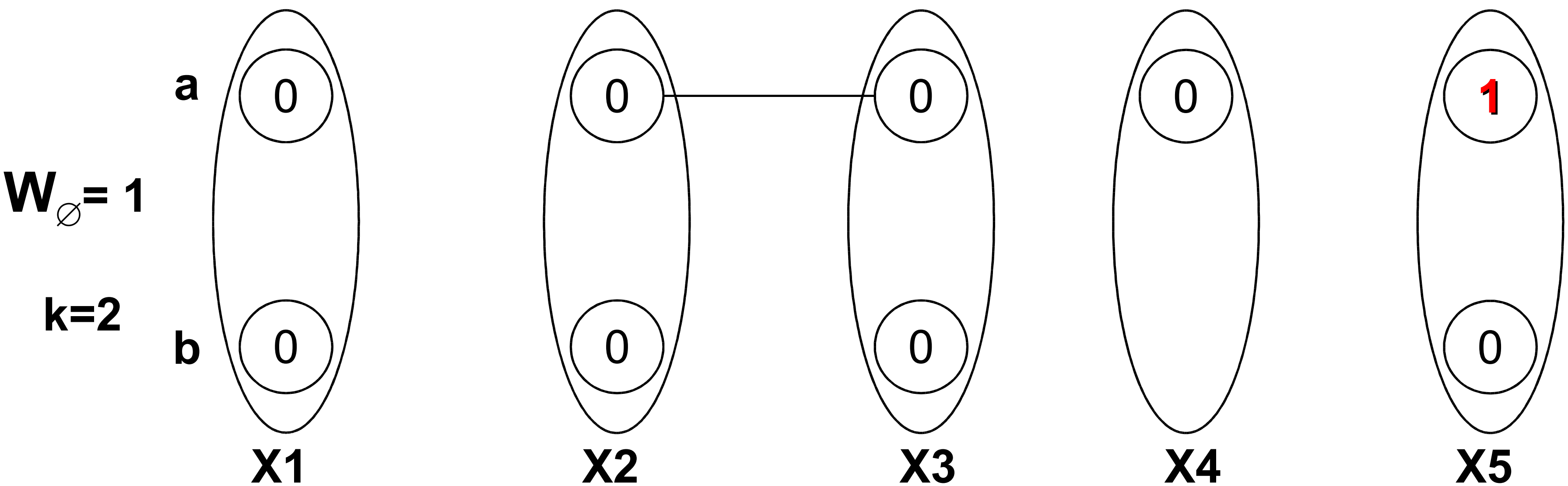
Value removal

Soft local consistency



Cost projection
from a binary to
a unary constraint

Soft local consistency

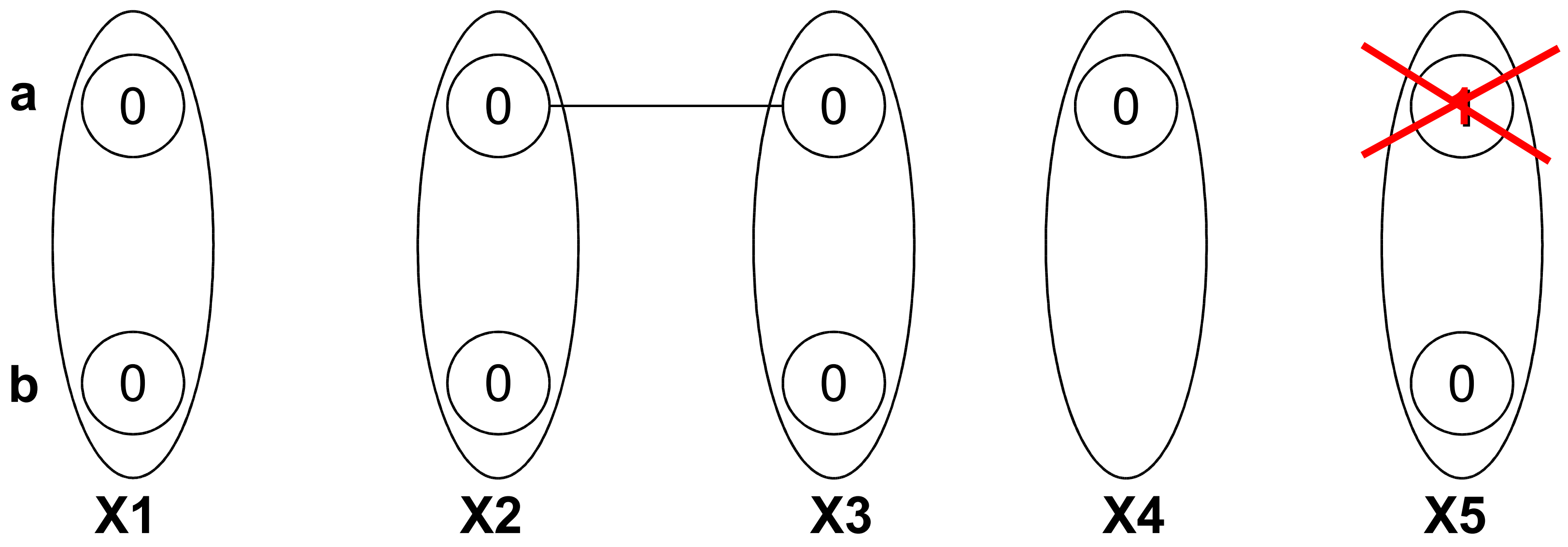


Cost projection
from a binary to
a unary constraint

Soft local consistency

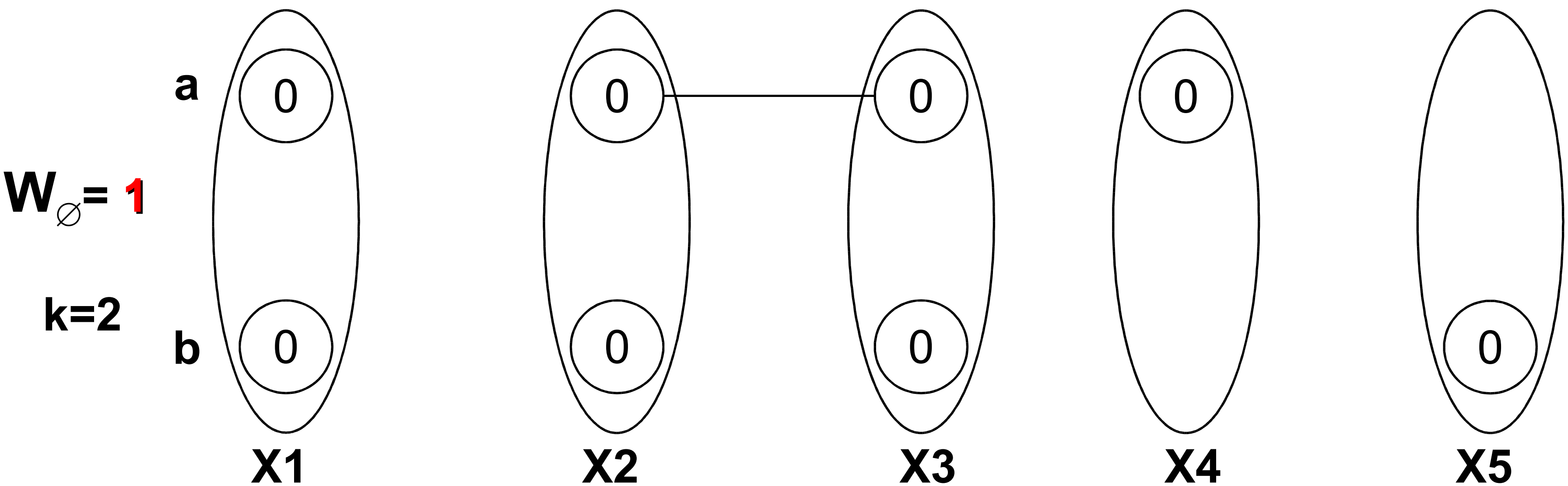
$W_\varnothing = 1$

k=2



Value removal

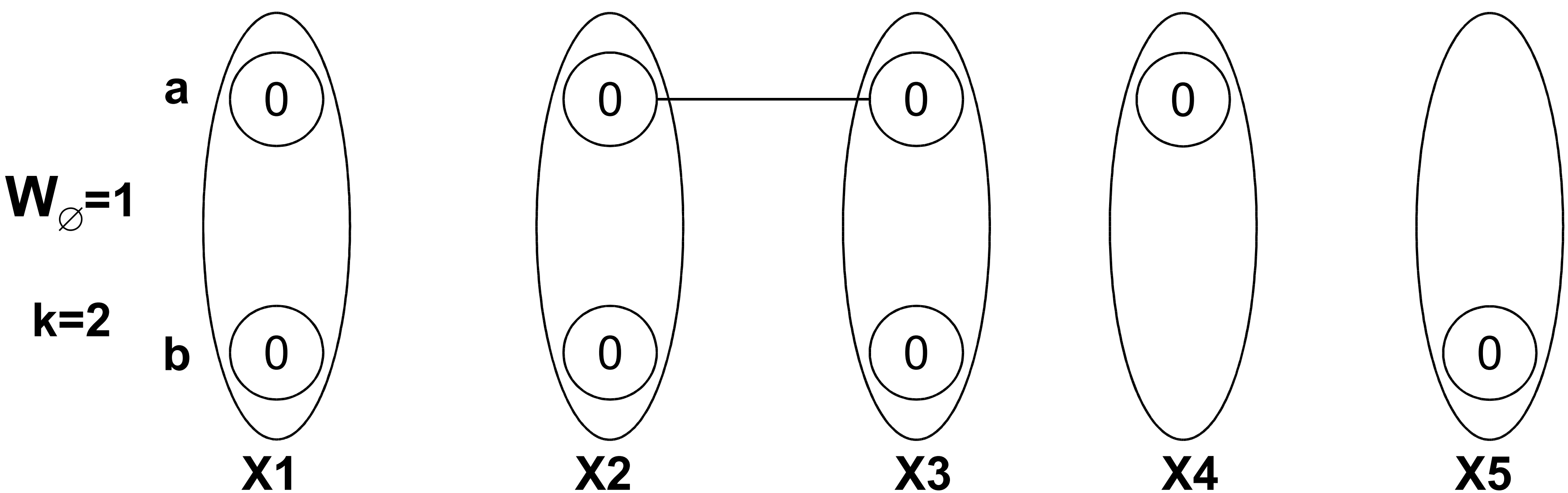
Soft local consistency



Value removal

Soft local consistency: various levels

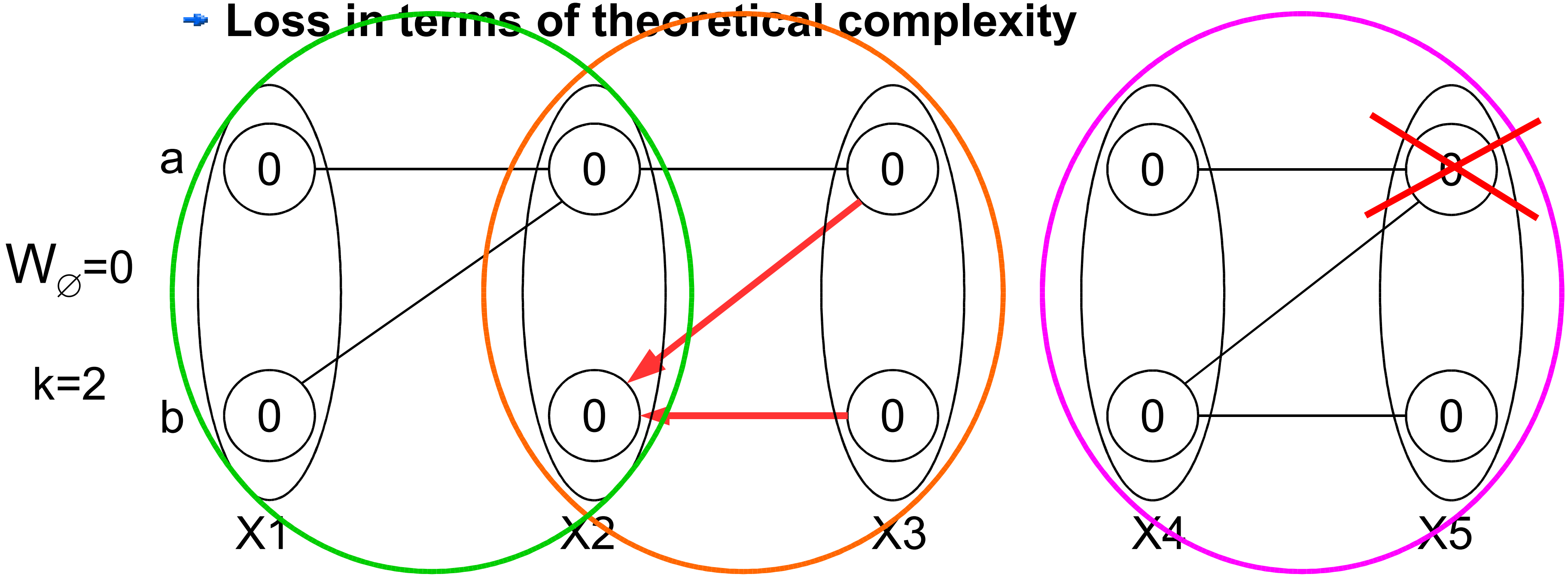
- **NC***: Node Consistency
- **AC***: Arc Consistency
- **DAC***: Directed Arc Consistency
- **FDAC***: Full Directed Arc Consistency



Tree decomposition and soft local consistency

Two main difficulties:

- **Costs are moving between clusters and towards $W_\varnothing$**
  - **Recorded subproblem lower bounds** may be **no longer valid**
- **Value removals may affect any cluster**
  - **No guarantee to improve the lower bound** when revisiting the same subproblem
  - **Loss in terms of theoretical complexity**



Three approaches considered

- **Limited form of soft local consistency (**forward-checking)
  - **FC-BTD** (Time: $O(d^w)$, Space: $O(d^s)$)
- **Limited soft arc consistency,** with corrected recorded lower bounds and value removals limited to the current subproblem
  - **FDAC-BTD+** (Time: $O(kd^w)$, Space: $O(d^s)$)
- **Unlimited soft arc consistency,** without learning
  - **FDAC-PTS** (Time: $O(d^h)$, Space: $O(nd)$)

Experimental results
Radio Link Frequency Allocation Problem

RLFAP	SUB_1		SUB_4		SCEN-06	
optimum	2669		3230		3389	
n, d, w, h	$14, 44, 13, 14$		$22, 44, 19, 21$		$100, 44, 19, 67$	
Method	time	#LB	time	#LB	time	#LB
FC-BTD	1197	0	-	0	-	-
NC-BTD+	490	0	-	0	-	-
FDAC-BTD+	14	0	929	0	10,309	326
FDAC-PTS	14	n/a	851	n/a	-	n/a
MFDAC	14	n/a	984	n/a	-	n/a

The first time the whole SCEN-06 instance is solved by a search algorithm

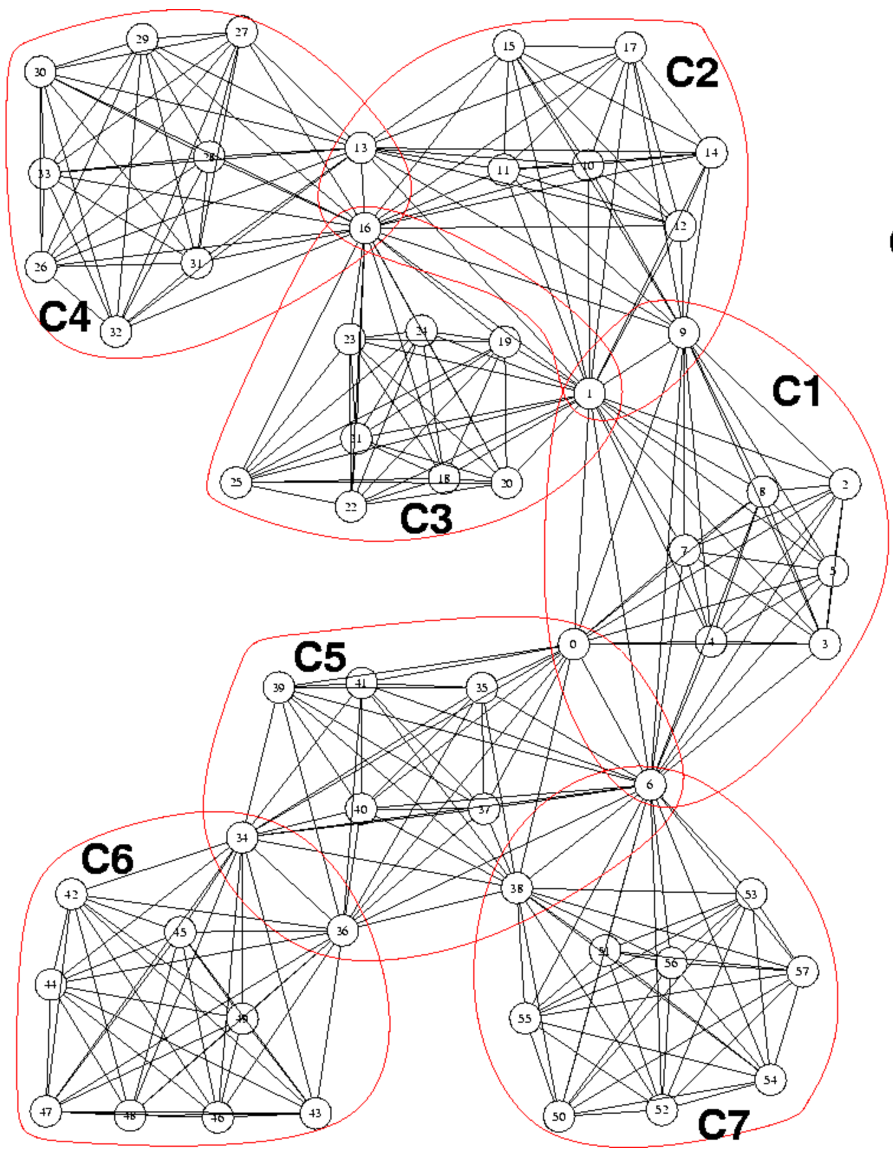
To be observed: small amount of memory required

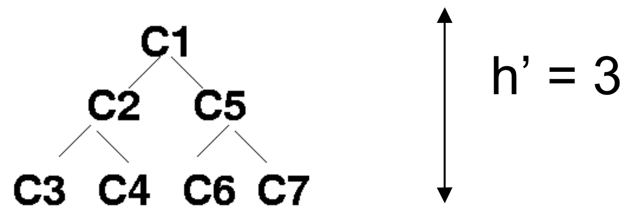
Conclusion

- **FDAC-BTD+**:
  - **Cluster tree problem decomposition**
  - **Tree search**
  - **Graph-based backjumping and learning**
  - **(Limited) soft arc consistency enforcing**
  - **Initial upper-bounds**

- **Cluster tree decomposition** and **soft local consistency** can be combined, but various technical options can be considered, and must be more widely experimented.

Random Binary Trees of Cliques





Parameters

- Clique size (w+1) = 10
- Separator size (s)
- Clique tree height (h') = 3
- Domain size (d) = 5
- Constraint tightness

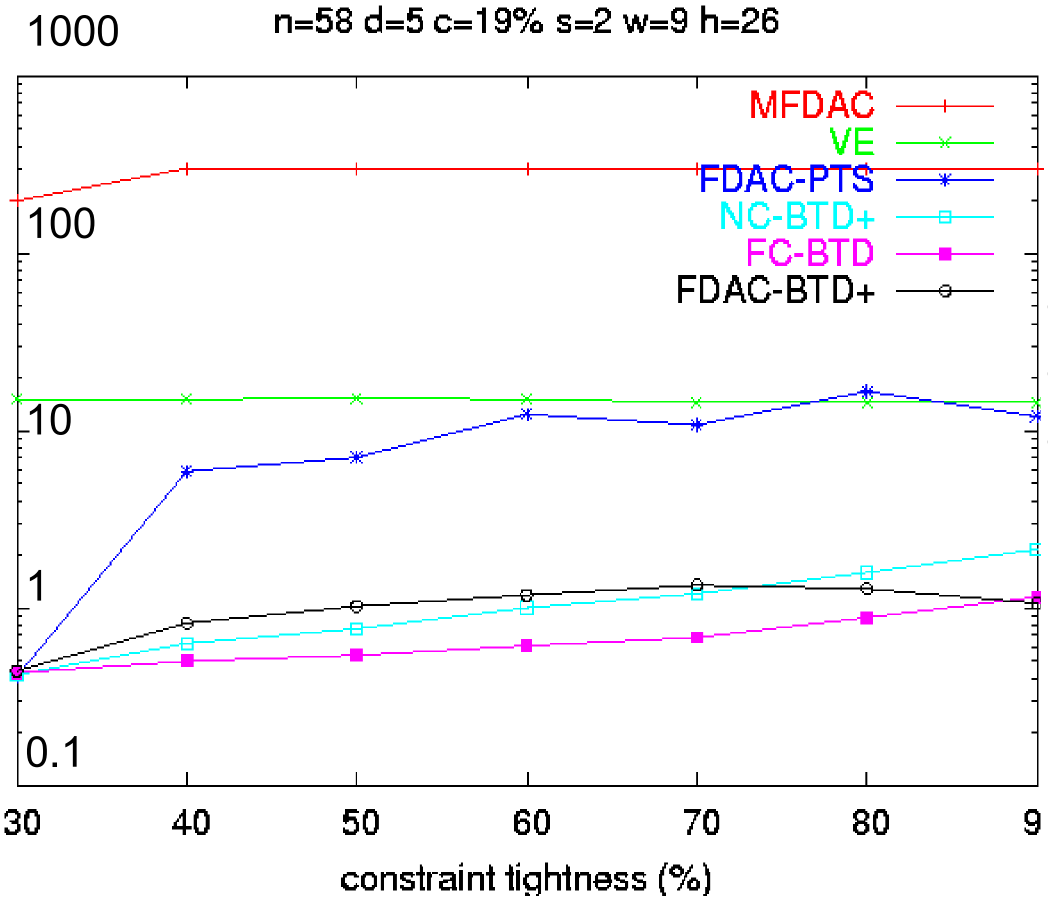
Tree decomposition based on Maximum Cardinality Search

Root selection minimizing tree-height (h)

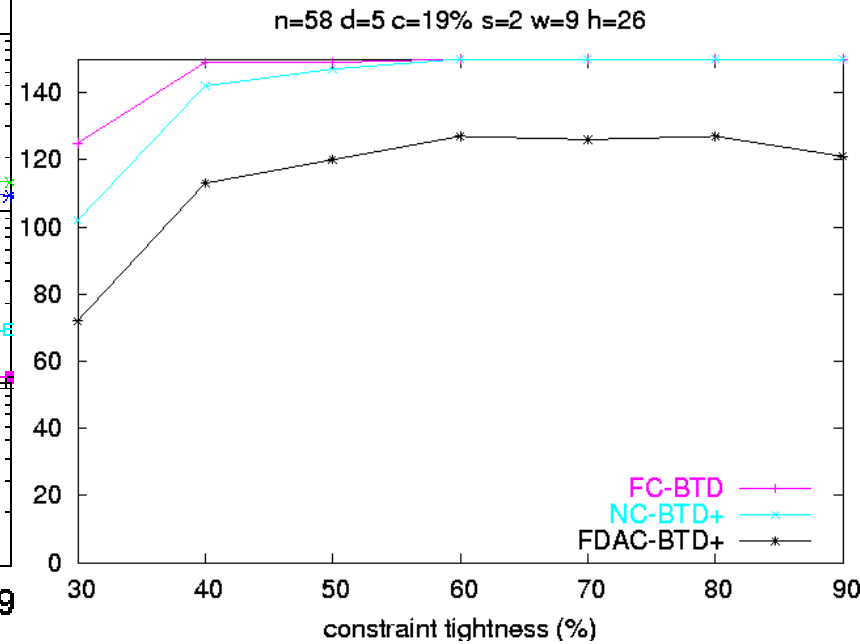
Random Binary Trees of Cliques

CPU-time (sec.)

Small-size separator s = 2



Memory space (#lb)

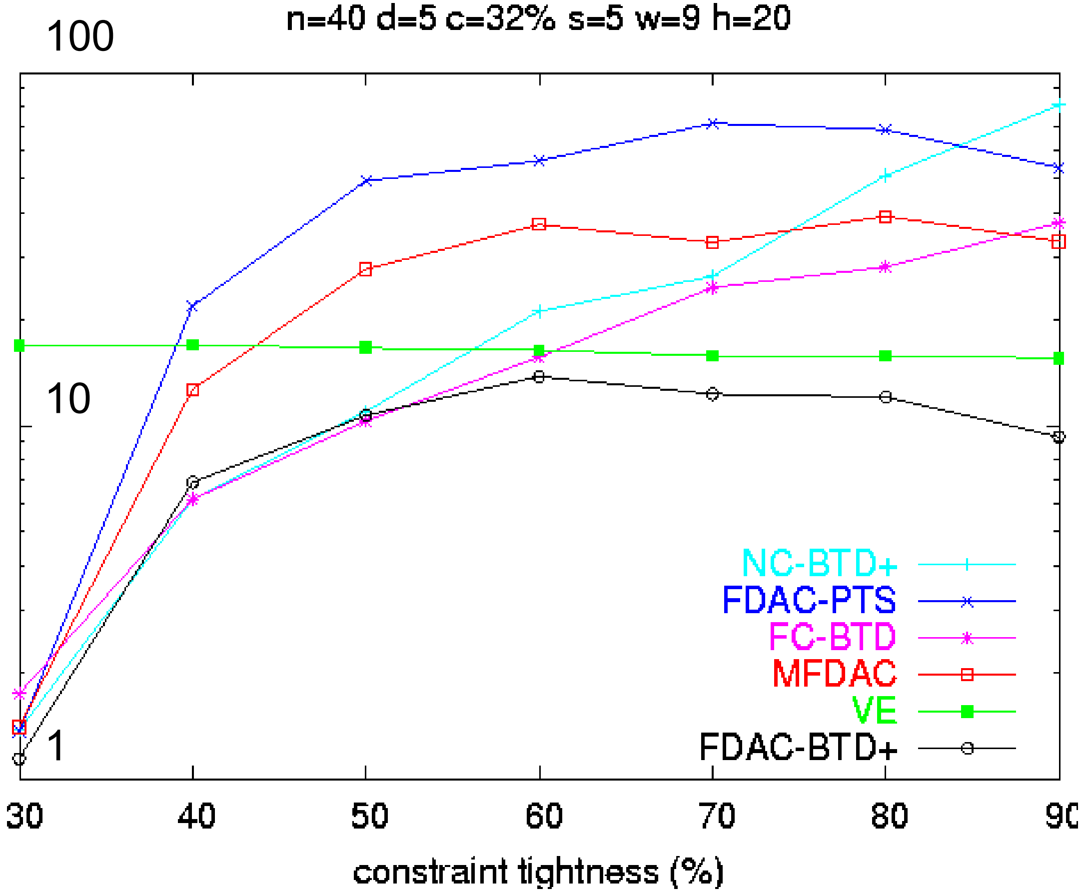


dom/deg dynamic variable ordering. CPU-time to find the optimum and prove its optimality

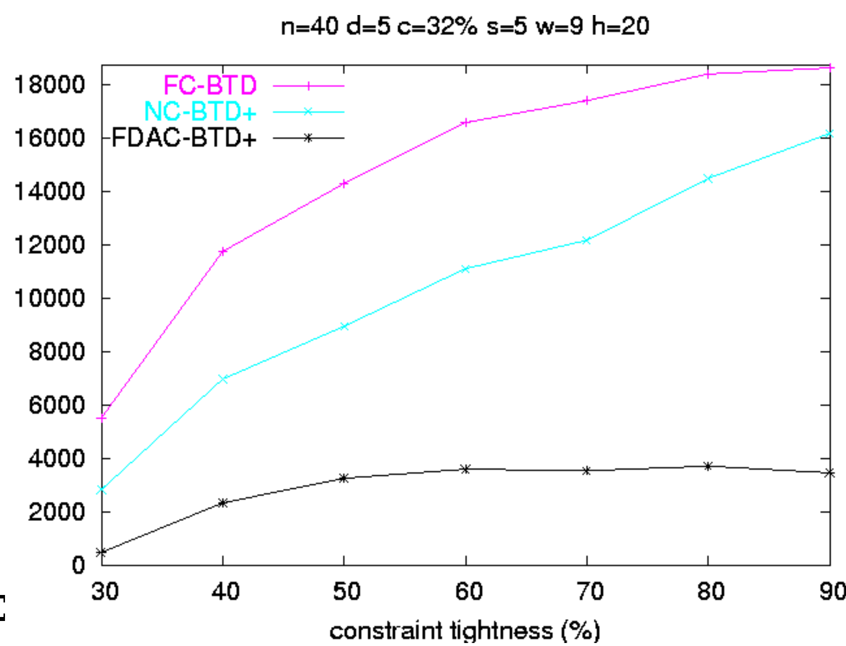
Random Binary Trees of Cliques

CPU-time (sec.)

Medium-size separator s = 5



Memory space (#lb)

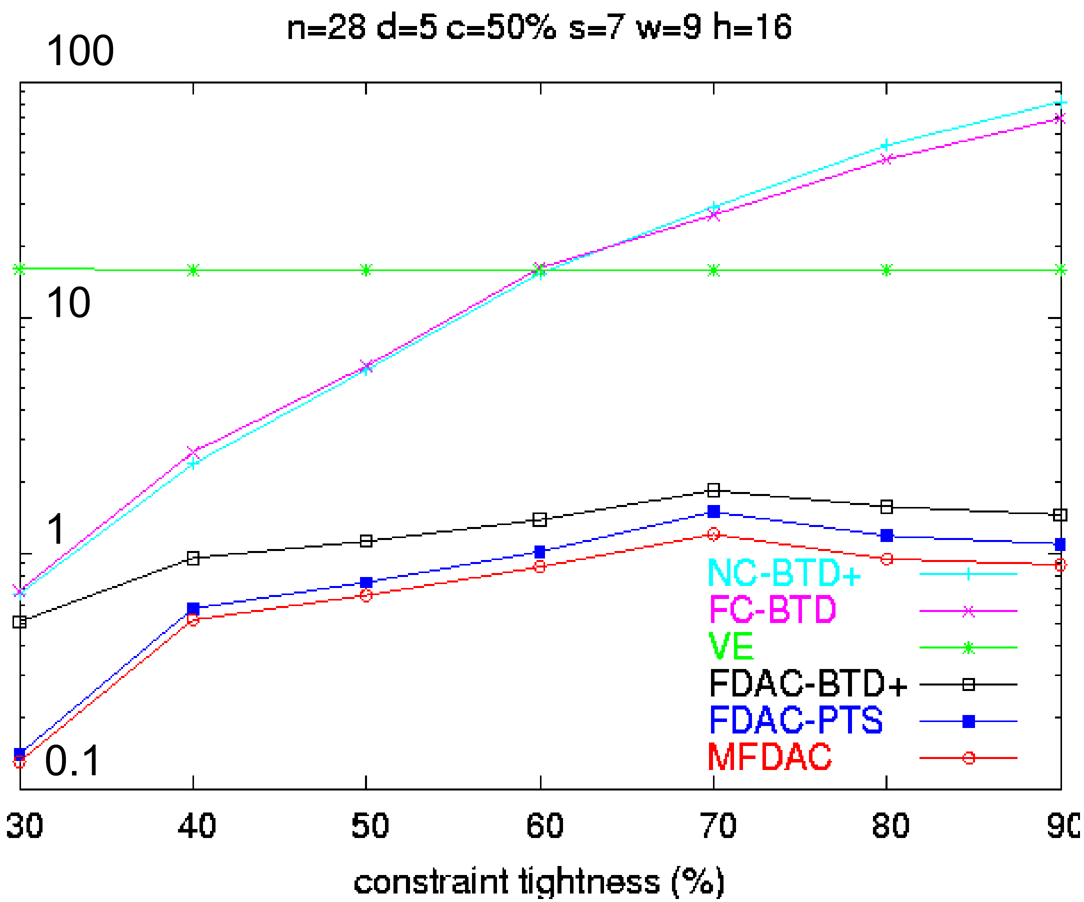


dom/deg dynamic variable ordering. CPU-time to find the optimum and prove its optimality

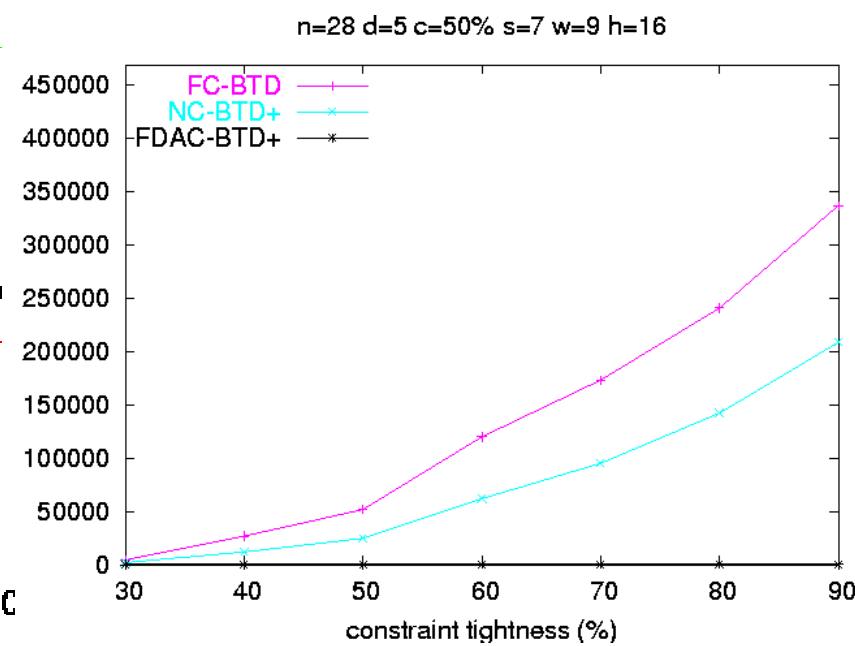
Random Binary Trees of Cliques

CPU-time (sec.)

Large-size separator s = 7



Memory space (#lb)



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