# Toulbar2 solver

Simon de Givry
INRAE, Toulouse

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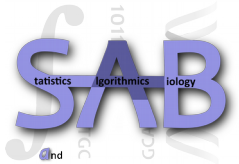



# Context

- Constraint Satisfaction Problems which are under/over constrained
  - Valued CSP (Schiex, Fargier, Verfaillie, IJCAI'1995) add preferences between solutions.

- Open-source branch-and-bound solver
  - *ToolBar (Toulouse & Barcelona)* (C code)

    (Schiex, Larrosa IJCAI'2003)
  - **Toulbar2** (C++ code)

    (Sanchez, Givry, Schiex, Constraints, 2008)

# Collaborations

Research in optimization algorithms


Applied research in *protein design*

# Optimization framework

$X$ set of discrete variables,

Minimize $F(X)$

such that $C(X)$ is satisfied

# Optimization framework

$X$ set of discrete variables,

$F$ set of cost functions, $f_S(X[S]) \in \mathbb{N}$

Minimize $\sum_{f_S \in F} f_S(X[S])$

such that $C(X)$ is satisfied

# Optimization framework

$X$ set of discrete variables,

$F$ set of cost functions,

$T$ infinite cost (forbidden assignment),

$$\text{Minimize} \sum_{f_S \in F} f_S(X[S])$$

Cost Function Network (CFN)

# Optimization framework

$X$ set of discrete variables,

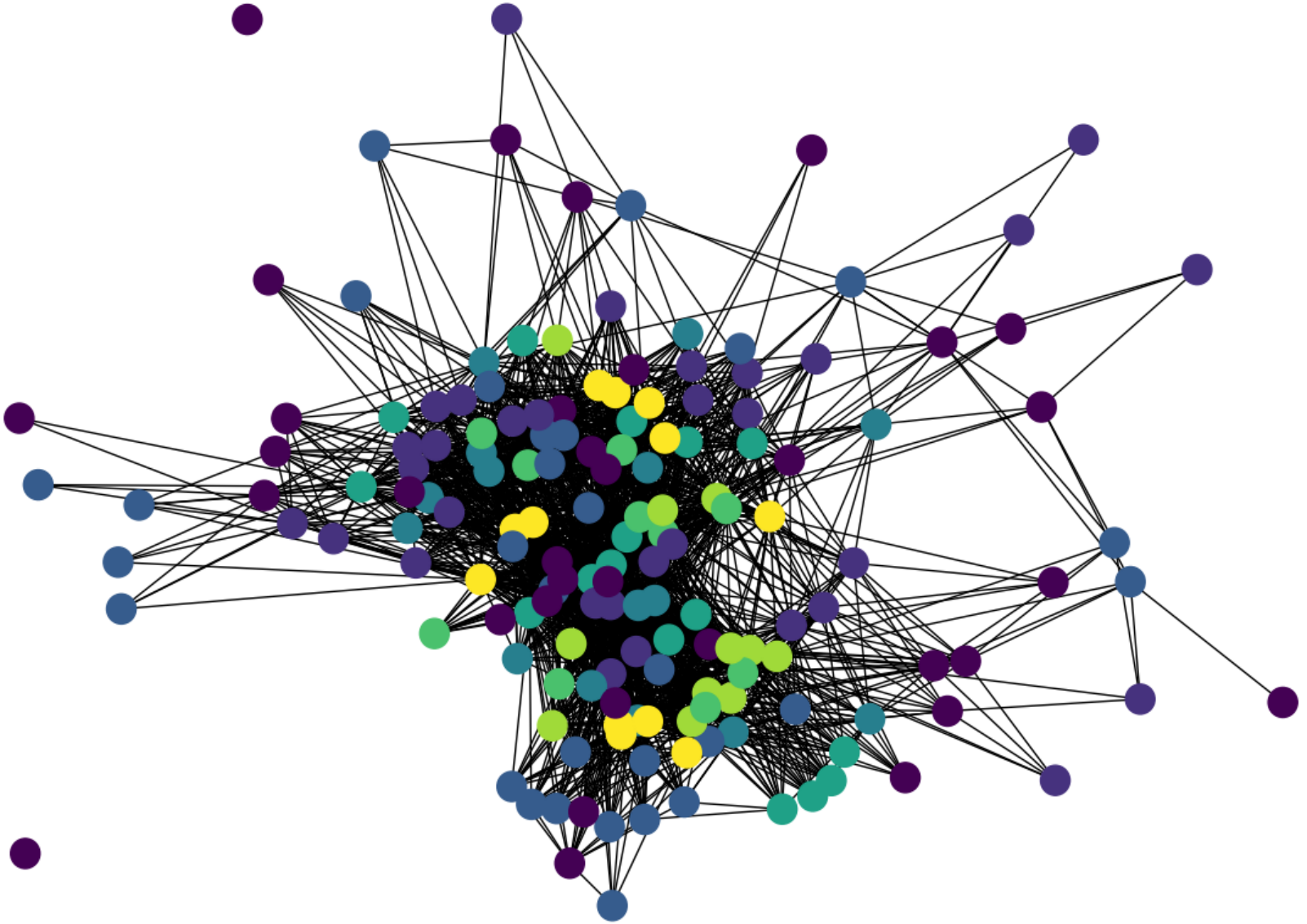
$F$ set of cost functions,

$T$ infinite cost (forbidden assignment),

Minimize $\sum_{f_S \in F} f_S(X[S])$

NP-hard problem

# Graphical Model



| X | f1(X) |
|---|---|
| a | 0 |
| c | 1 |
| g | 1 |
| t | 1 |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | 1 |
| | g | 2 |
| | t | 0 |
| c | a | 1 |
| | c | ∞ |
| | g | 0 |
| | t | 2 |
| g | a | 2 |
| | c | 0 |
| | g | ∞ |
| | t | 1 |
| t | a | 0 |
| | c | 2 |
| | g | 1 |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | 1 |
| g | 1 |
| t | 1 |

Minimize f1(X) + f2(X,Y) + f3(Y)

# Cost function in extension

| X | Y | Z | f(X,Y,Z) |
|---|---|---|---|
| a | a | a | ∞ |
| | a | c | 1 |
| | a | g | 2 |
| | a | t | 0 |
| a | c | a | 1 |
| | c | c | 1 |
| | c | g | 0 |
| | c | t | 2 |
| a | g | a | 2 |
| | g | c | 0 |
| | g | g | 2 |
| | g | t | 1 |
| a | t | a | 0 |
| | t | c | 2 |
| | t | g | 1 |
| | t | t | 0 |

… 4*4*4 = 64 assignments!

# Cost function using a compact table

| X1 | X2 | X3 | X4 | X5 | X6 | X7 | f(X1,X2,X3,X4,X5, X6,X7) |
|---|---|---|---|---|---|---|---|
| g | a | t | t | a | c | a | 1 |
| Other tuples (i.e., default cost): | | | | | | | 0 |

Here, it is a soft clause.

# Cost function in intention

**alldiff**(*X*), **among**(*X*,params), **gcc**(*X*,params), **regular**(*X*,rules), **grammar**(*X*,rules), **clique**(*X*,params), **knapsack**(*X*,weights)

# *knapsack* constraints

- Pseudo-Boolean linear constraint $X_i \in \{0,1\}$

$$2 * X_1 + 3 * X_2 + 4 * X_3 + 5 * X_4 \geqslant 10$$

**knapsack**({$X_1$, $X_2$, $X_3$, $X_4$}, ‘10 2 3 4 5’)

- Linear constraint $X_i \in \{0,1,2\}$

$$2 * X_1 + 3 * X_2 + 4 * X_3 + 5 * X_4 \leqslant 10$$

**knapsackp**({$X_1$, $X_2$, $X_3$, $X_4$}, ‘-10 2 1 -2 2 -4 2 1 -3 2 -6 2 1 -4 2 -8 2 1 -5 2 -10’)

# *knapsack* constraints

- Multiple-choice knapsack constraint $X_i \in D$

**knapsackp**($X$, <capacity> (<nbval> (<value> <weight>)*)*)

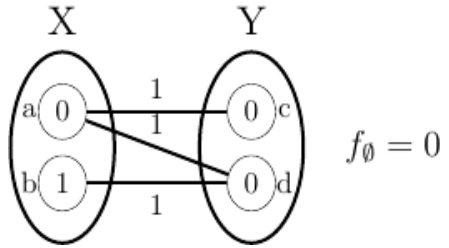
$$weight \in \mathbb{Z}$$

- Hamming distance constraint

$$(X_1 = 2) + (X_2 = 3) + (X_3 = 4) + (X_4 = 5) \leqslant 2$$

**knapsackp**({$X_1$, $X_2$, $X_3$, $X_4$}, '-2 1 2 -1 1 3 -1 1 4 -1 1 5 -1')

# Exact solving methods




Branch-and-bound combined with equivalence preserving transformations

- FDAC (Schiex, Larrosa, IJCAI'2003)
- EDAC (Heras, Larrosa, Givry, Zytnicki, IJCAI'2005)
- VAC (Cooper, Givry, Sanchez, Schiex, Zytnicki, AAAI'2008) (Trösser, Givry, Katsirelos, CPAIOR'2020)
- OSAC (Cooper, Givry, Schiex, IJCAI'2007)
- EDmaxRPC (Nguyen, Bessiere, Givry, Schiex, Constraints 2017)
- *VSAC-SR* (Dlask, Werner Givry, CP'2021)

| X | f1(X) |
|---|---|
| a | 0 |
| c | 1 |
| g | 1 |
| t | 1 |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | 1 |
| | g | 2 |
| | t | **1** |
| c | a | 1 |
| | c | ∞ |
| | g | 0 |
| | t | **3** |
| g | a | 2 |
| | c | 0 |
| | g | ∞ |
| | t | **2** |
| t | a | 0 |
| | c | 2 |
| | g | 1 |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | 1 |
| g | 1 |
| t | **0** |

| X | f1(X) |
|---|---|
| a | **1** |
| c | 1 |
| g | 1 |
| t | 1 |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | **0** |
| | g | **1** |
| | t | **0** |
| c | a | 1 |
| | c | ∞ |
| | g | 0 |
| | t | **3** |
| g | a | 2 |
| | c | 0 |
| | g | ∞ |
| | t | **2** |
| t | a | 0 |
| | c | 2 |
| | g | 1 |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | 1 |
| g | 1 |
| t | **0** |

| X | f1(X) |
|---|---|
| a | **1** |
| c | 1 |
| g | 1 |
| t | 1 |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | **0** |
| | g | **2** |
| | t | **0** |
| c | a | 1 |
| | c | ∞ |
| | g | **1** |
| | t | **3** |
| g | a | 2 |
| | c | 0 |
| | g | ∞ |
| | t | **2** |
| t | a | 0 |
| | c | 2 |
| | g | **2** |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | 1 |
| g | **0** |
| t | **0** |

| X | f1(X) |
|---|---|
| a | **1** |
| c | **2** |
| g | 1 |
| t | 1 |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | **0** |
| | g | **2** |
| | t | **0** |
| c | a | **0** |
| | c | ∞ |
| | g | **0** |
| | t | **2** |
| g | a | 2 |
| | c | 0 |
| | g | ∞ |
| | t | **2** |
| t | a | 0 |
| | c | 2 |
| | g | **2** |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | 1 |
| g | **0** |
| t | **0** |

| X | f1(X) |
|---|---|
| a | **1** |
| c | **2** |
| g | 1 |
| t | 1 |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | **1** |
| | g | **2** |
| | t | **0** |
| c | a | **0** |
| | c | ∞ |
| | g | **0** |
| | t | **2** |
| g | a | 2 |
| | c | **1** |
| | g | ∞ |
| | t | **2** |
| t | a | 0 |
| | c | 3 |
| | g | **2** |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | **0** |
| g | **0** |
| t | **0** |

| X | f1(X) |
|---|---|
| a | **1** |
| c | **2** |
| g | **2** |
| t | 1 |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | **1** |
| | g | **2** |
| | t | **0** |
| c | a | **0** |
| | c | ∞ |
| | g | **0** |
| | t | **2** |
| g | a | **1** |
| | c | **0** |
| | g | ∞ |
| | t | **1** |
| t | a | 0 |
| | c | 3 |
| | g | **2** |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | **0** |
| g | **0** |
| t | **0** |

| X | f1(X) |
|---|---|
| a | **0** |
| c | **1** |
| g | **1** |
| t | **0** |

+

| X | Y | f2(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | **1** |
| | g | **2** |
| | t | **0** |
| c | a | **0** |
| | c | ∞ |
| | g | **0** |
| | t | **2** |
| g | a | **1** |
| | c | **0** |
| | g | ∞ |
| | t | **1** |
| t | a | 0 |
| | c | 3 |
| | g | **2** |
| | t | ∞ |

+

| Y | f3(Y) |
|---|---|
| a | 0 |
| c | **0** |
| g | **0** |
| t | **0** |

$f_{\varnothing} = 1$

$f_{\varnothing}$ **is the current problem lower bound !**

$f_{X_i}$ **used by value and variable heuristics**

$T = 1$

| X | f(X) |
|---|---|
| a | **0** |
| c | **∞** |
| g | **∞** |
| t | **0** |

+

| X | Y | g(X,Y) |
|---|---|---|
| a | a | ∞ |
| | c | **∞** |
| | g | **∞** |
| | t | **0** |
| c | a | **0** |
| | c | ∞ |
| | g | **0** |
| | t | **∞** |
| g | a | **∞** |
| | c | **0** |
| | g | ∞ |
| | t | **∞** |
| t | a | 0 |
| | c | **∞** |
| | g | **∞** |
| | t | ∞ |

+

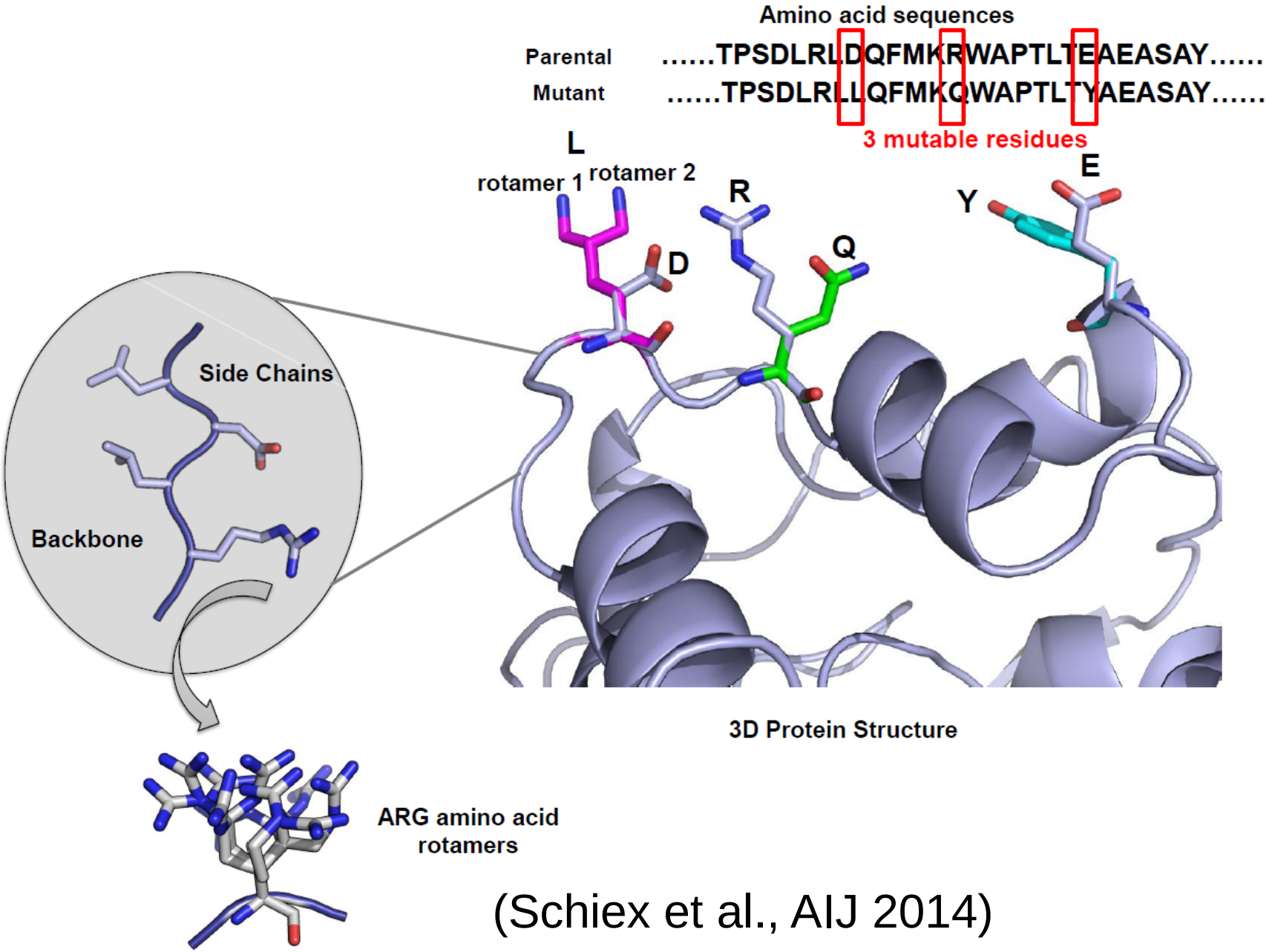
| Y | f(Y) |
|---|---|
| a | 0 |
| c | **0** |
| g | **0** |
| t | **0** |

$f_{\varnothing} = 1$

**$T$ is decreasing at each new solution found!**

**More value pruning!!**

# Protein design




(Schiex et al., AIJ 2014)

# Protein design

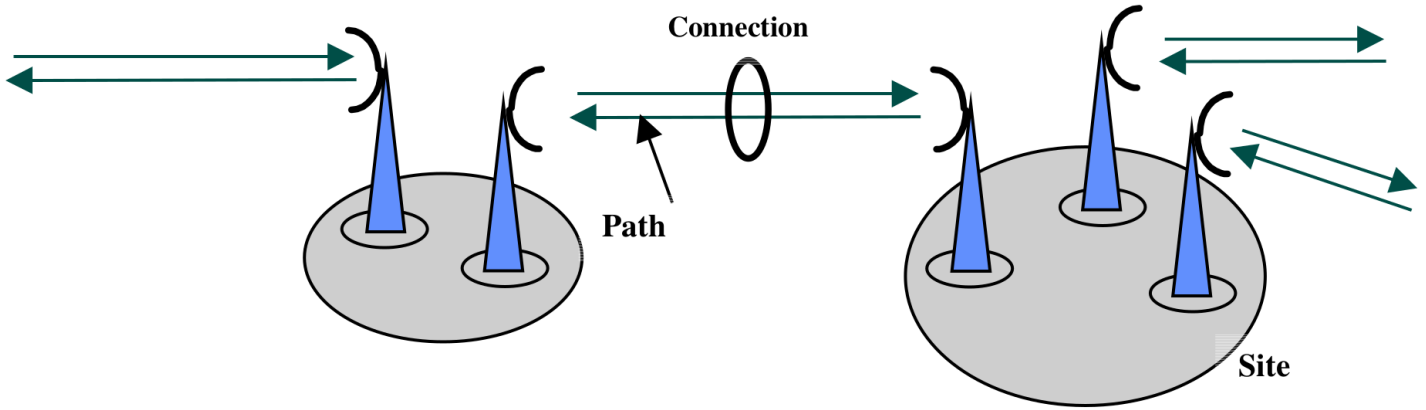
$$E = E_{\varnothing} + \sum_i E(i_r) + \sum_i \sum_{j>i} E(i_r, j_s)$$

| | CPLEX 20.1 | toulbar2 1.1.1 |
|---|---|---|
| 1BK2.matrix.24p.17aa | 42.84 sec (0 node)<br><br>Reduced MIP has<br>16,090 rows<br>284,424 columns<br>584,143 nonzeros<br>805 binaries | 0.37 sec (38 nodes)<br><br>\|X\|=24<br>max(\|D\|)=182<br>\|F\|=300 |

(Intel Xeon 2.5GHz with 256GB RAM)

# Frequency Assignment Problem with Polarization

## Challenge ROADEF 2001




The problem is to assign a pair (frequency f, polarization p) to every path.

$$1 \le f \le \sim 151 \quad p \in \{-1,1\}$$

Constraints are:

(I) two paths must use equal or different frequencies
(II) the absolute difference between two frequencies should exactly be equal or different to a given number
(III) two paths must use equal or different polarizations
(IV) the absolute difference between two frequencies should be greater at a relaxation level *l* (0 to 10) than a given number *g_l* (resp. *d_l*) if polarization are equal (resp. different, usually *g_l* > *d_l*)

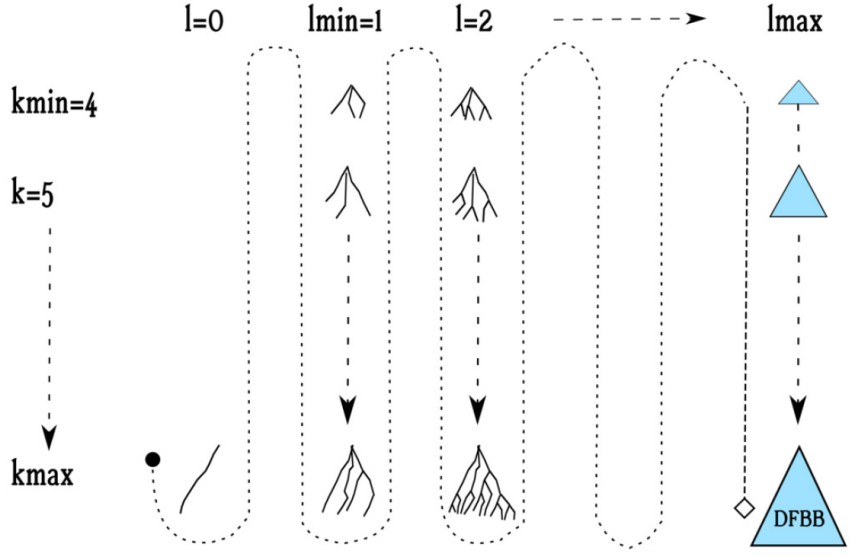
Find a feasible assignment with the smallest relaxation level *l*
and which minimizes the number of violations of (IV) at lower levels

# Frequency Assignment Problem with Polarization

- Combine frequency with polarization into a single variable
  - Better propagation (only binary cost functions)

- Use Parallel Variable Neighborhood Search
  (Ouali et al., AIJ 2020)

- See our CP'2020 tutorial



# Frequency Assignment Problem with Polarization

| Instance | k | ROADEF'2001 Best Results | toulbar2 | (time to best solution) |
|---|---|---|---|---|
| fapp01_0200 | 4 | 35,758,830 | **35,758,826** | (1.333s) |
| fapp02_0250 | 2 | 40,370,567 | **40,370,566** | (16.689s) |
| fapp03_0300 | 7 | 294,380,136 | **294,380,135** | (26.155s) |
| fapp04_0300 *\|X\|=300 max(\|D\|)=270 \|F\|=2100* | 1 | 24,648,190 | **24,616,970** | (3.282s) *(optimality proof in 27 seconds!)* |
| fapp05_0350 *\|X\|=350 max(\|D\|)=270 \|F\|=2839* | 11 | **521,348,004** | 521,348,429 | (53.496s) *(521,348,004 in 89.788 seconds)* |
| test01_0150 | 4 | 18,292,591 | **18,292,585** | (1.691s) |

ROADEF competitors: 1 hour on Intel Pentium III 500MHz 128MB
Toulbar2: 1 minute on 21 cores of Intel Xeon 2.5GHz 256GB

# Grid operation-based outage maintenance planning

## Challenge ROADEF 2020

Find an optimal planning regarding a risk-based objective

$$18 \leqslant |I| \leqslant 706$$

Planning horizon within a year (day or week discretization)

$$53 \leqslant H \leqslant 365$$

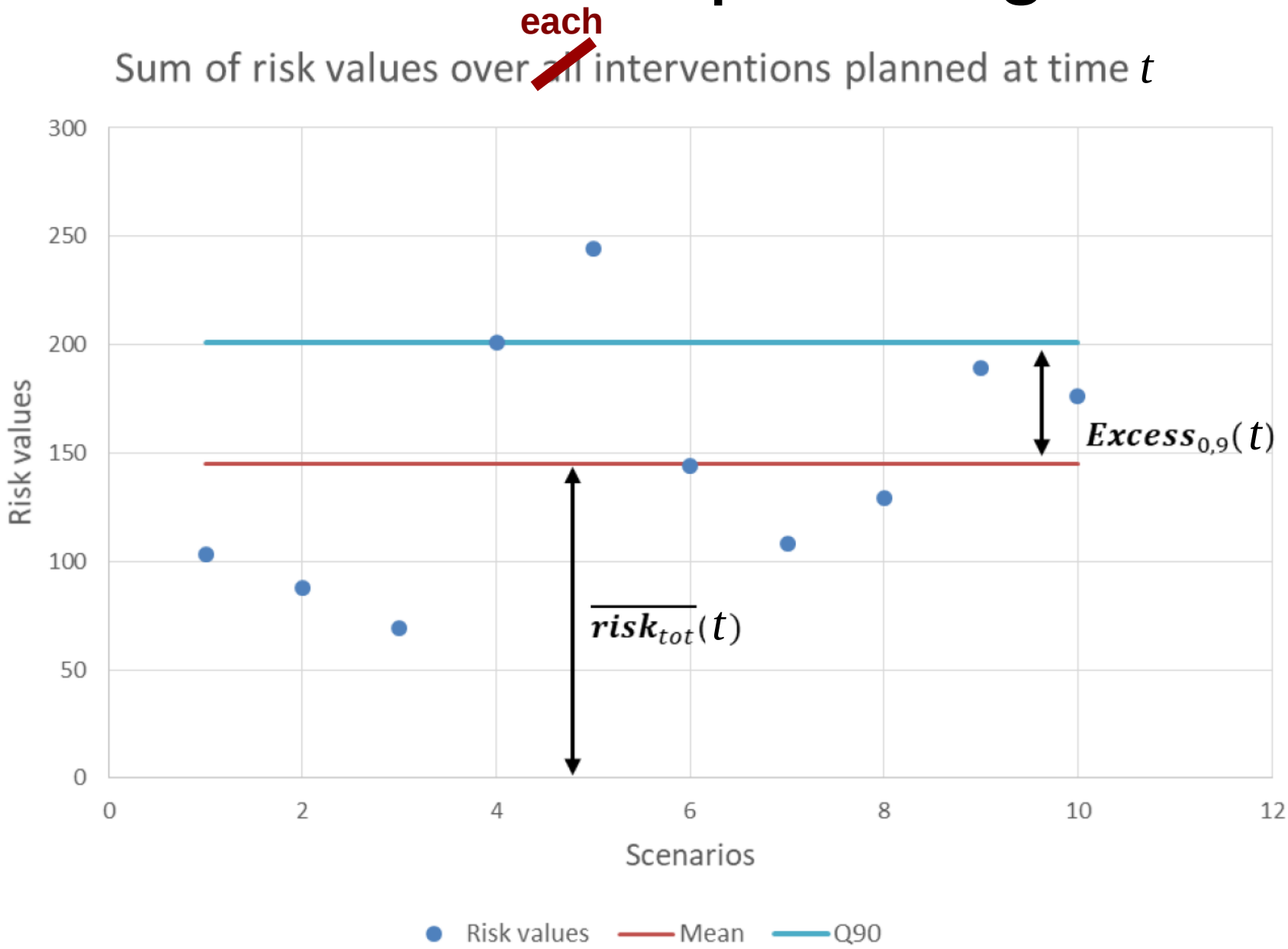
# Grid operation-based outage maintenance planning

## CFN model

- $|I|$ variables (start time of each intervention) with domain size $H$

- Two approximations
  - Resource constraint (using knapsackp)
    - Integer approximation of continuous weights (r multiplier)
  - Risk-based objective decomposed into $|I|$ cost functions for every intervention

# Grid operation-based outage maintenance planning

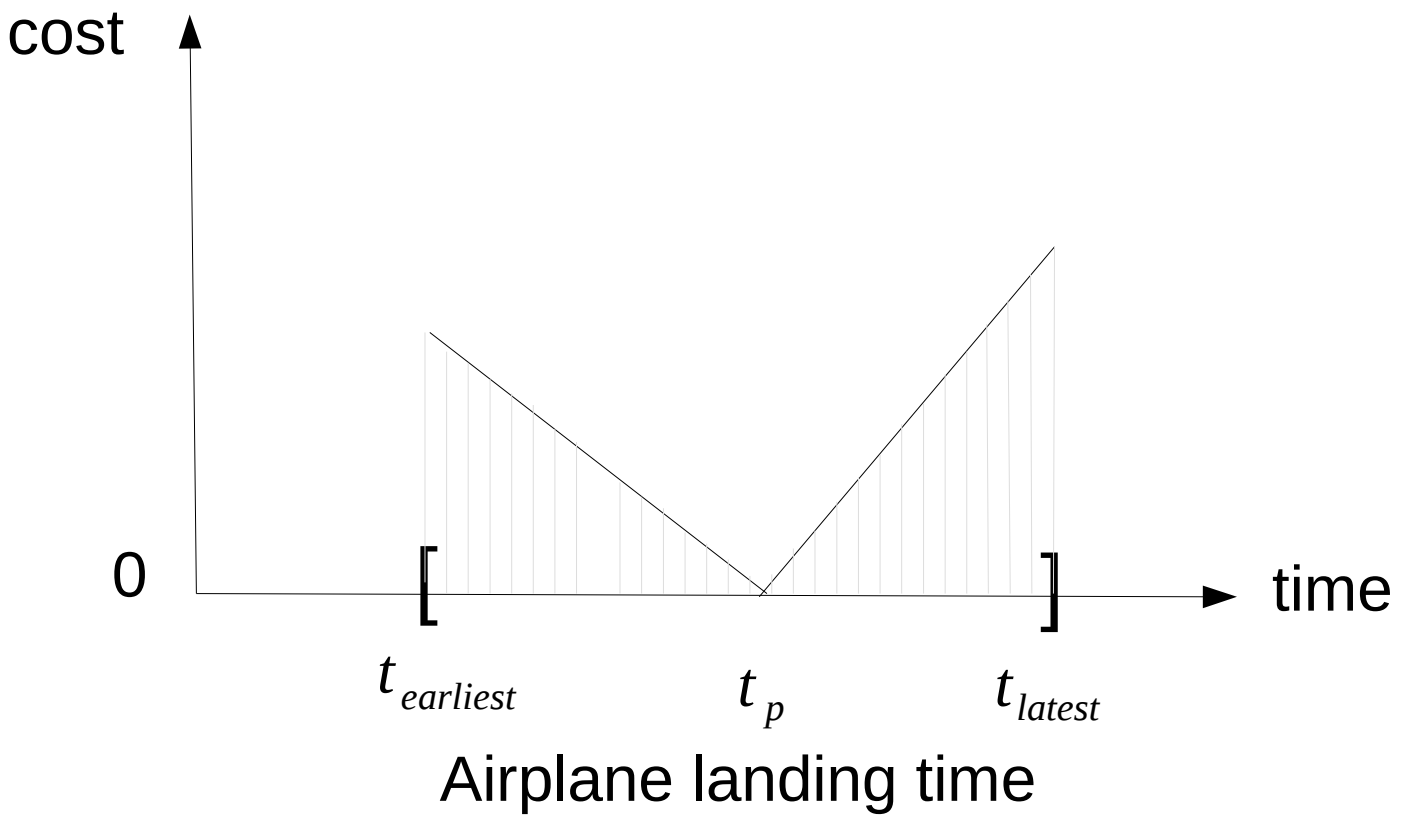


# Grid operation-based outage maintenance planning

| Instance | Best Results 15 min | toulbar2 15min *r=10* | Best Results 90 min | toulbar2 1hour *r=100* |
|---|---|---|---|---|
| A_01(\|I\|=181,H=90) | 1,767.82 | 1,770.15 | 1,767.82 | 1,769.50 |
| A_02(\|I\|=89,H=90) | 4,671.38 | 4,732.05 *(9.870s)* | 4,671.38 | 4,732.02 *(9.132s)* |
| A_03(\|I\|=91,H=90) | 848.18 | 848.18 *(5.060s)* | 848.18 | 848.18 *(4.325s)* |
| A_04(\|I\|=706,H=365) | 2,085.88 | - | 2,085.88 | - |
| A_05(\|I\|=180,H=182) | 635.22 | 651.49 *(50.63s)* | 635.22 | 651.24 *(49.59s)* |
| A_06(\|I\|=180,H=182) | 590.62 | - | 590.62 | 640.69 |
| A_07(\|I\|=36,H=17) | 2,272.78 | 2,280.16 *(0.103s)* | 2,272.78 | - |
| A_08(\|I\|=18,H=17) | 744.29 | 750.03 *(0.269s)* | 744.29 | 750.03 *(0.252s)* |
| A_09(\|I\|=18,H=17) | 1,507.28 | 1,507.32 *(0.063s)* | 1,507.28 | 1,507.32 *(0.061s)* |
| A_10(\|I\|=108,H=53) | 2,994.85 | 3,022.76 *(1.452s)* | 2,994.85 | 3,022.76 *(1.623s)* |
| A_11(\|I\|=54,H=53) | 495.26 | 504.95 *(2.544s)* | 495.26 | 504.95 *(2.452s)* |
| A_12(\|I\|=54,H=53) | 789.63 | 828.88 *(0.668s)* | 789.63 | 793.04 *(1.289s)* |
| A_13(\|I\|=179,H=90) | 1,998.66 | 2,009.79 | 1,998.66 | 2,009.66 |
| A_14(\|I\|=108,H=53) | 2,264.12 | 2,302.56 *(90.13s)* | 2,264.12 | 2,295.65 *(1,215s)* |
| A_15(\|I\|=108,H=53) | 2,268.57 | 2,310.25 *(113.4s)* | 2,268.57 | 2,305.29 *(1,036s)* |
| *Distance to best results (except A04,A06,A07):* | | *1.39%* | | *0.96%* |

# Some Tips

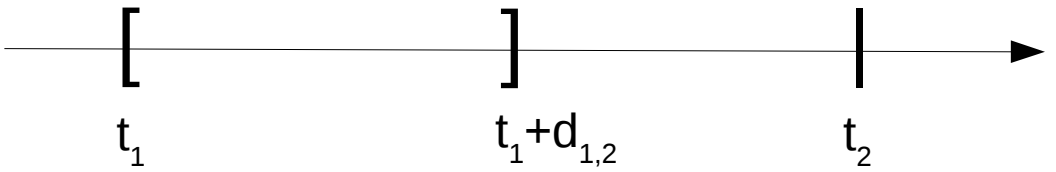
1) No continuous variable, you must discretize all!




Trade-off between accuracy and memory

# Some Tips

2) Avoid 0/1 variables, use domain variables




| $f(t_1,t_2)$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | ∞ | ∞ | 0 | 0 | 0 | 0 |
| 2 | ∞ | ∞ | ∞ | 0 | 0 | 0 |
| 3 | 0 | ∞ | ∞ | ∞ | 0 | 0 |
| 4 | 0 | 0 | ∞ | ∞ | ∞ | 0 |
| 5 | 0 | 0 | 0 | ∞ | ∞ | ∞ |
| 6 | 0 | 0 | 0 | 0 | ∞ | ∞ |

disjunctive constraint (non-linear function)

# Some Tips

3) Keep scopes of cost functions small

- toulbar2 propagates scopes of size 2 or 3,
  the rest is delayed

- except for clauses, cliques and knapsack
  but it remains local reasoning
  - Approximate dual feasible solution of a linear program
  - Does not solve a relaxed linear system

# Toulbar2 platform

https://forgemia.inra.fr/thomas.schiex/cost-function-library (+18.281 instances)

http://genoweb.toulouse.inra.fr/~degivry/evalgm (3.026 instances)

https://miat.inrae.fr/toulbar2/ (main site, documentations, tutorials, examples)

*Debian and ubuntu packages v1.1.1*

*Github latest source and release v1.1.1 (linux, macos, win exe)*
*Q&A User discussions*

Python interface (pip install pytoulbar2)
*– to be documented*
*– compatible with numpy and pytorch (future work)*