

Optimisation combinatoire dans les réseaux de fonctions de coût

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ALIMENTATION
AGRICULTURE
ENVIRONNEMENT



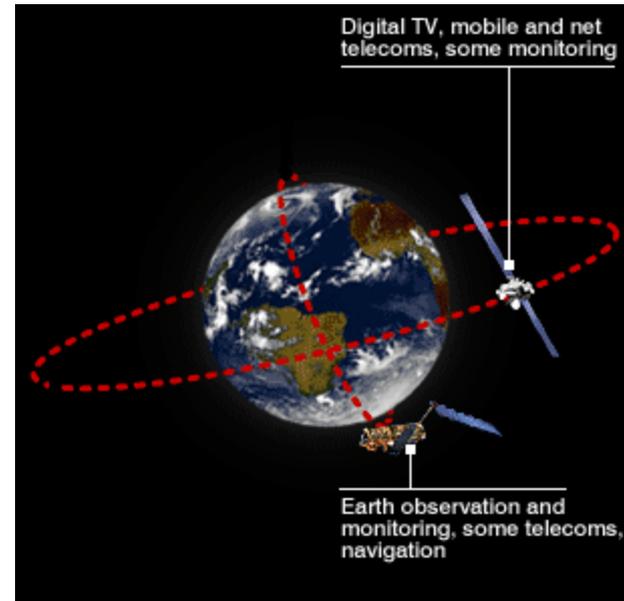
Optimisation combinatoire



Allocation de fréquences à des liens radio minimisant les interférences entre les liens

$n \leq 458, e \leq 5000$

CELAR



Sélection de prises de vue maximisant un revenu sous contraintes de compatibilité des instruments optiques

$n \leq 364, e \leq 10108$

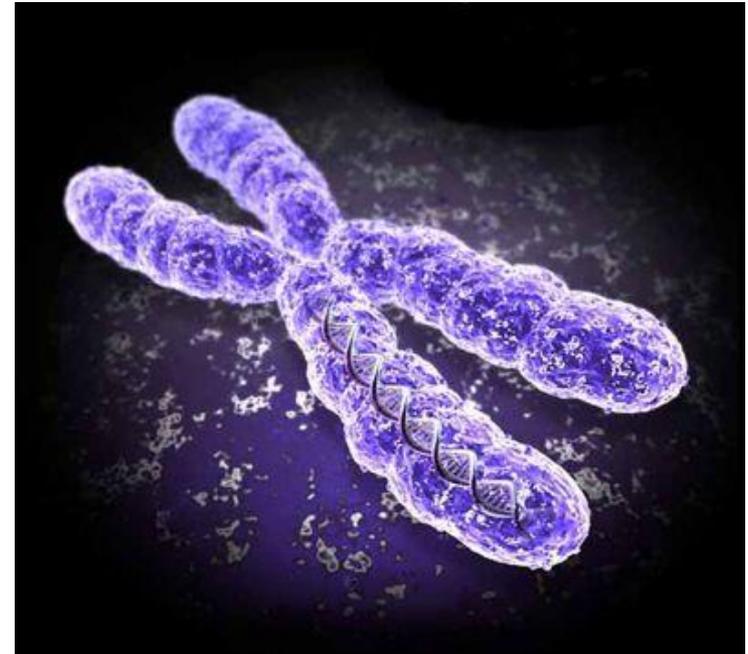
SPOT5

Optimisation combinatoire et génétique



Diagnostic. Sélection d'un sous-ensemble minimum/le plus probable d'observations de génotypes tel que leur retrait restaure la cohérence avec les lois de l'hérédité

MENDEL $n \leq 20000$, $e \leq 30000$



Couverture. Sélection d'un sous-ensemble minimum de marqueurs couvrant au mieux l'ensemble du chromosome

TAGSNP $n \leq 1500$, $e \leq 150000$

Cadre général des réseaux de fonctions de coût (WCSP)

(Shapiro, Haralick, IEEE PAMI 81)

(Freuder, Wallace, AIJ 92)

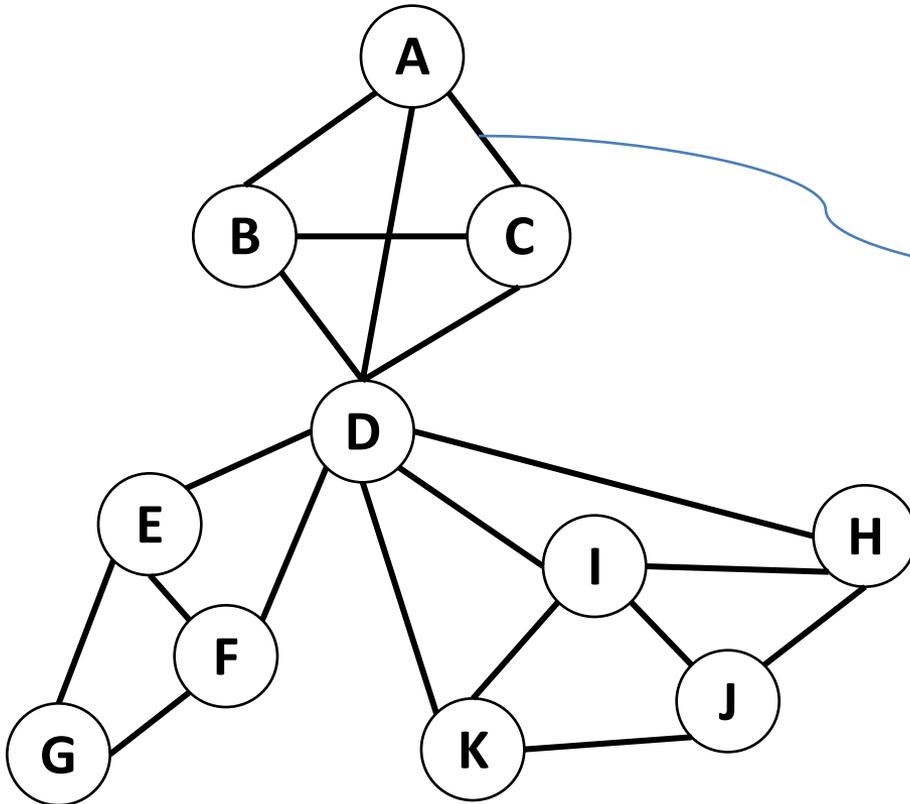
(Schiex, Fargier, Verfaillie, IJCAI 95)

- (X, D, F)
 - $X = \{X_1, \dots, X_n\}$ **n** variables
 - $D = \{D_1, \dots, D_n\}$ **n** domaines finis de taille maximum **d**
 - $F = \{f_{S_1}, \dots, f_{S_e}\}$, **e** fonctions de coût
 - f_{S_i} : fonction associant une valeur positive finie ou infinie à chaque tuple/affectation de l'ensemble de variables S_i
- Objectif : trouver une affectation A de toutes les variables qui minimise

$$\sum_{f_S \in F} f_S (A[S])$$

Problème NP-difficile

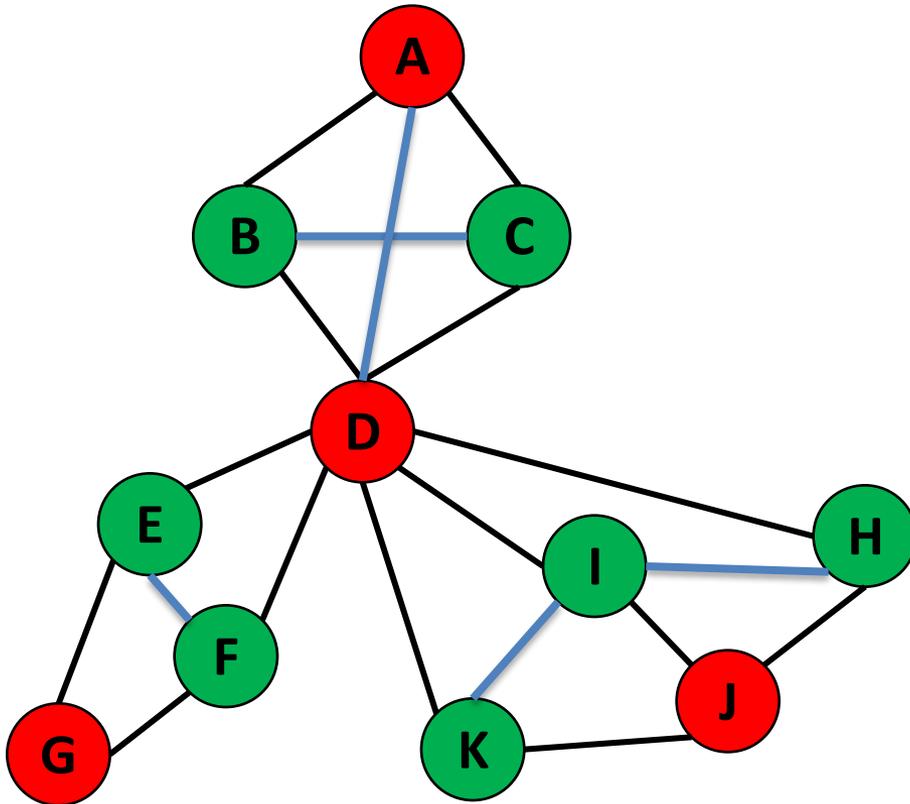
Exemple min-2coloriage



graphe primal

A	C	$f_{\{A,C\}}$
Red	Red	1
Red	Green	0
Green	Red	0
Green	Green	1

Exemple min-2coloriage

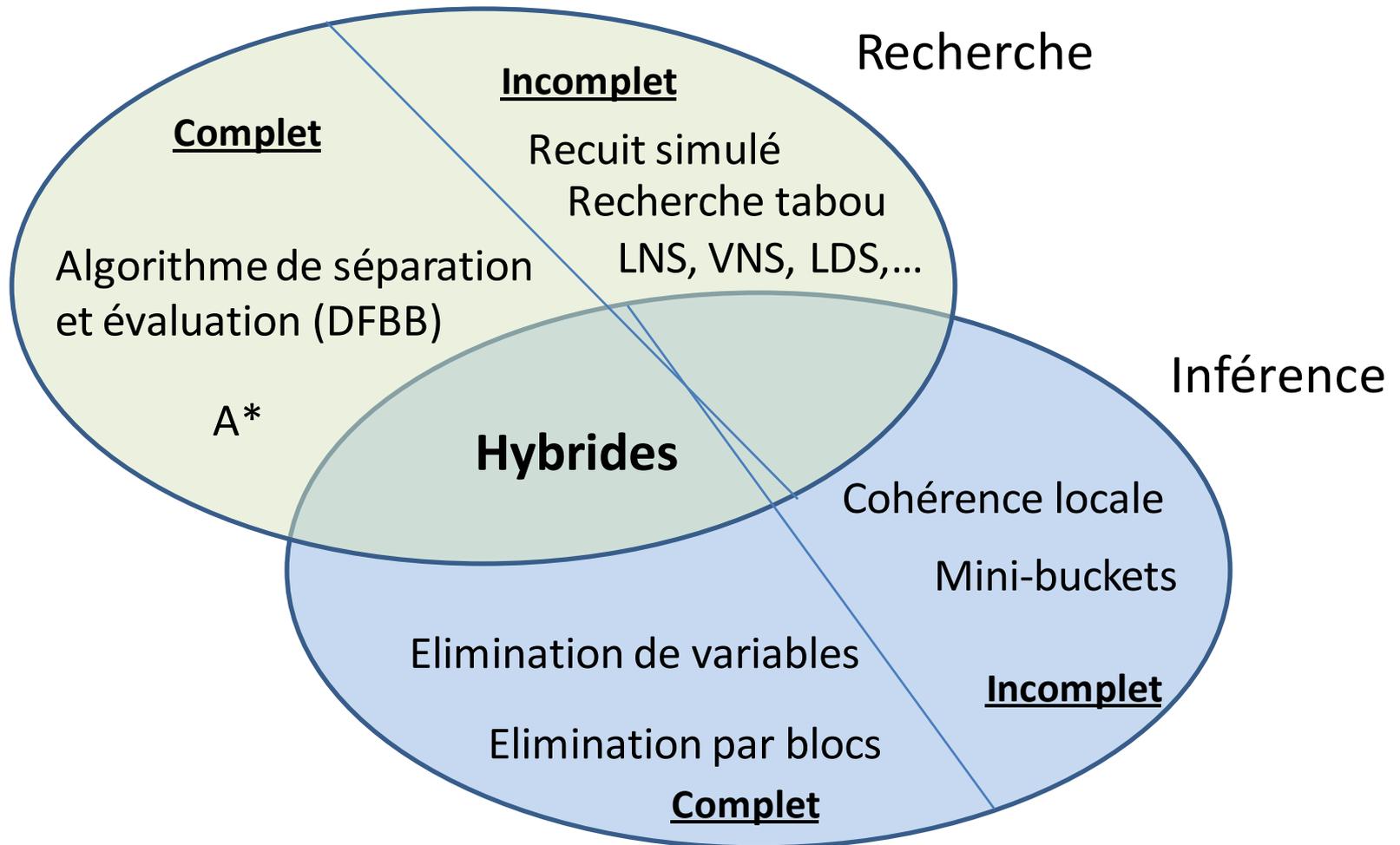


Solution optimale de coût 5

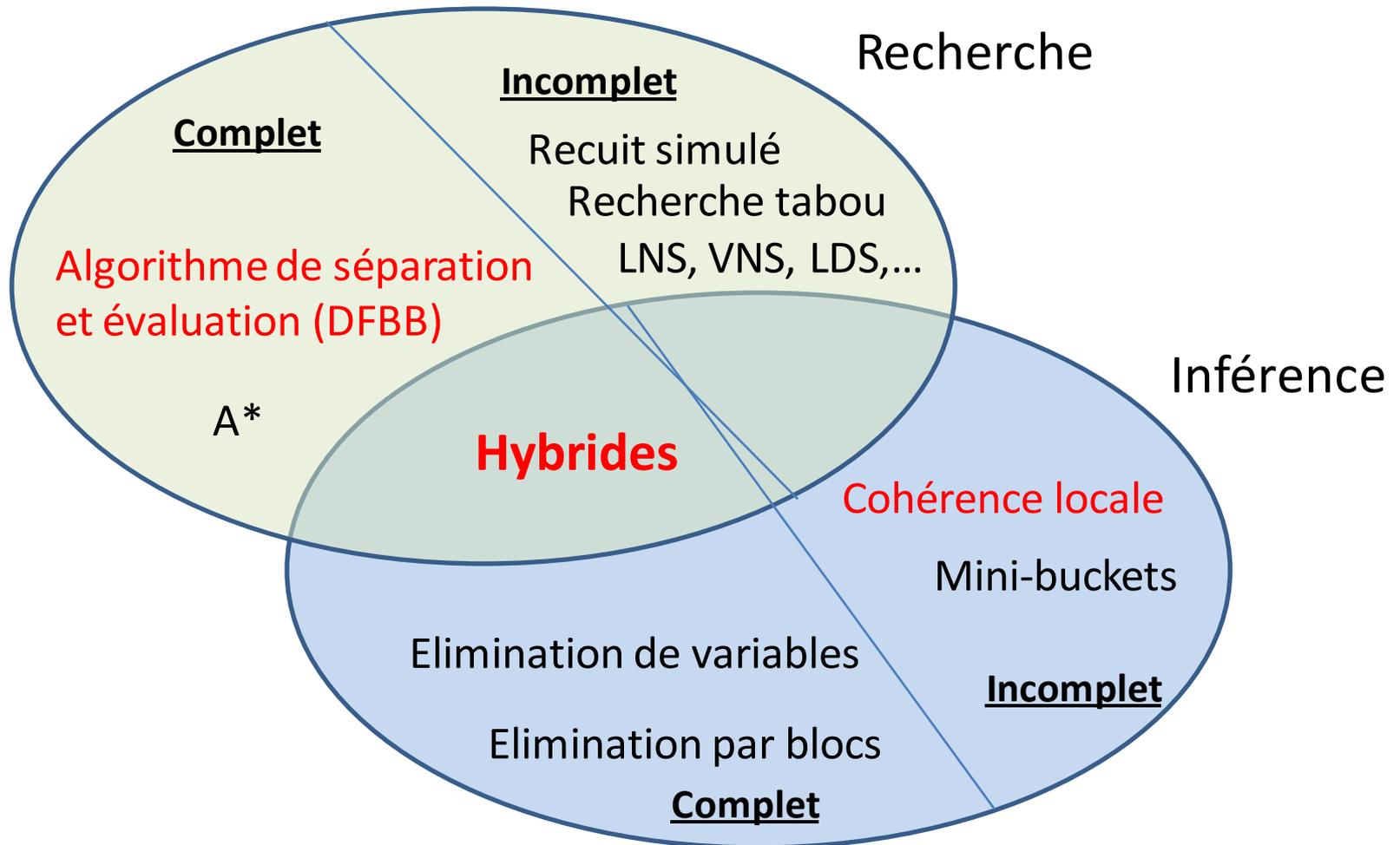
Liens vers d'autres formalismes

- **Problème de satisfaction de contraintes** (X', D, C)
 - Ajout d'une variable de coût par fonction
Exemple : $Z_{\{A,C\}} \equiv (A=C)$
 - Ajout d'une variable objectif
 $Z = \sum_{f_S \in F} Z_S$
- **Programmation linéaire en nombres entiers**
 - Ajout d'une variable 0/1 par tuple
Exemple : $A + C + Z_{\{A=0,C=0\}} \geq 1$
 - Ajout d'une variable objectif
 $Z = \sum_{f_S \in F} \sum_{t \in L(S)} Z_{S=t}$

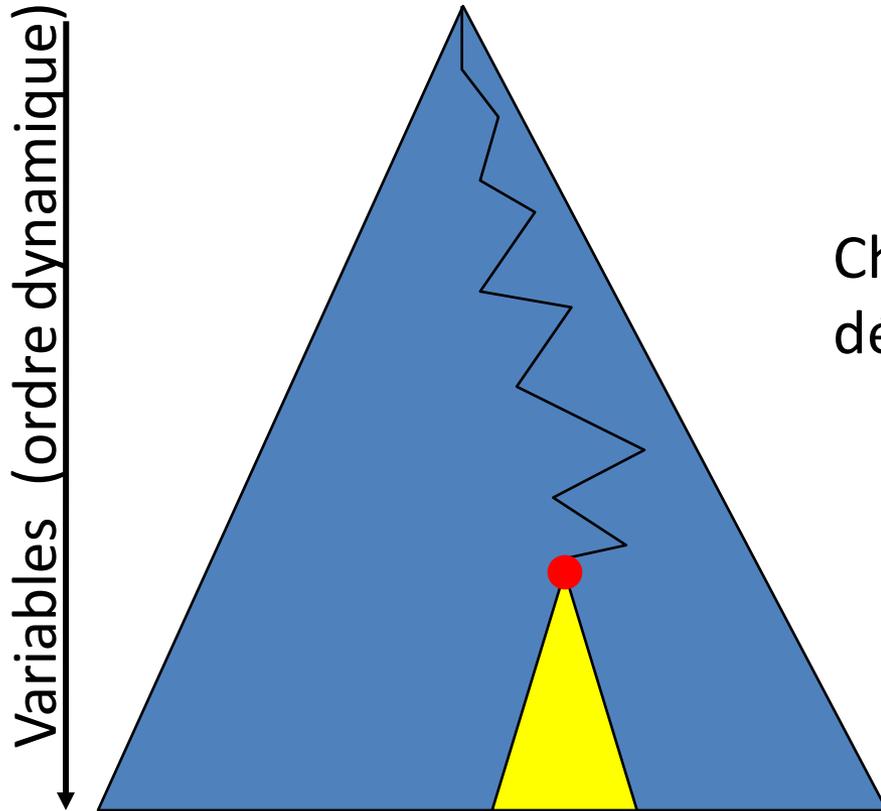
Panorama des méthodes de résolution



Panorama des méthodes de résolution

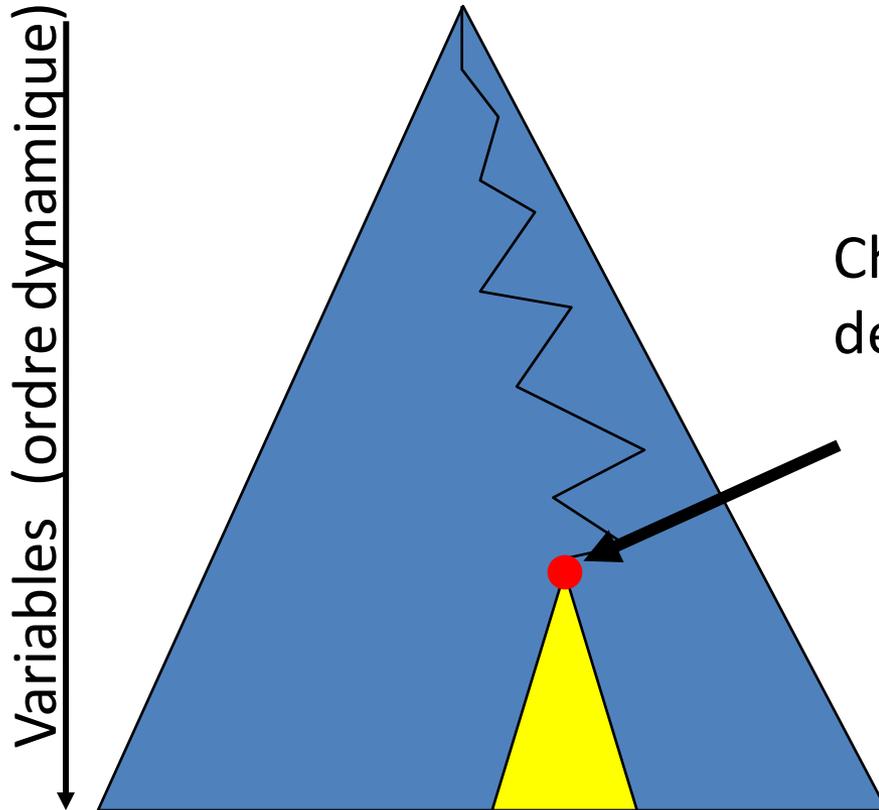


Algorithme de séparation et évaluation



Chaque nœud de l'arbre est un WCSP défini par l'affectation courante

Algorithme de séparation et évaluation

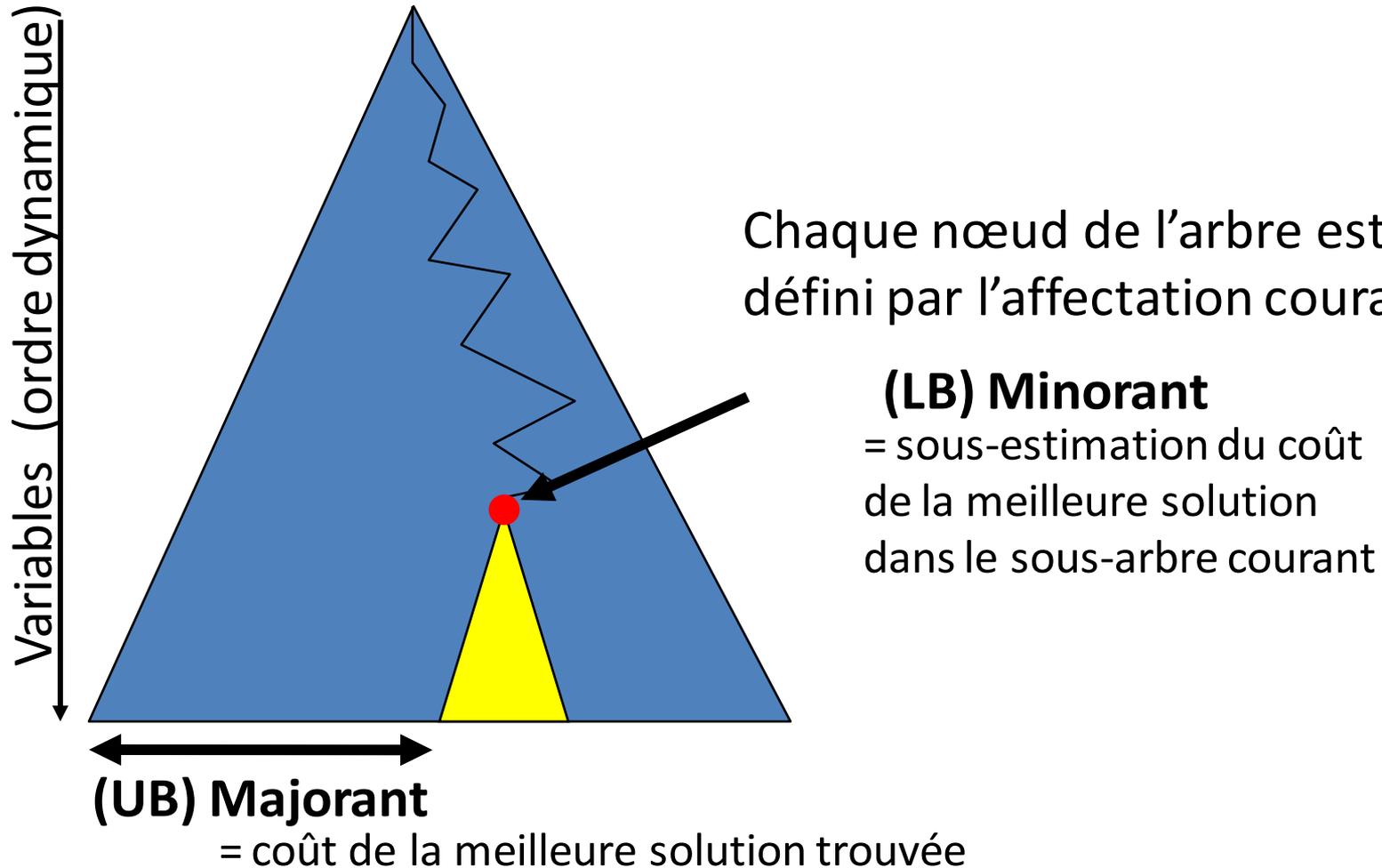


Chaque nœud de l'arbre est un WCSP défini par l'affectation courante

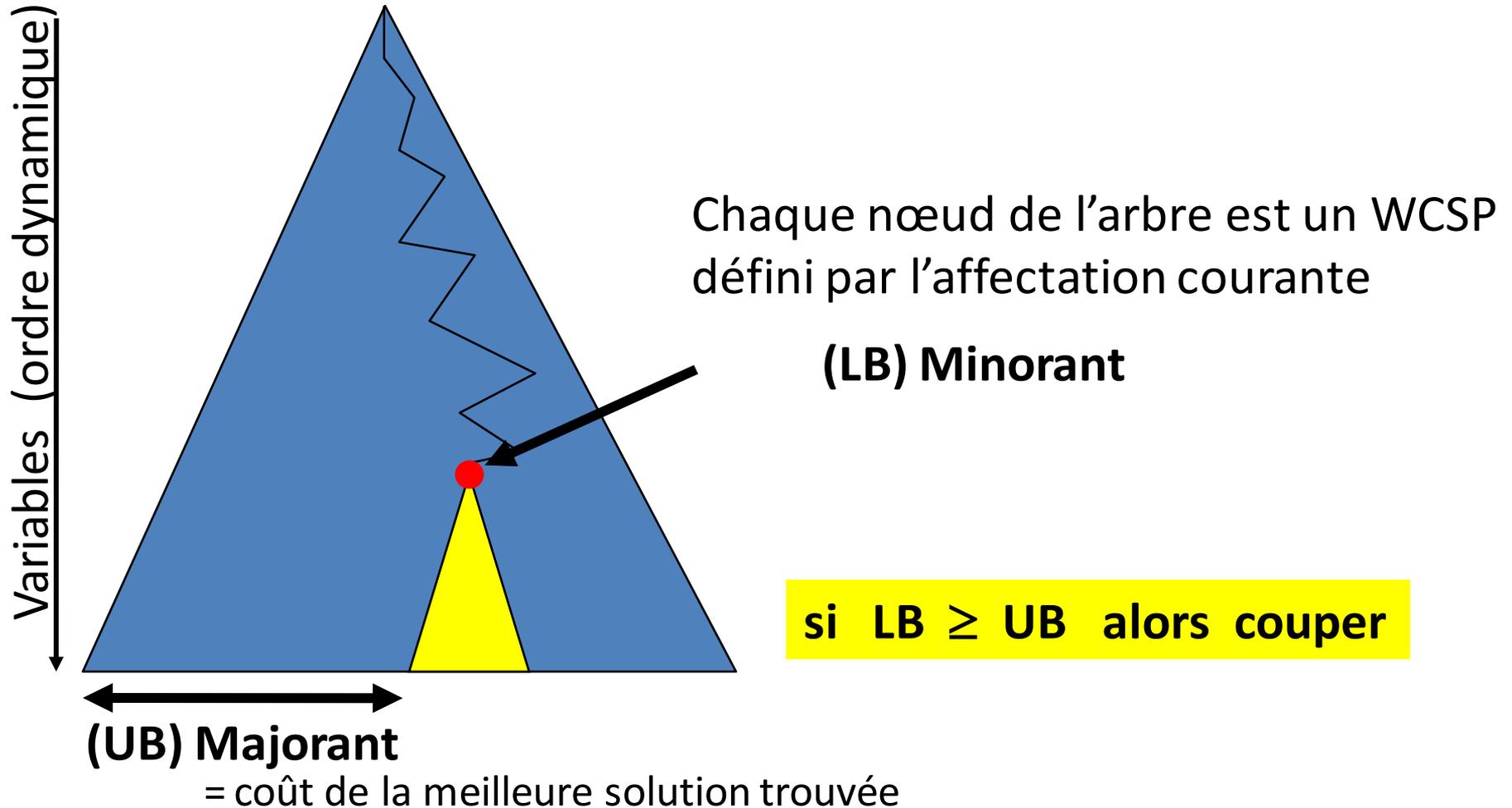
(LB) Minorant

= sous-estimation du coût de la meilleure solution dans le sous-arbre courant

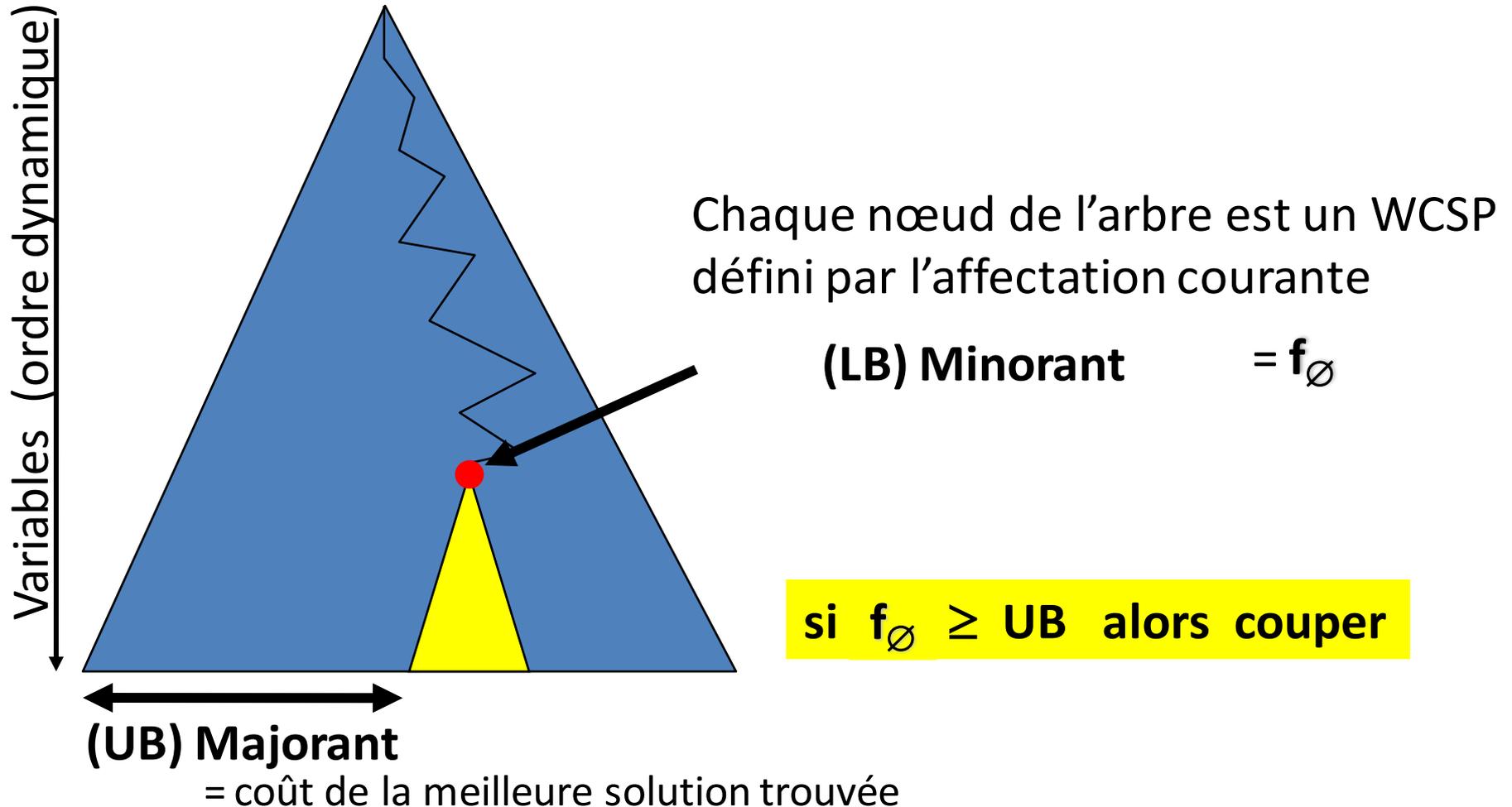
Algorithme de séparation et évaluation



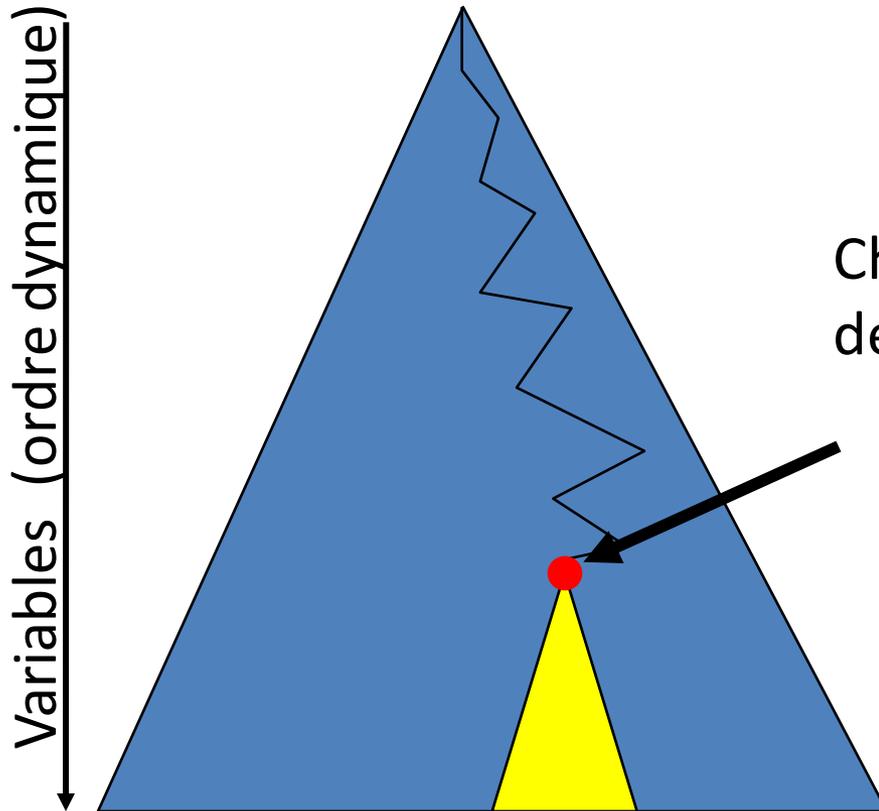
Algorithme de séparation et évaluation



Algorithme de séparation et évaluation



Algorithme de séparation et évaluation



Chaque nœud de l'arbre est un WCSP défini par l'affectation courante

(LB) Minorant = f_{\emptyset}

si $f_{\emptyset} \geq k$ alors couper

(UB) Majorant

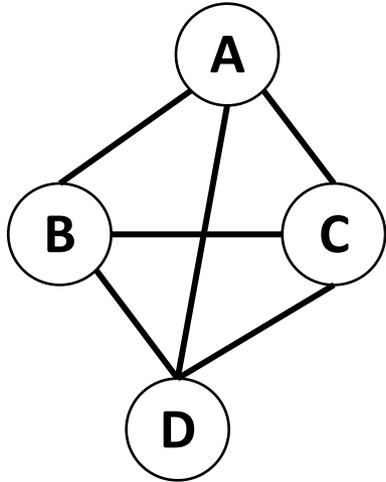
= coût de la meilleure solution trouvée = k

$$\alpha +_k \beta = \min \{k, \alpha + \beta\}$$

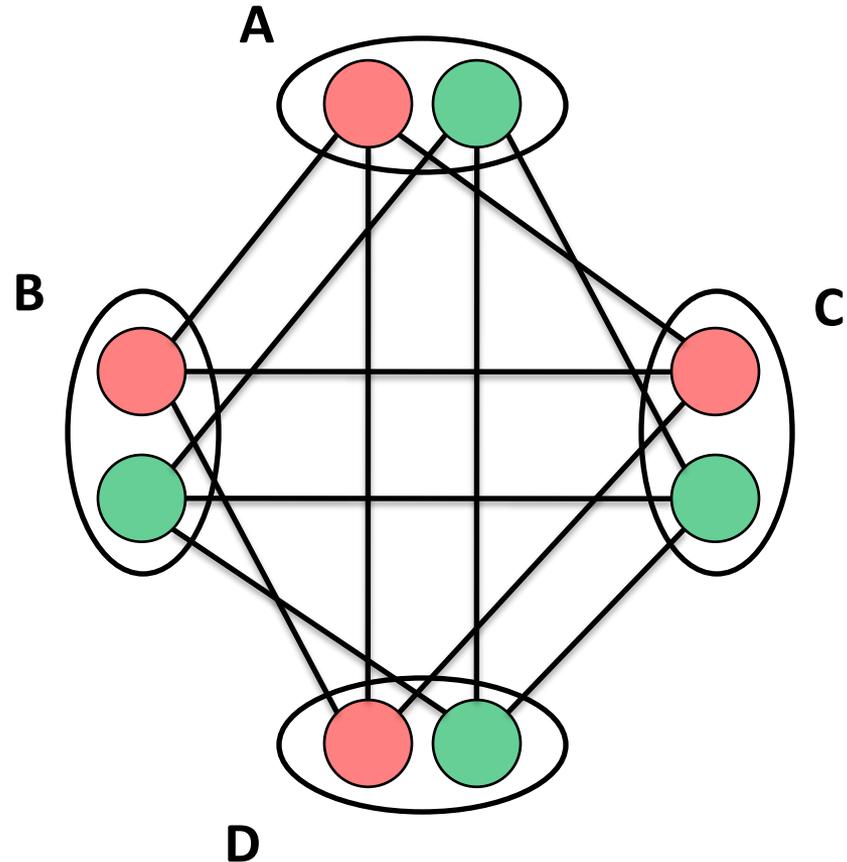
Cohérence locale souple

- Objectif : produire un WCSP plus simple vérifiant une propriété locale (incrémental)
- Renforcement en temps polynomial de la propriété via des **reformulations en problèmes équivalents** (*i.e.* même fonction objectif)
- Similaire à
 - Cohérences locales dans les CSP (Mackworth, AIJ 77)
 - Equivalent transformation in 2D grammars (Schlesinger, Kibernetika 76)
 - Roof dual in QPBO (Boros, Hammer, Discr. Ap. Math 02)
 - Equivalent Reparameterization in MRF (Wainwright, Jaakkola, Willsky, IEEE Trans. on I.T. 03)

Exemple min-2coloriage

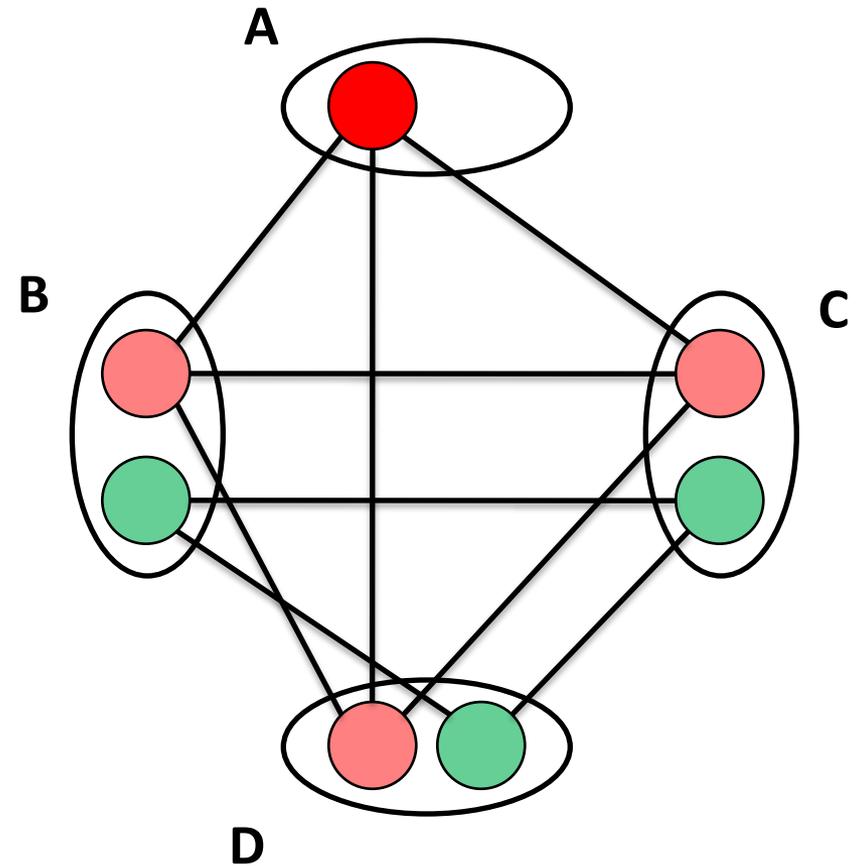
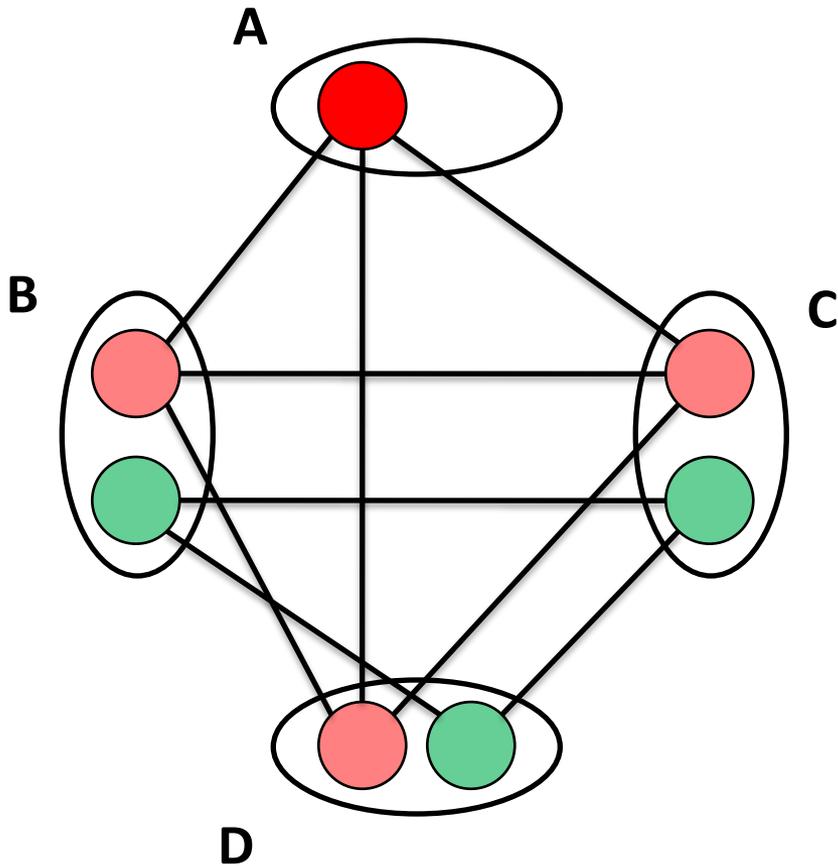


graphe primal

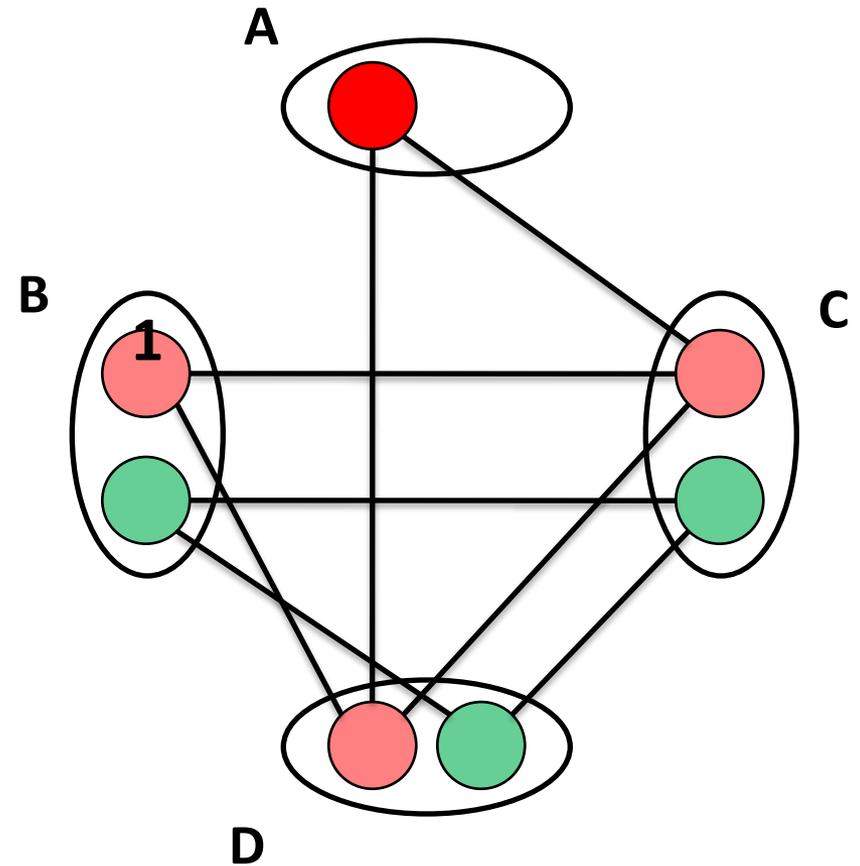
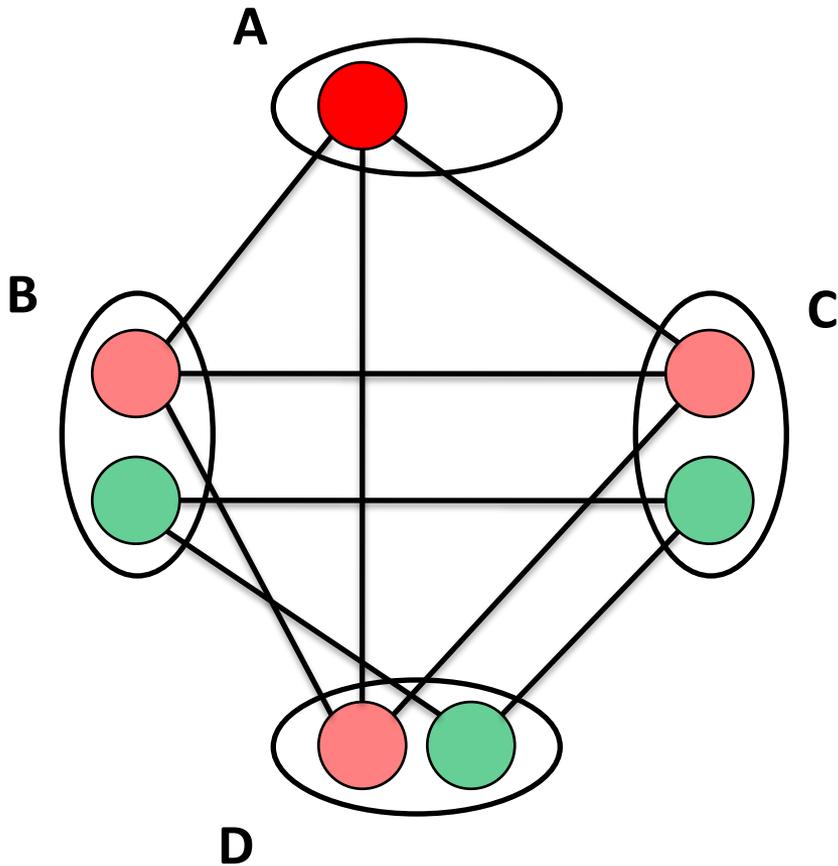


micro-structure

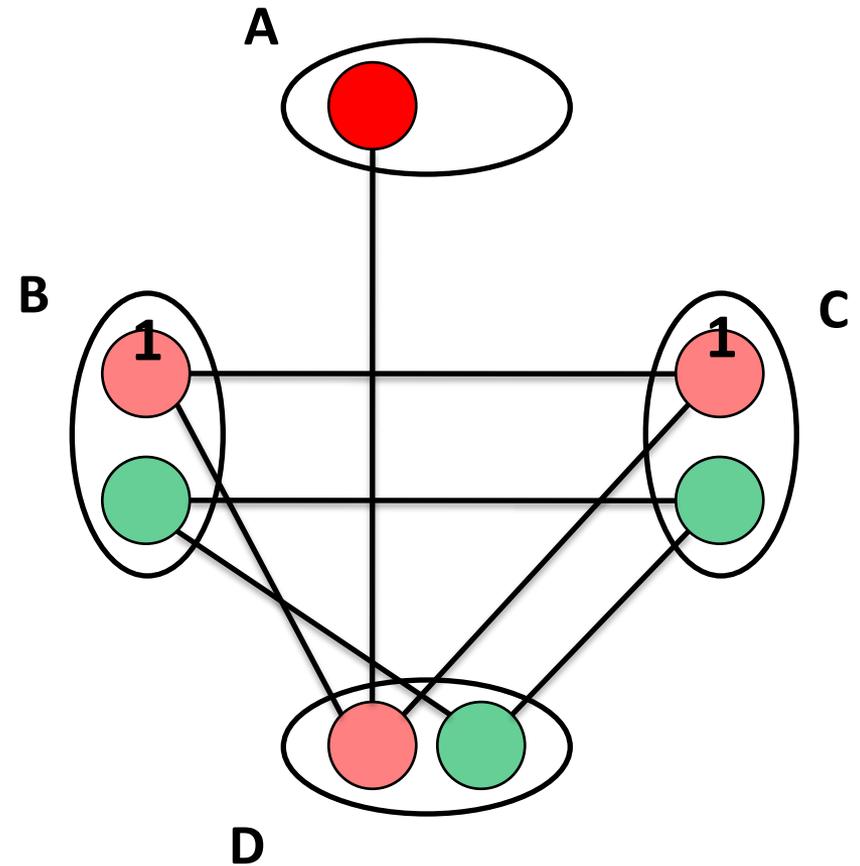
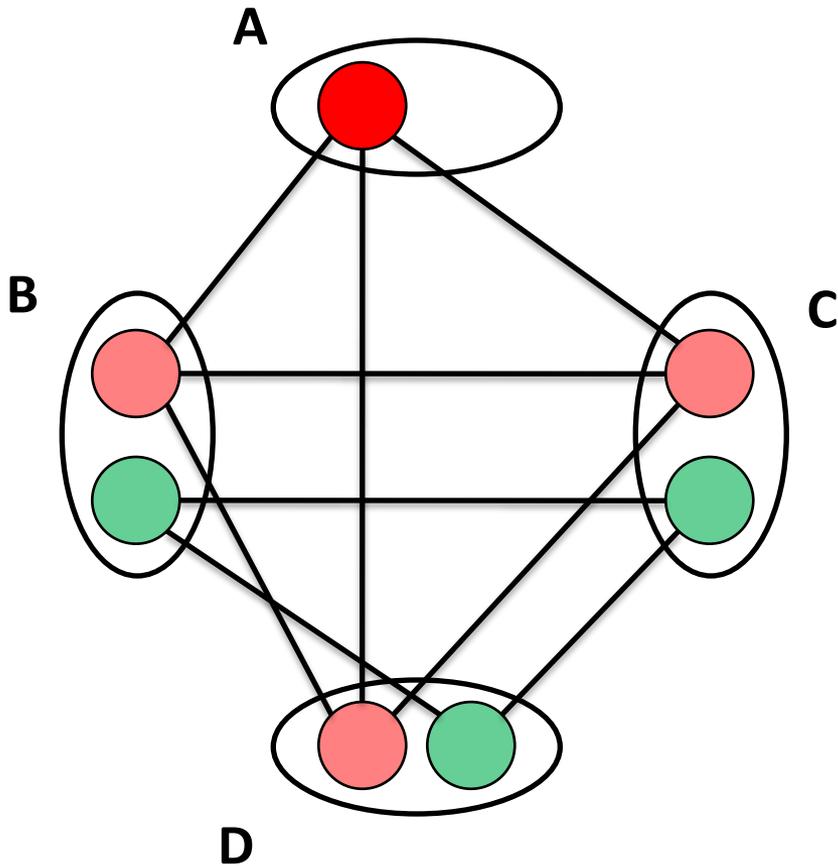
Reformulation en problème équivalent



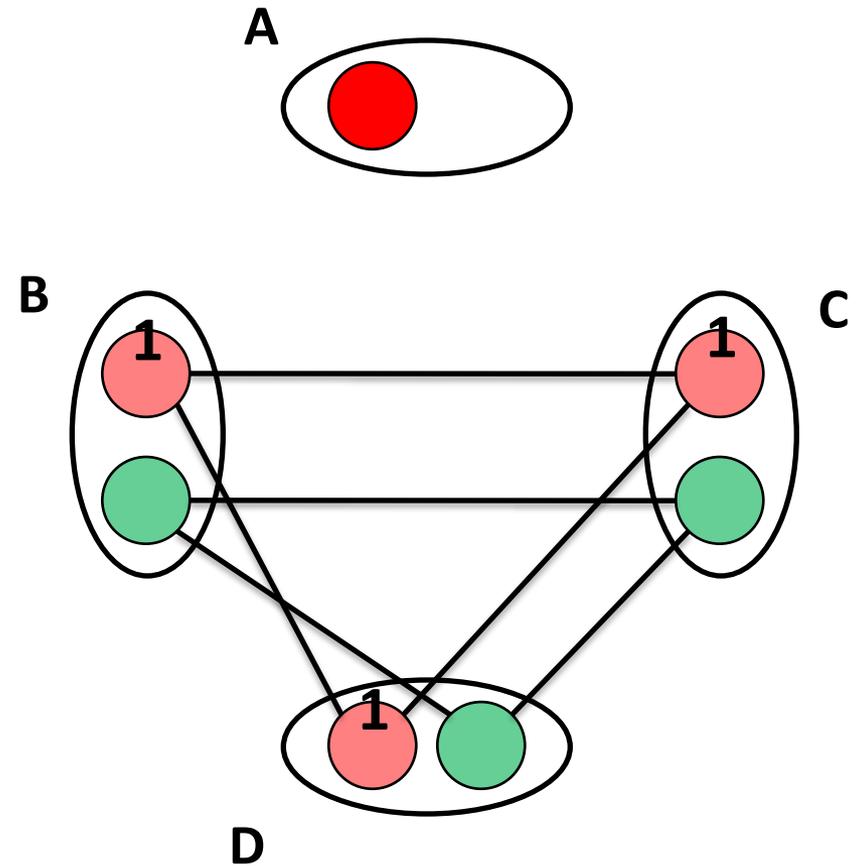
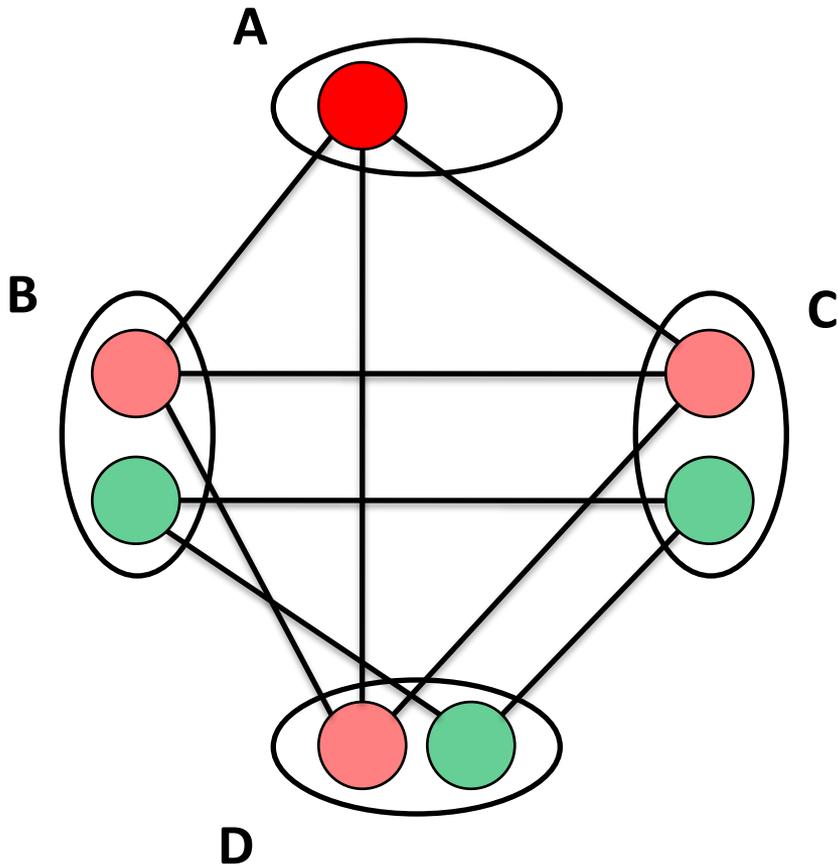
Reformulation en problème équivalent



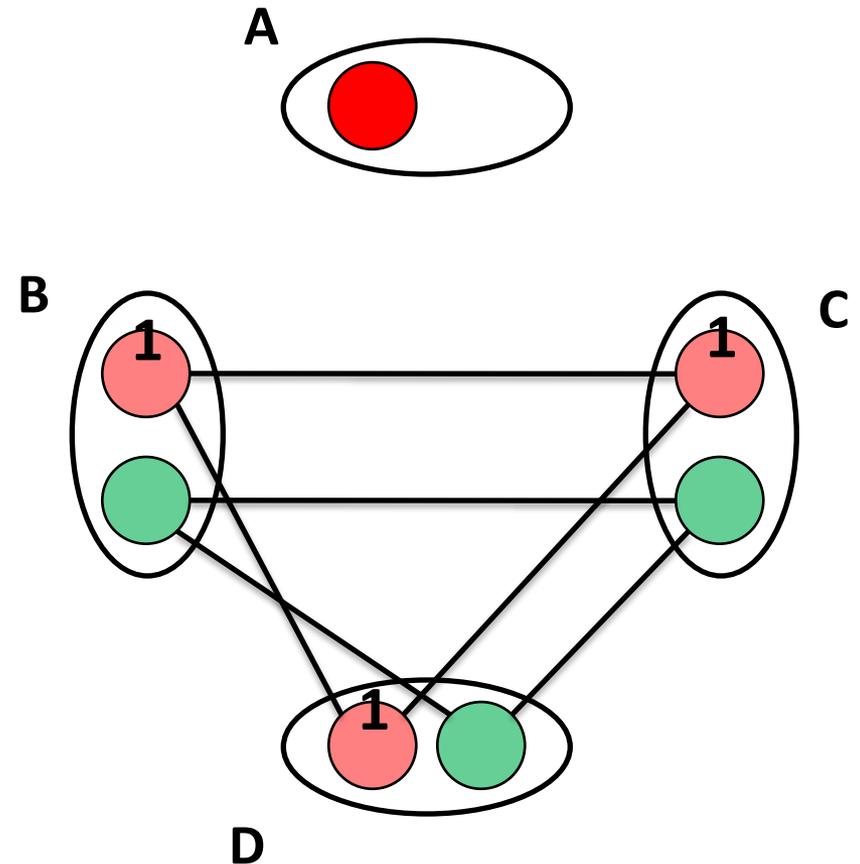
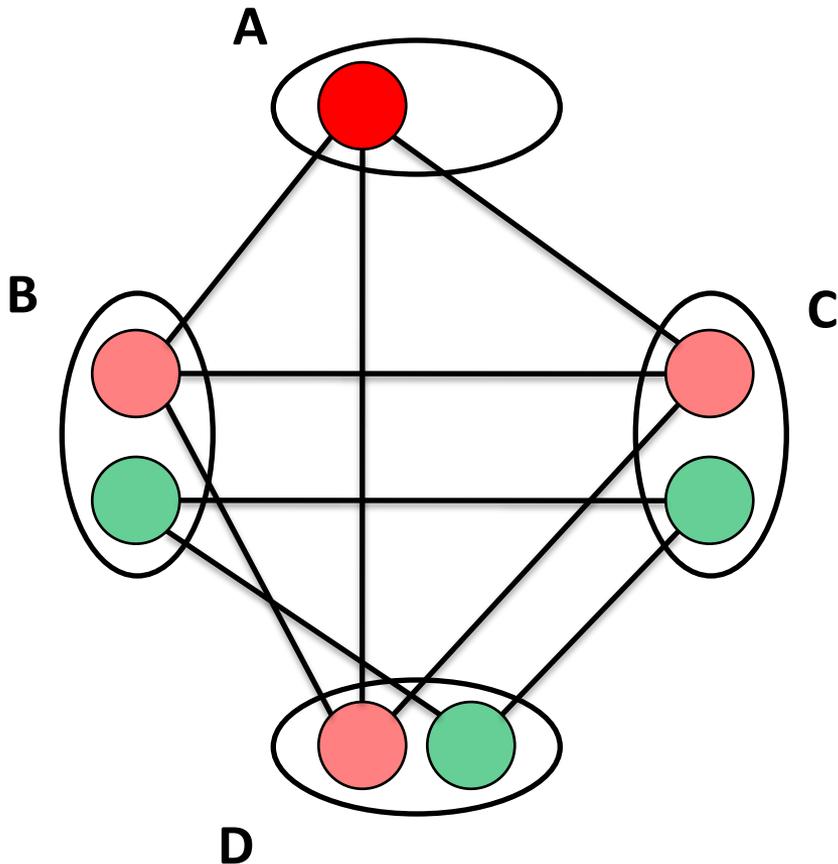
Reformulation en problème équivalent



Reformulation en problème équivalent



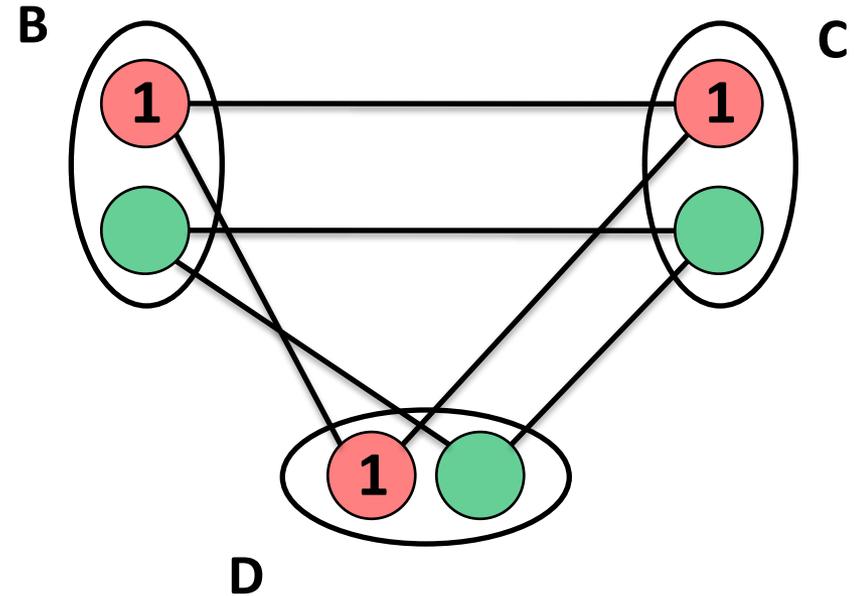
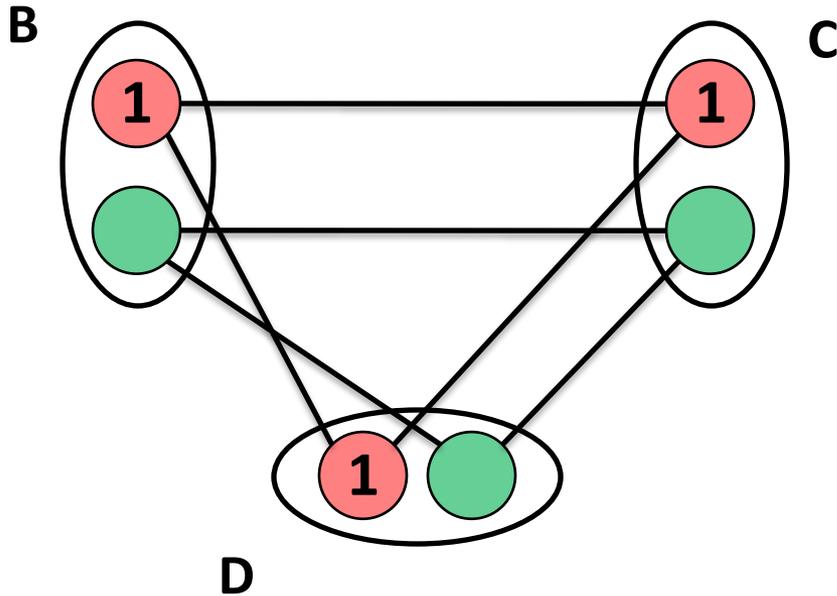
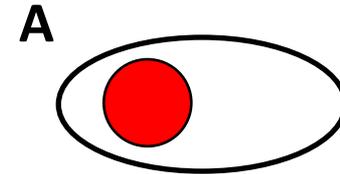
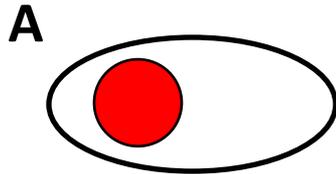
Reformulation en problème équivalent



Problème arc-cohérent (AC)

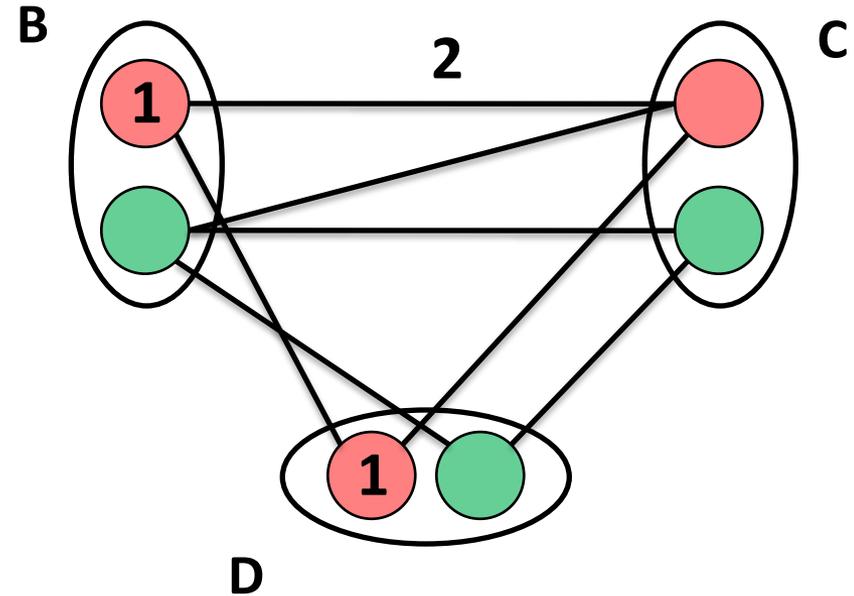
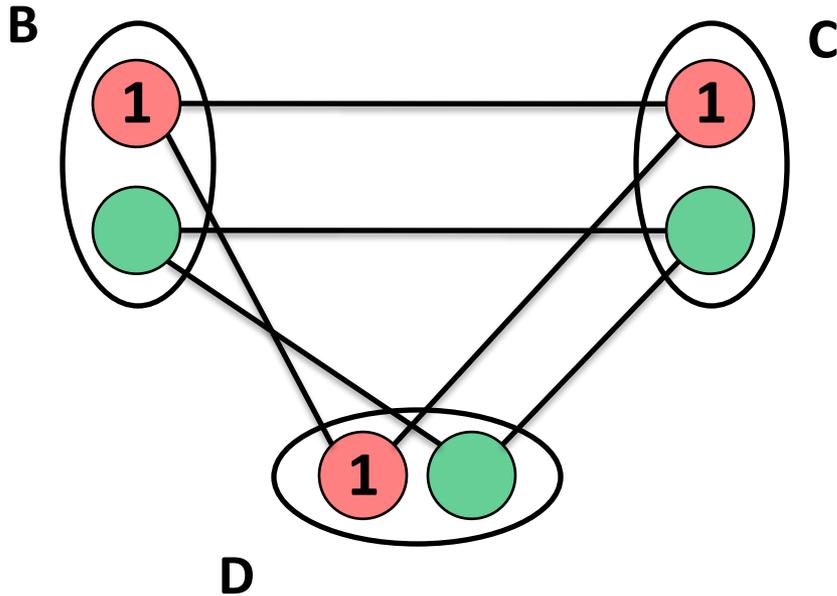
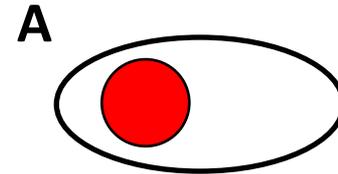
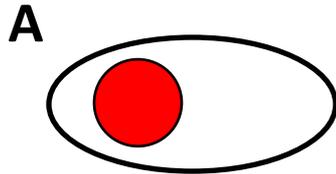
(Schiex, CP 00)

Reformulations chaotiques



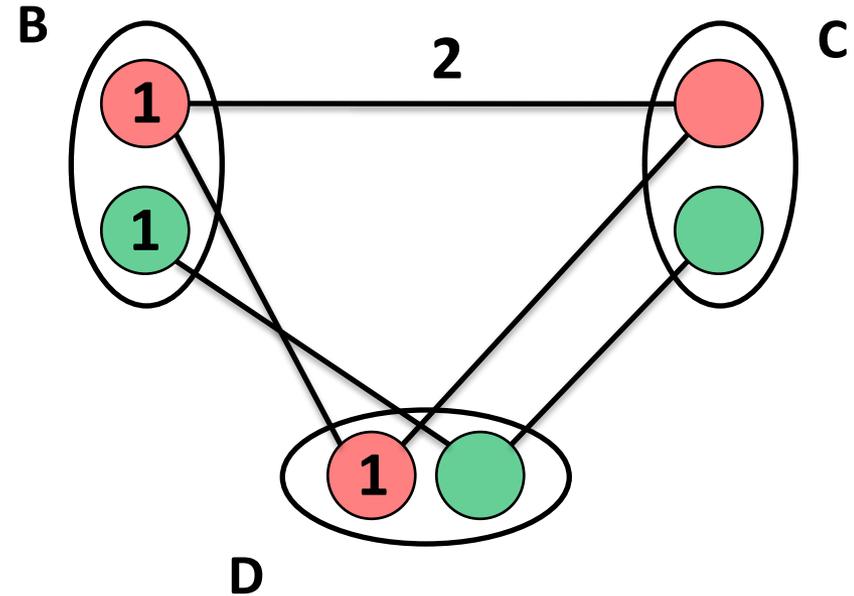
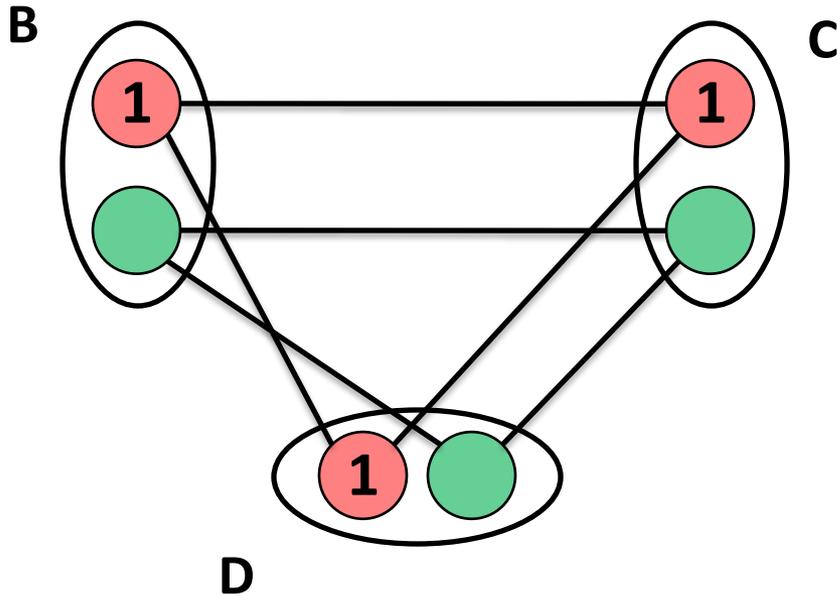
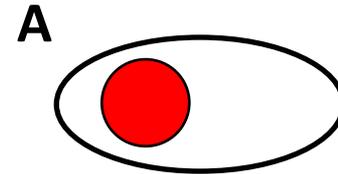
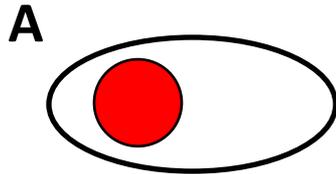
Problème AC

Reformulations chaotiques



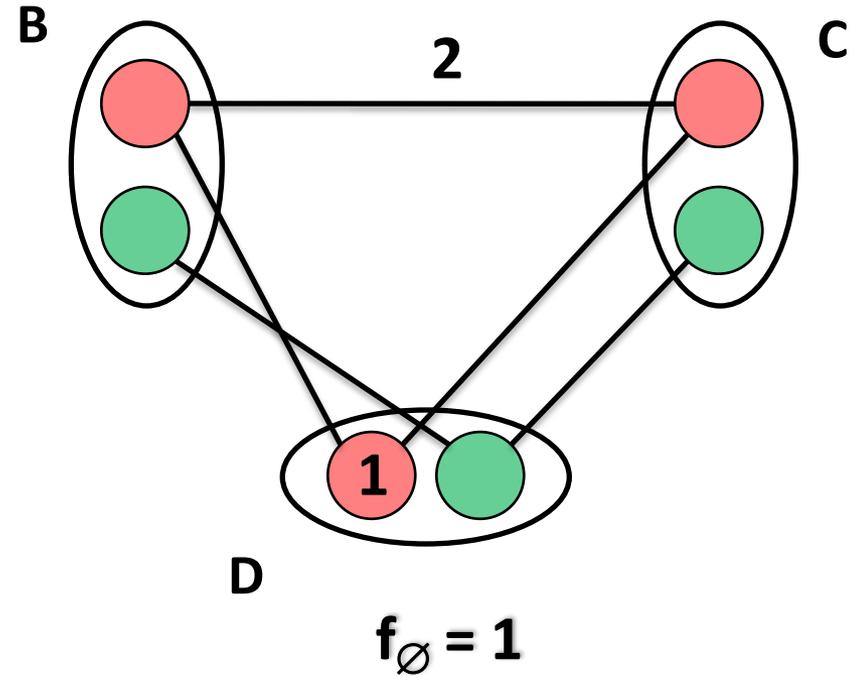
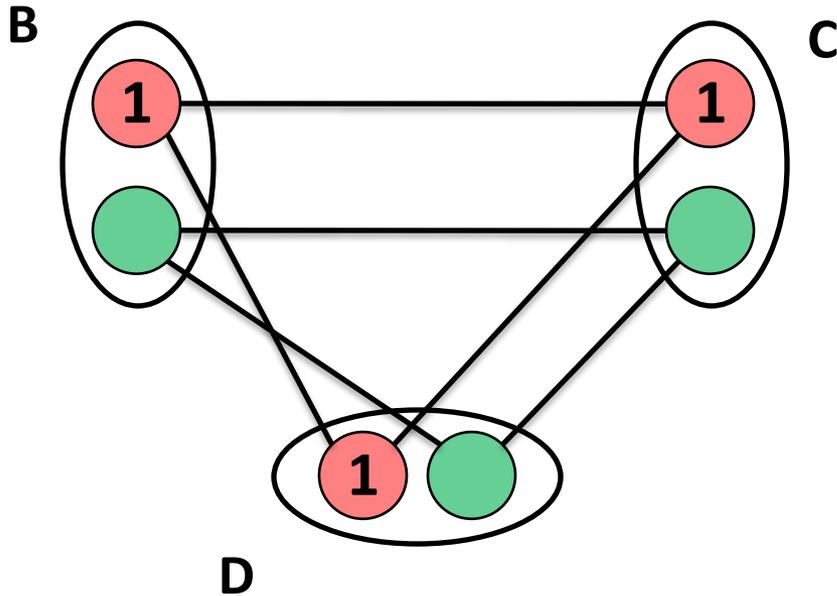
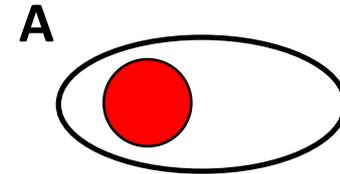
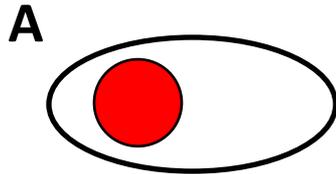
Problème AC

Reformulations chaotiques



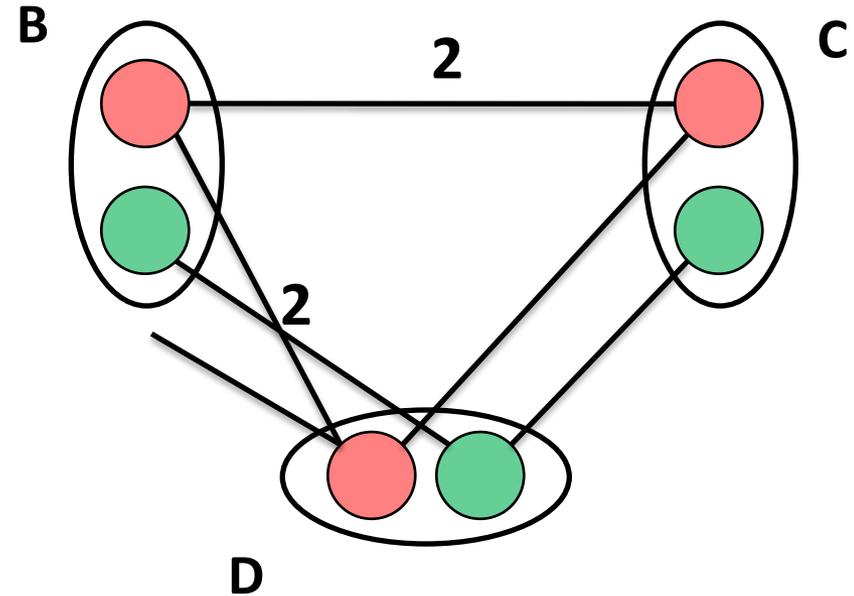
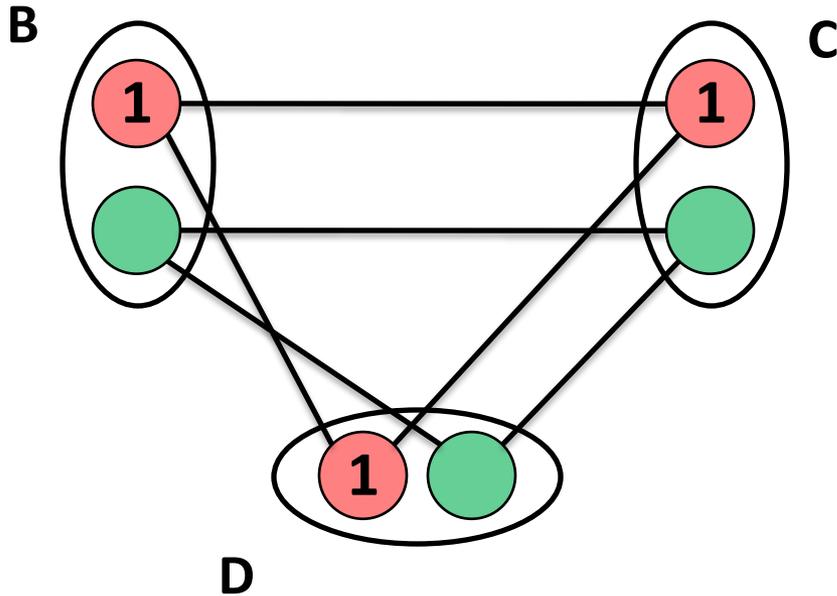
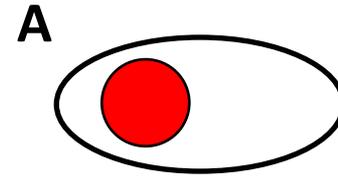
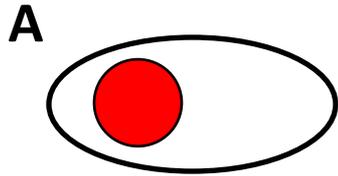
Problème AC

Reformulations chaotiques



Problème AC

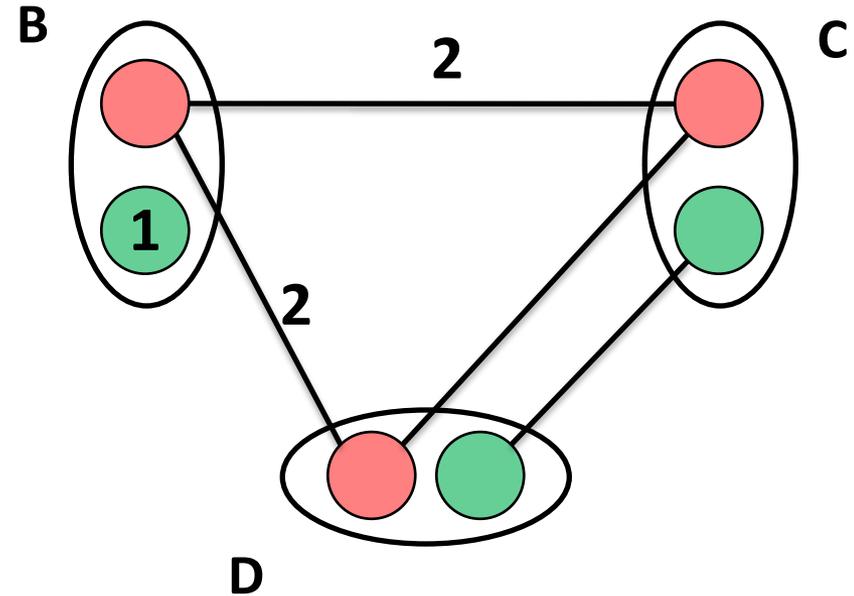
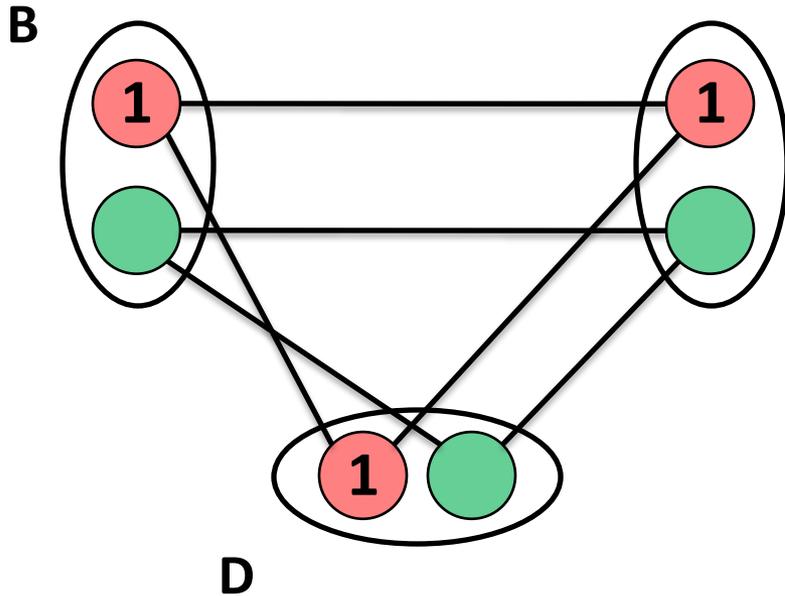
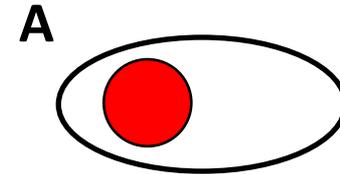
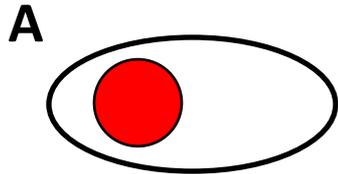
Reformulations chaotiques



Problème AC

$f_{\emptyset} = 1$
 $A < B < C < D$

Reformulations chaotiques

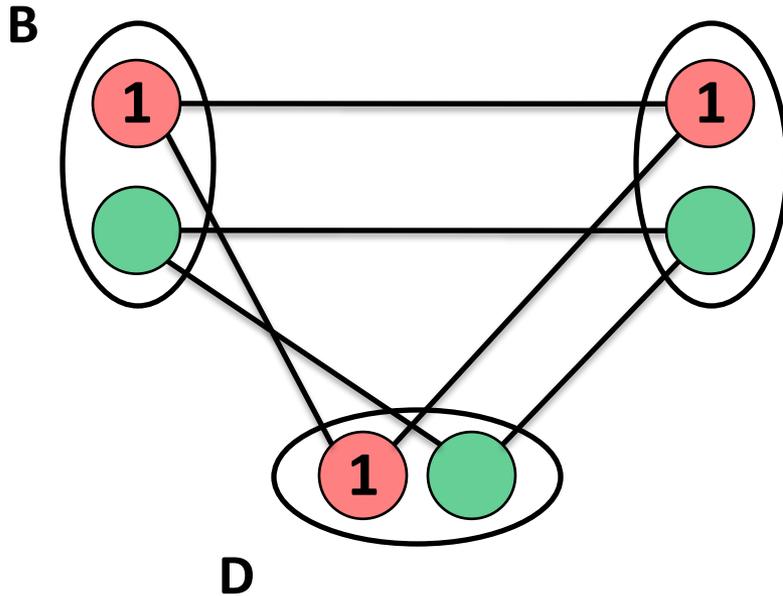
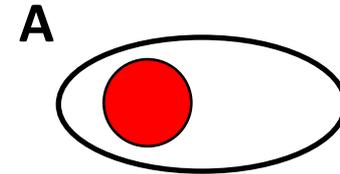
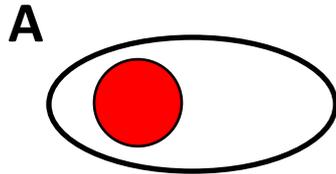


Problème AC

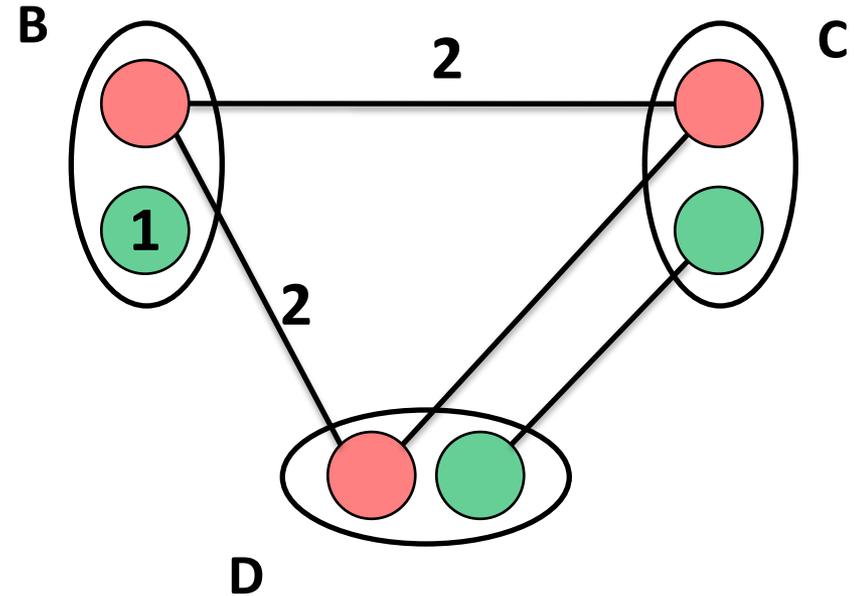
Problème arc-cohérent directionnel (DAC)

$f_{\emptyset} = \mathbf{1}$ (Cooper, FSS 03)

Reformulations chaotiques



C



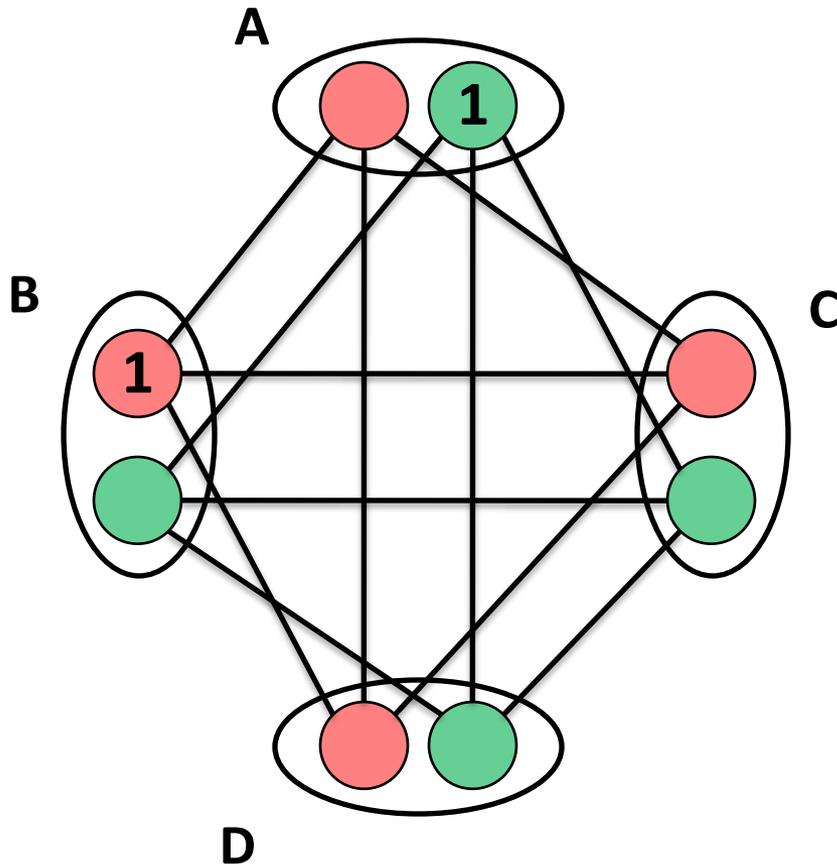
$f_{\emptyset} = \mathbf{1}$ (Cooper, FSS 03)

Problème AC

Problème arc-cohérent directionnel (DAC)

➔ Application à Max-SAT (de Givry et al, CP 03) (Larrosa, Heras, de Givry, AIJ, 08)

Reformulations chaotiques



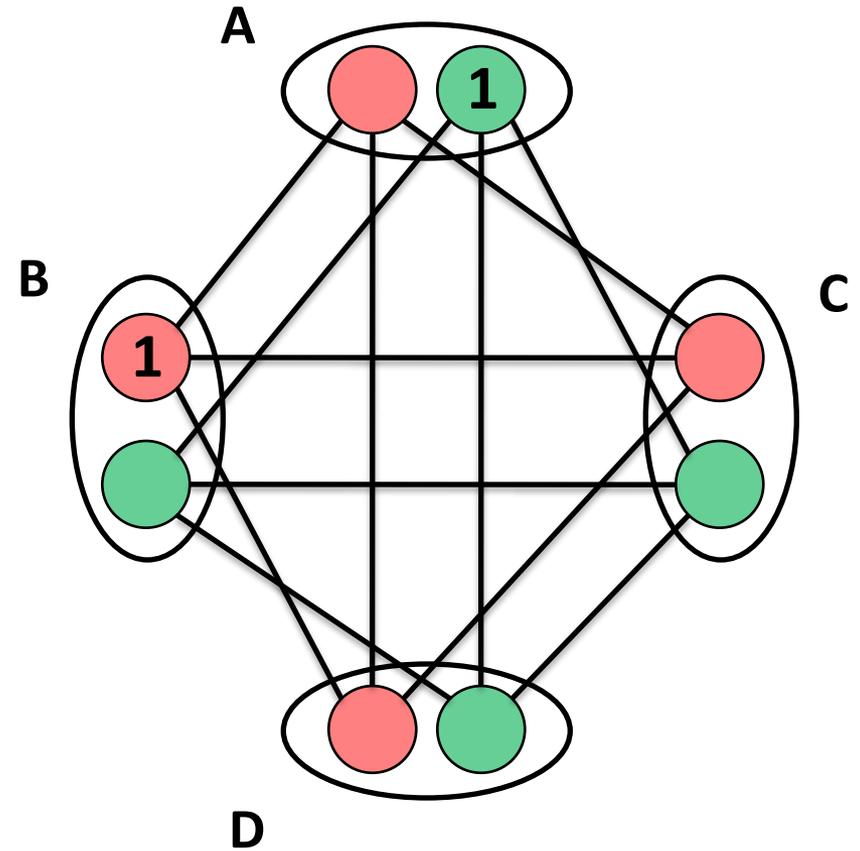
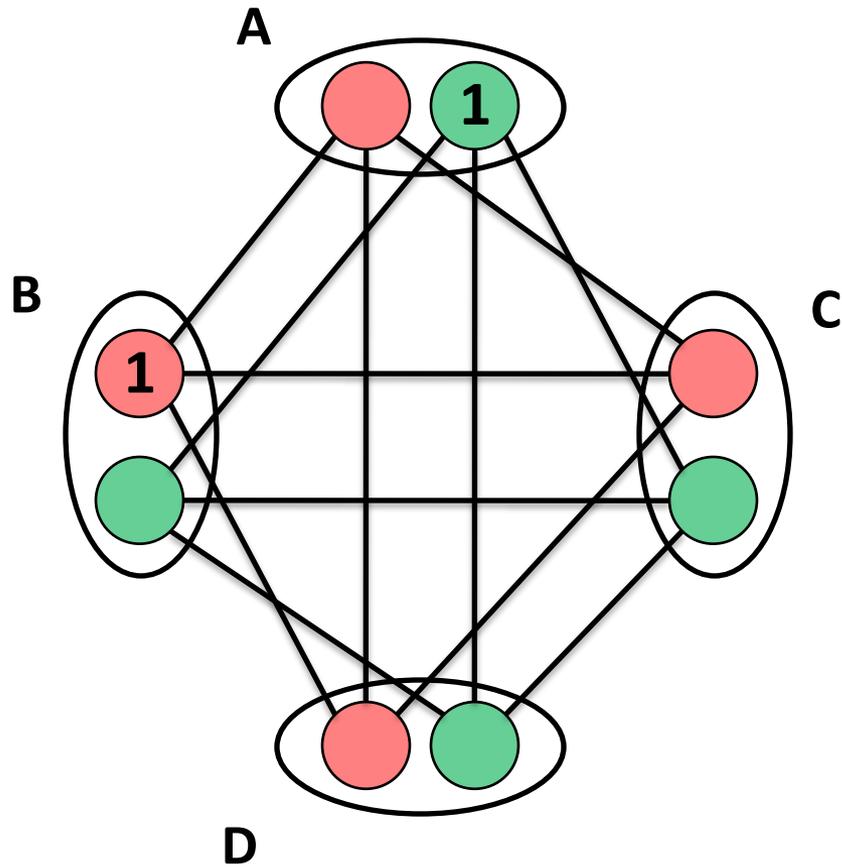
Propriété EAC

$\forall X_i \in X, \exists a \in D_i$ telque

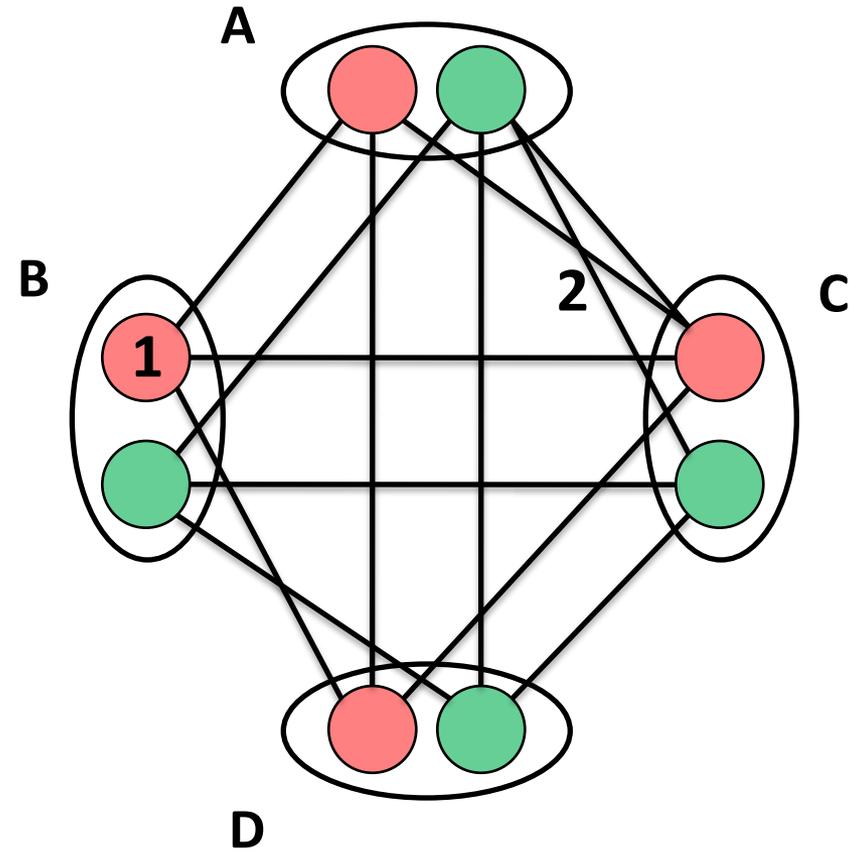
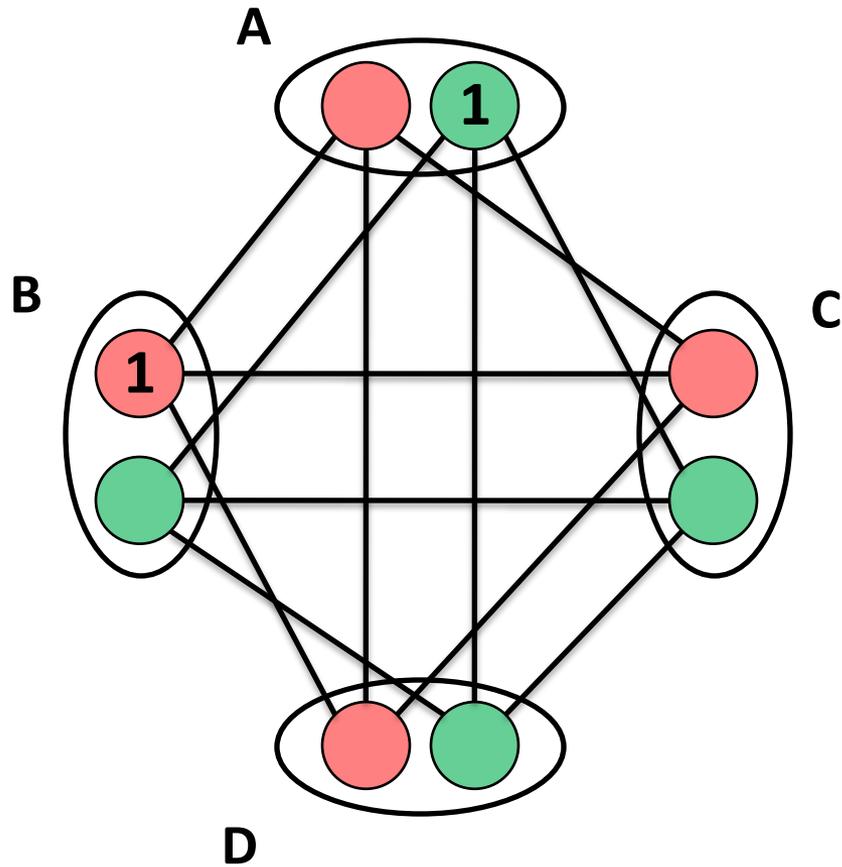
- $f_{\{X_i\}}(a) = 0$

- $\forall f_{\{X_i, X_j\}}, \exists b \in D_j$ telque
 $f_{\{X_i, X_j\}}(a, b) = f_{\{X_j\}}(b) = 0$

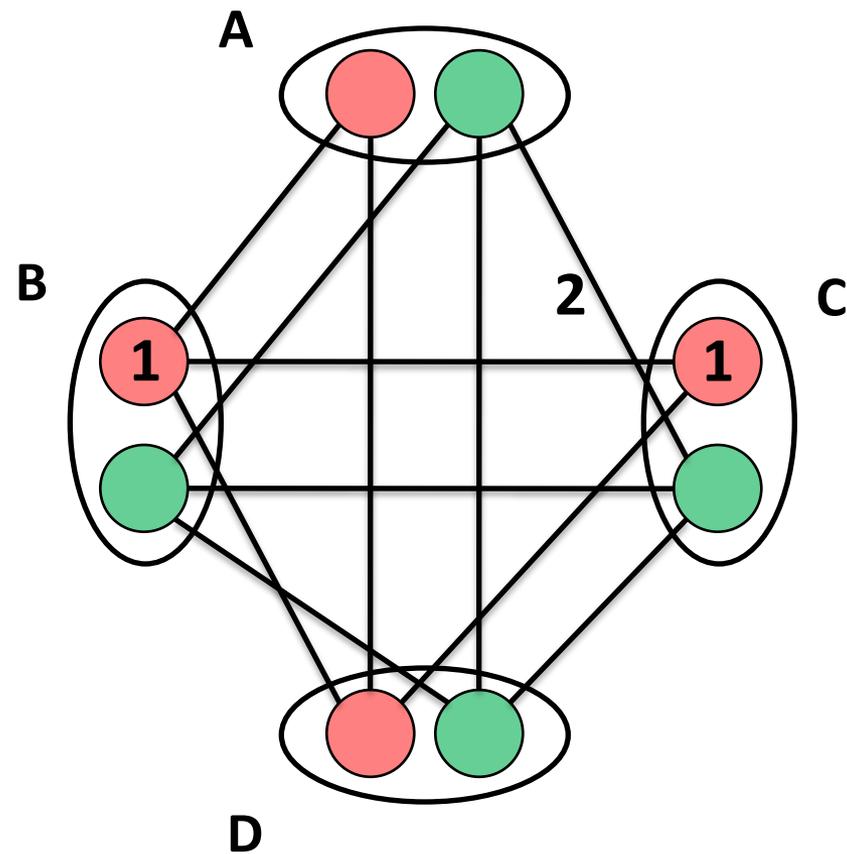
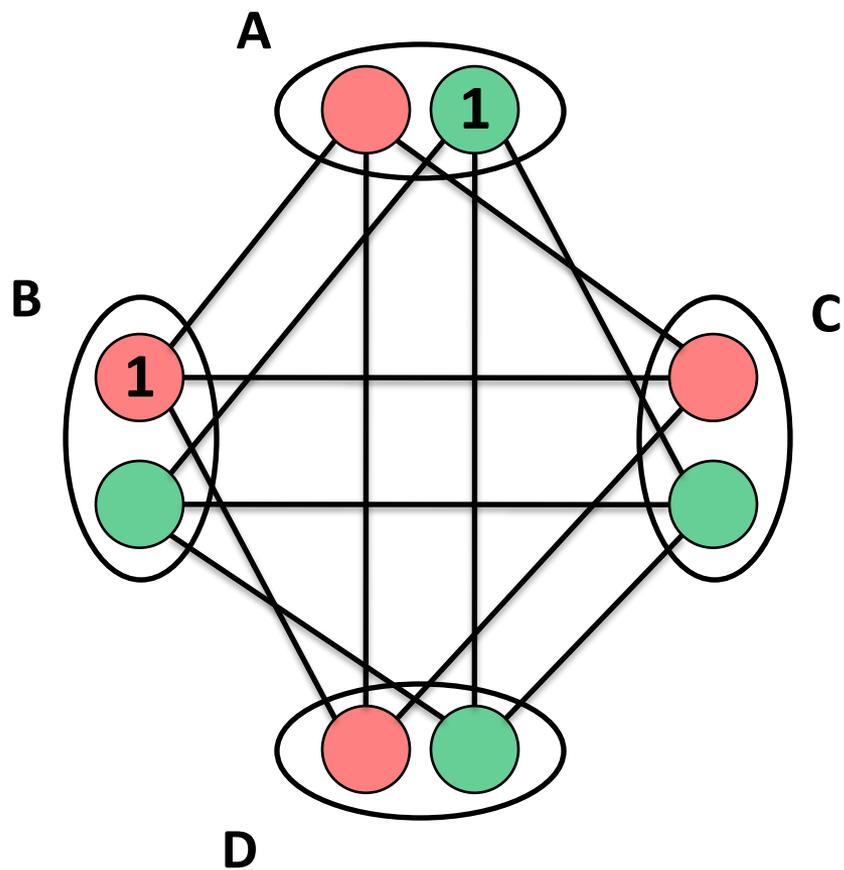
Reformulations chaotiques



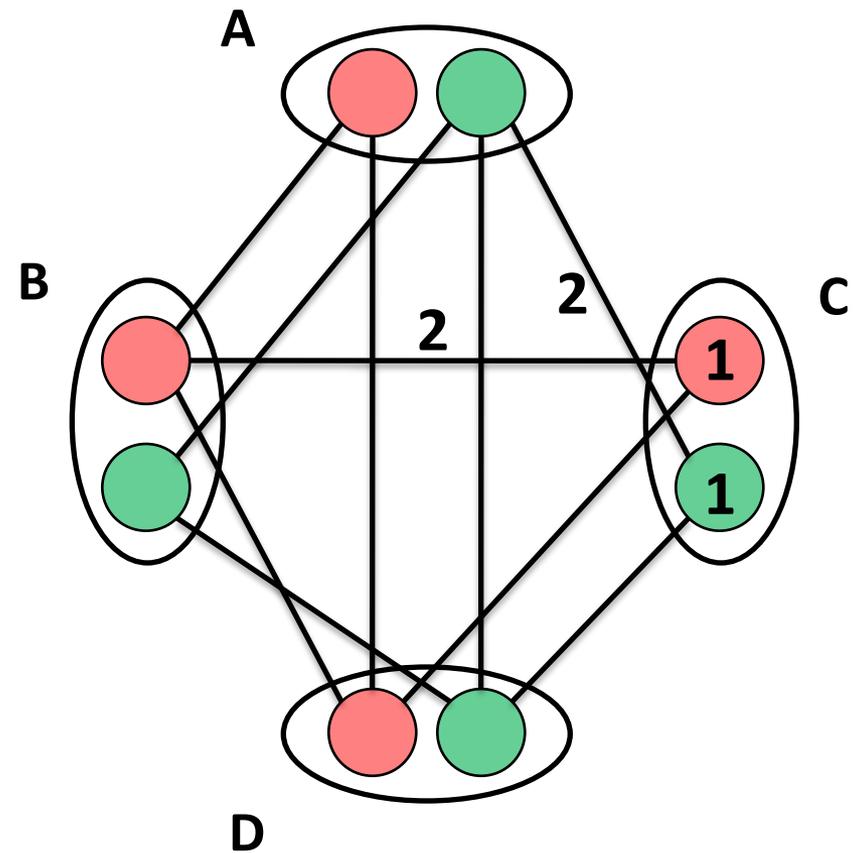
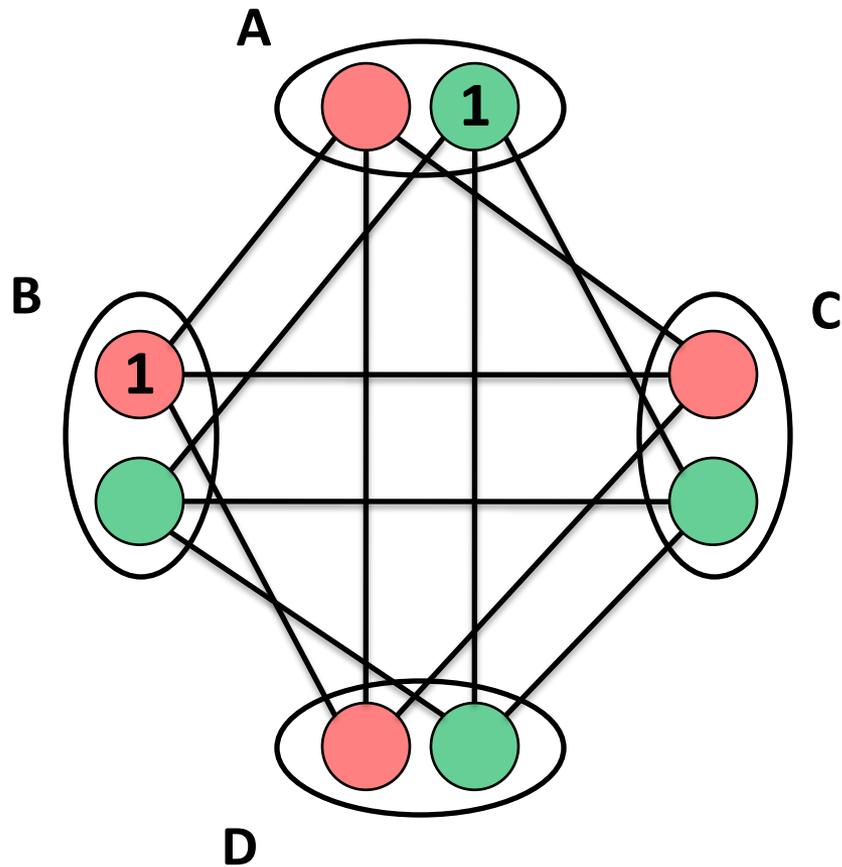
Reformulations chaotiques



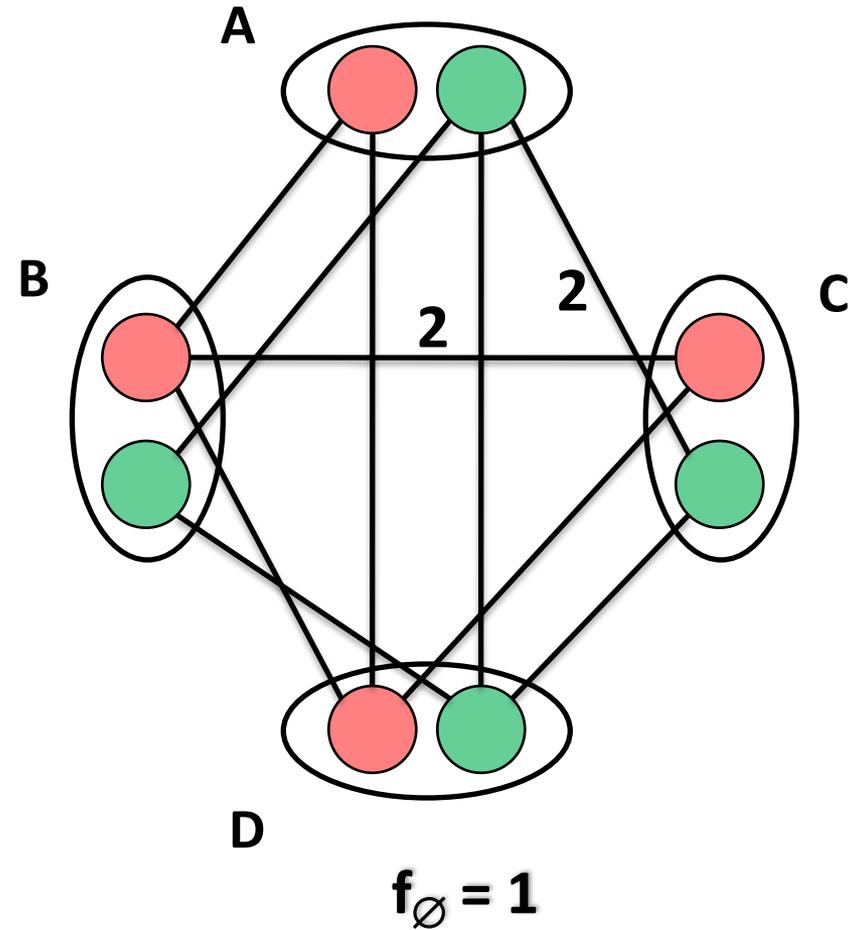
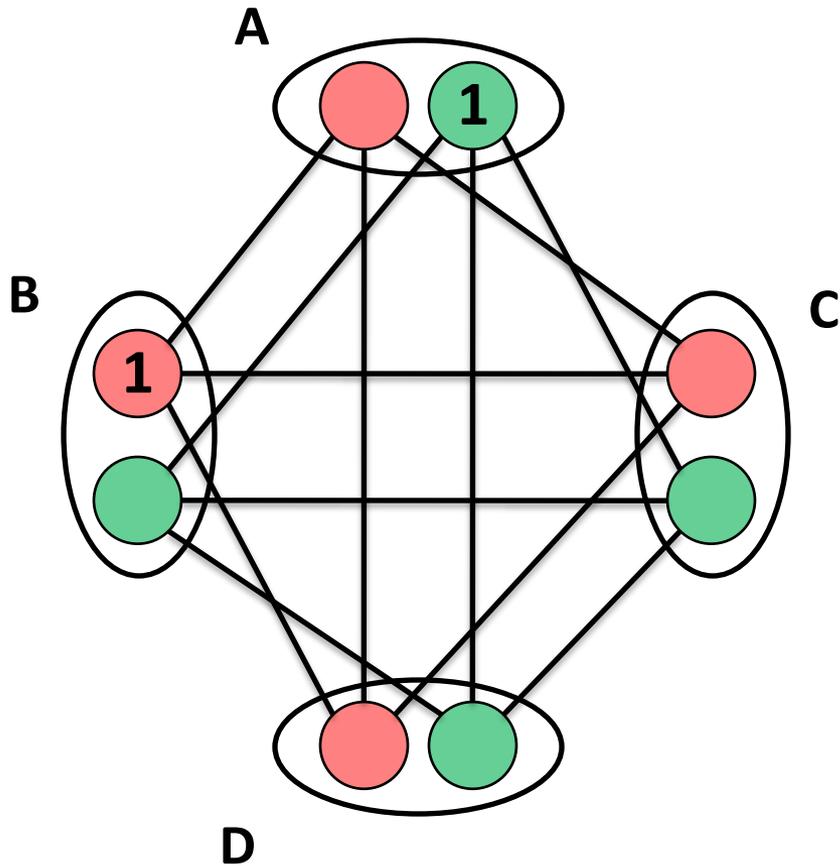
Reformulations chaotiques



Reformulations chaotiques

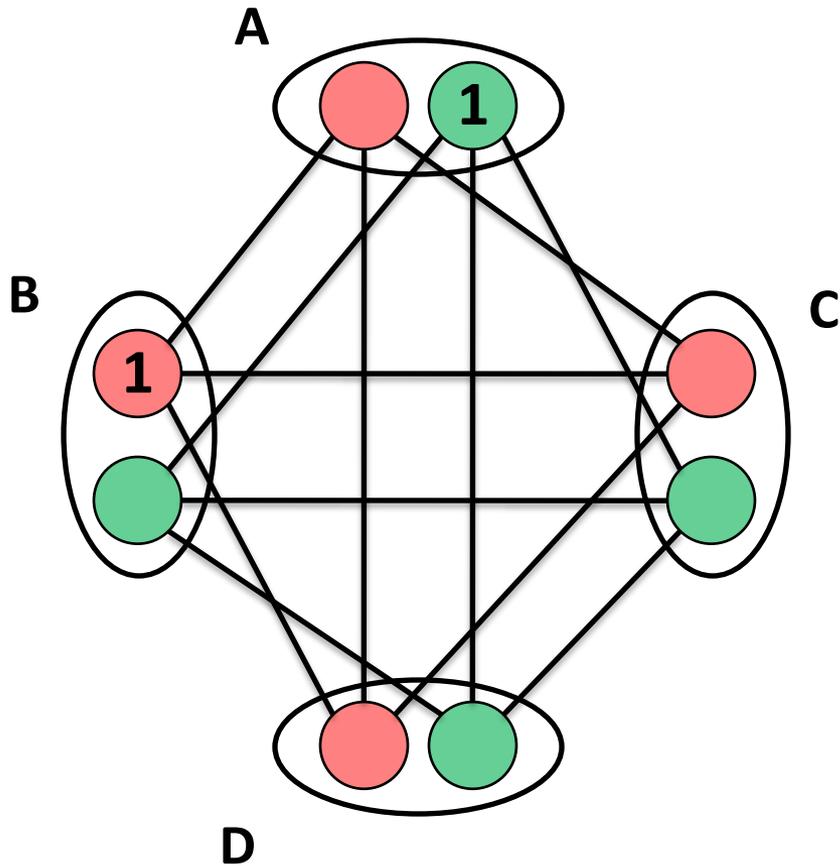


Reformulations chaotiques

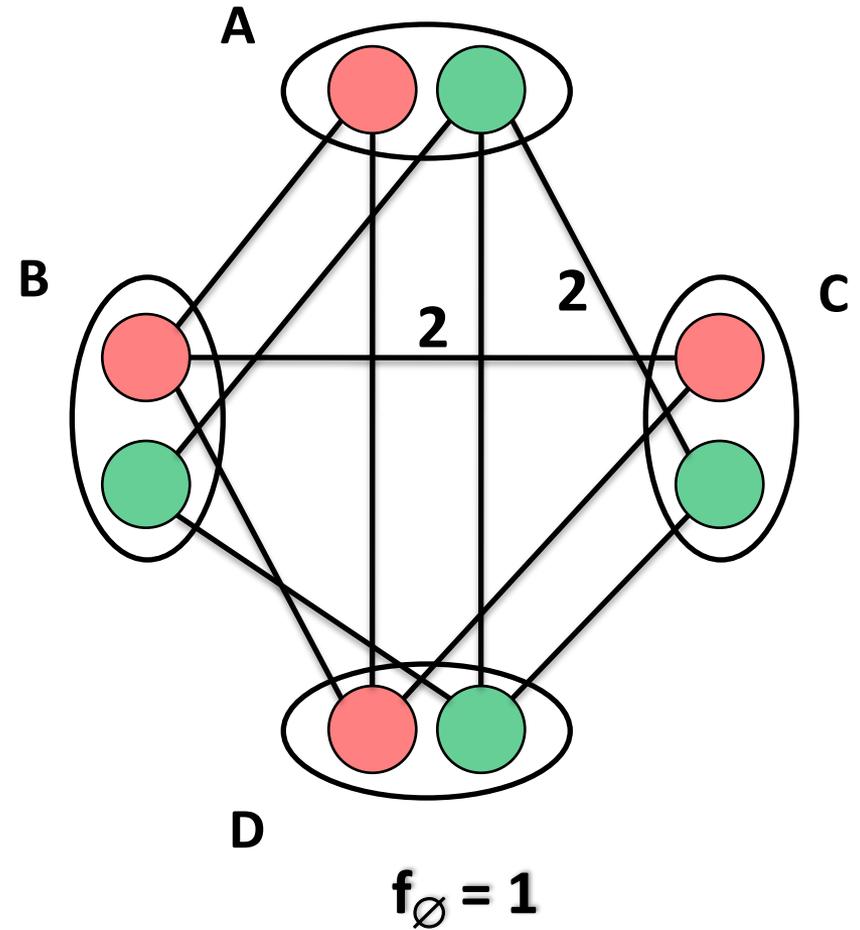


Problème arc-cohérent existentiel (EAC)

Reformulations chaotiques



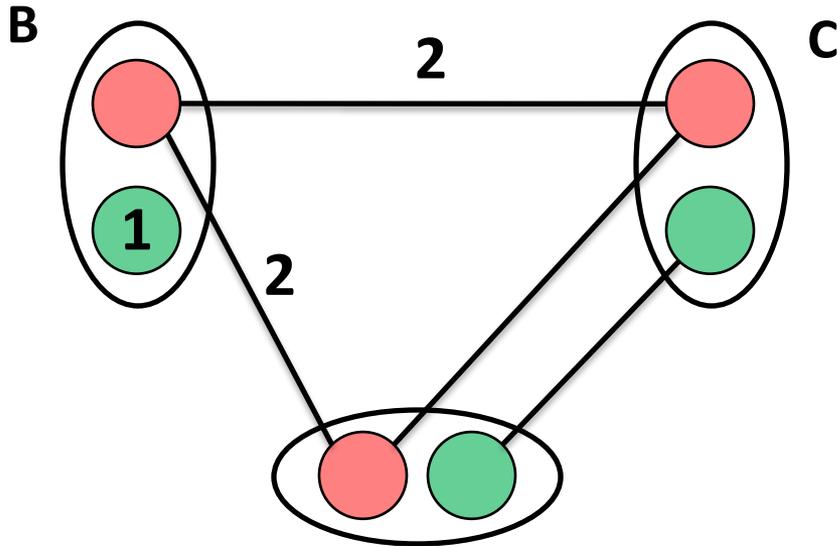
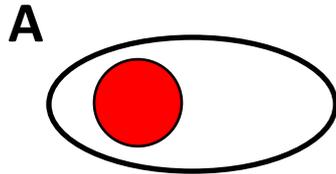
C



Problème arc-cohérent existentiel (EAC)

→ WAREHOUSE (de Givry et al, IJCAI 05) MENDEL (Sanchez, de Givry, Schiex, Constraints 08)

Reformulation optimale



D

$$f_{\emptyset} = 1$$

Problème EDAC

Hypothèse : **coûts rationnels**

Programme linéaire :

ensemble des opérations

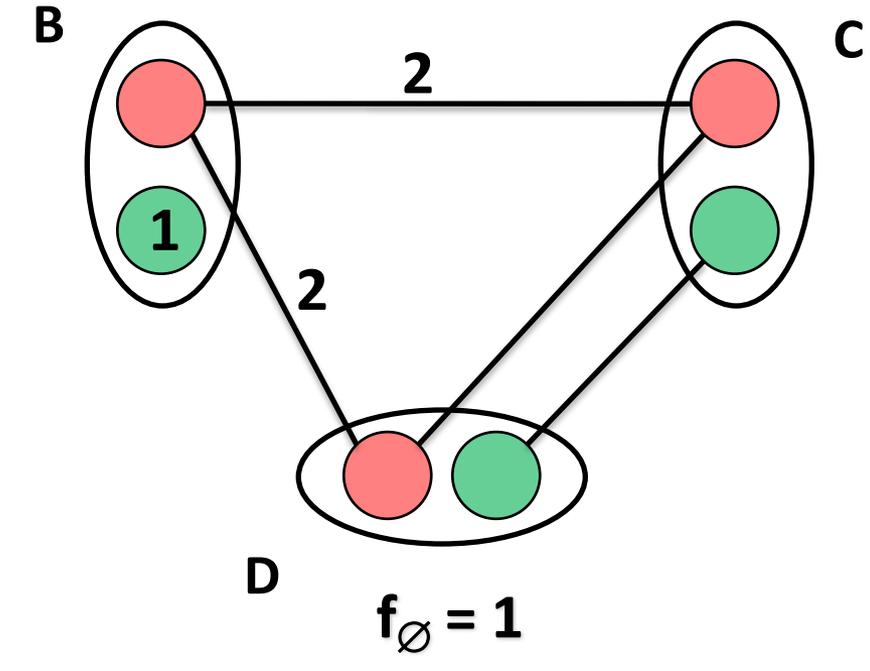
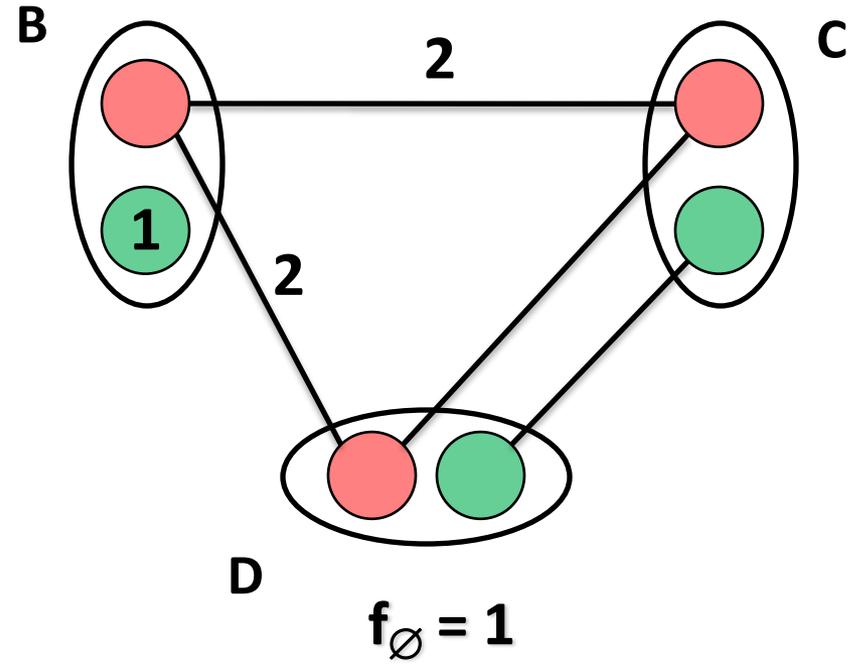
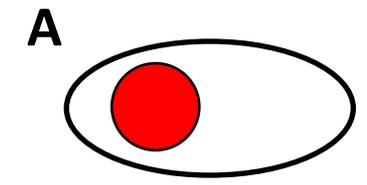
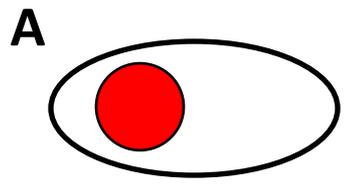
AC/DAC appliquées

simultanément maximisant f_{\emptyset}

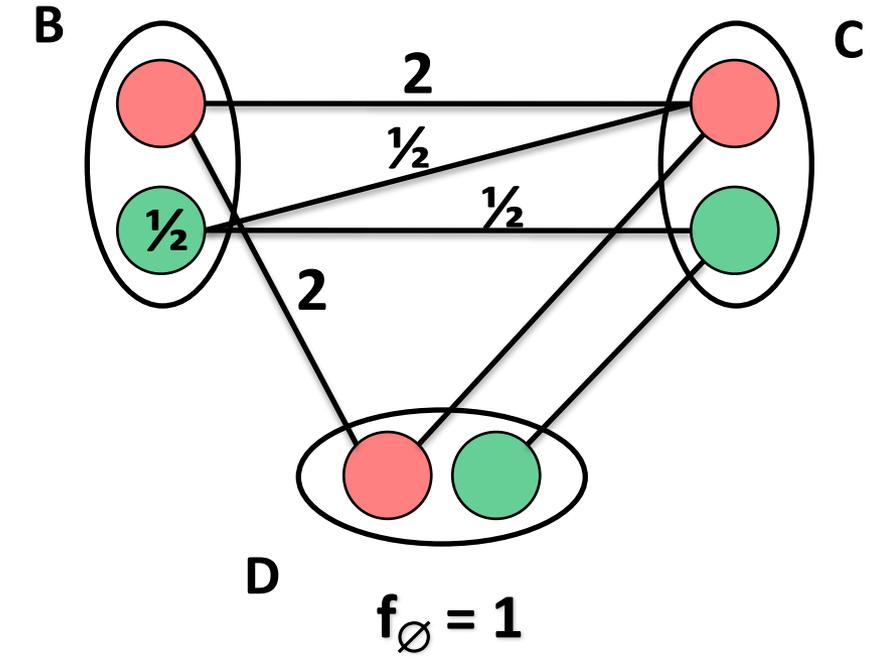
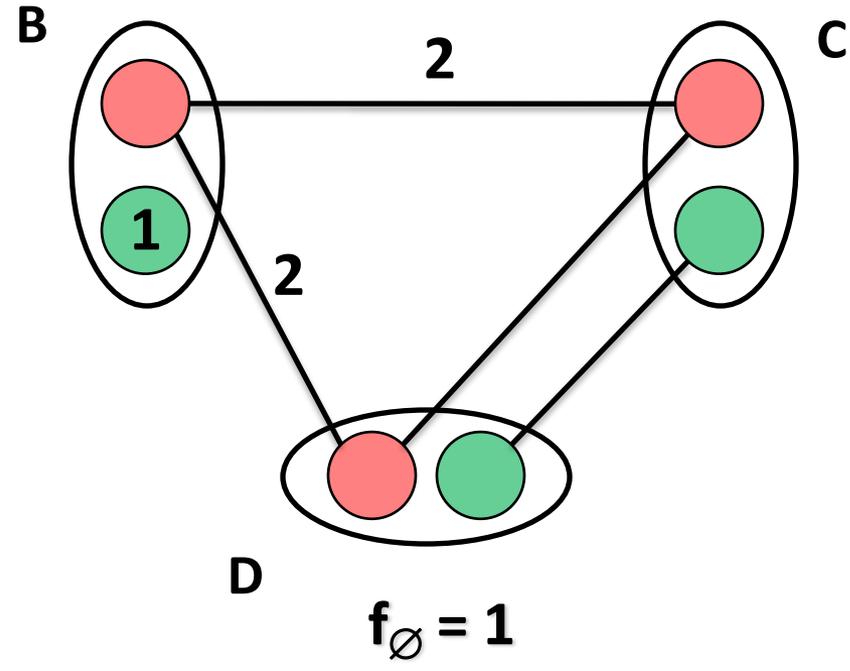
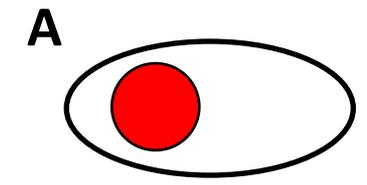
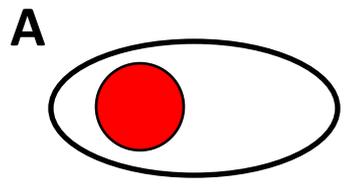
sous contraintes de conserver

des coûts positifs

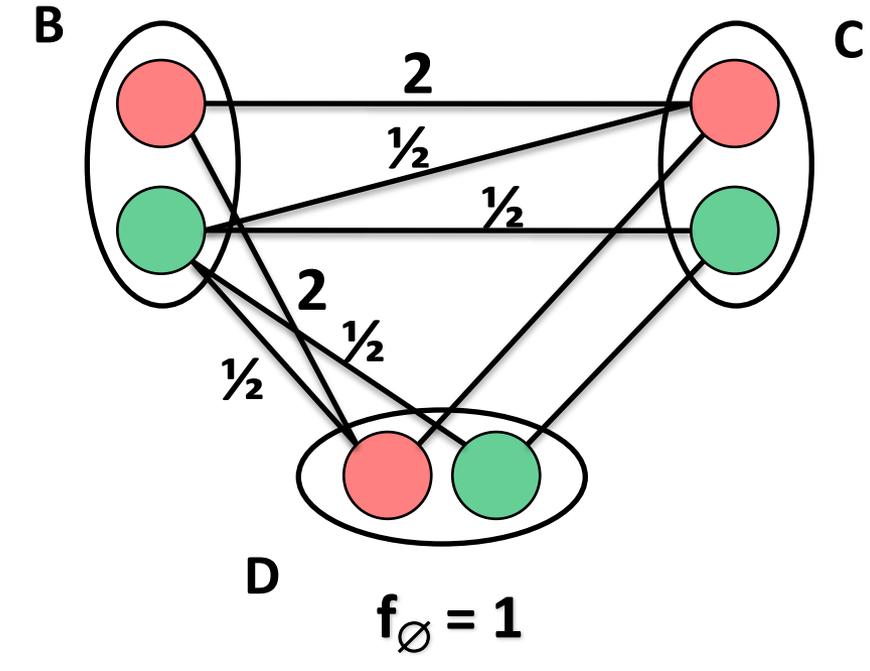
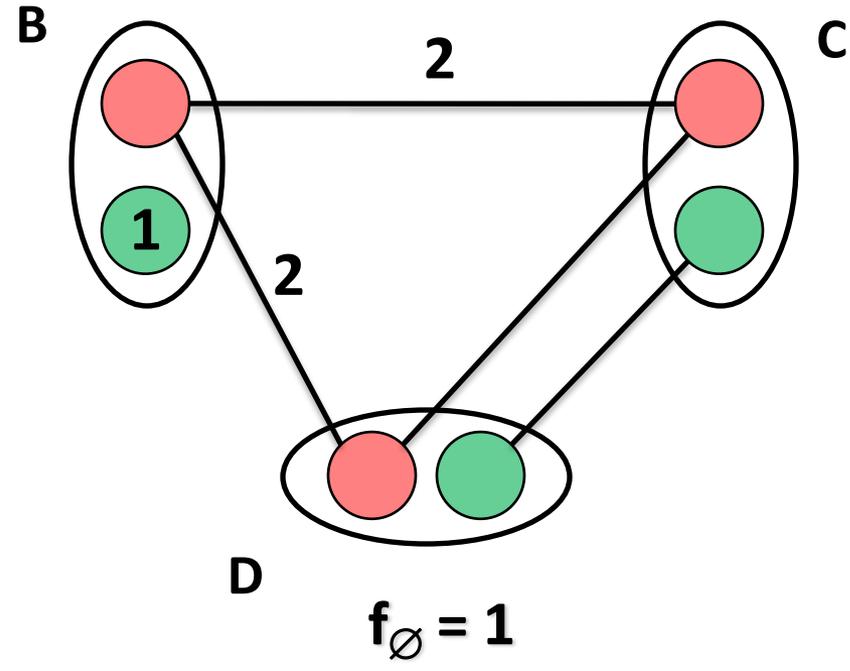
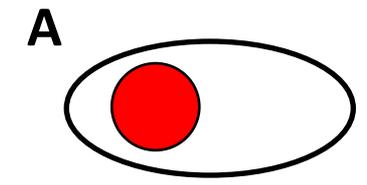
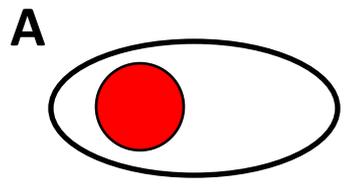
Reformulation optimale



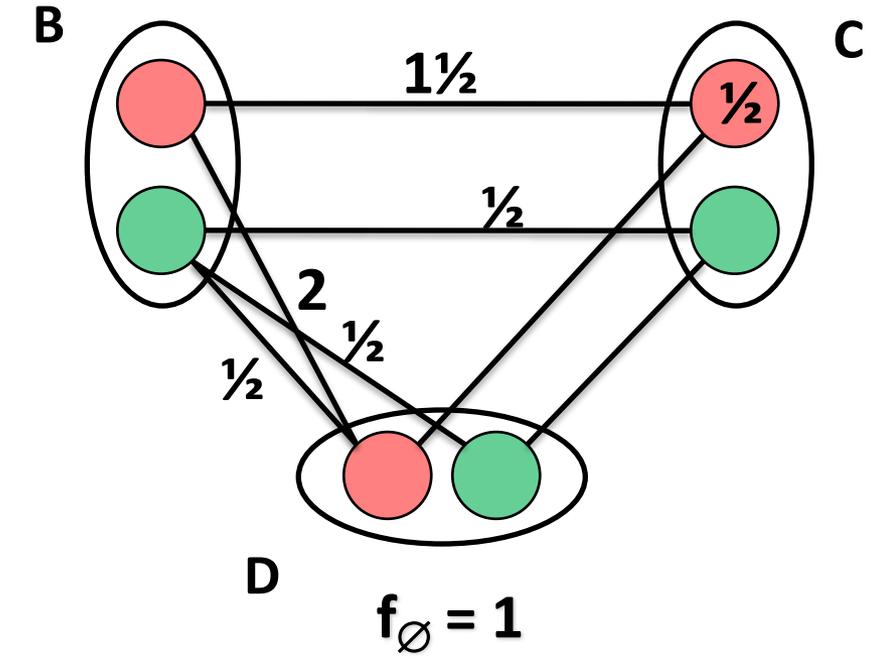
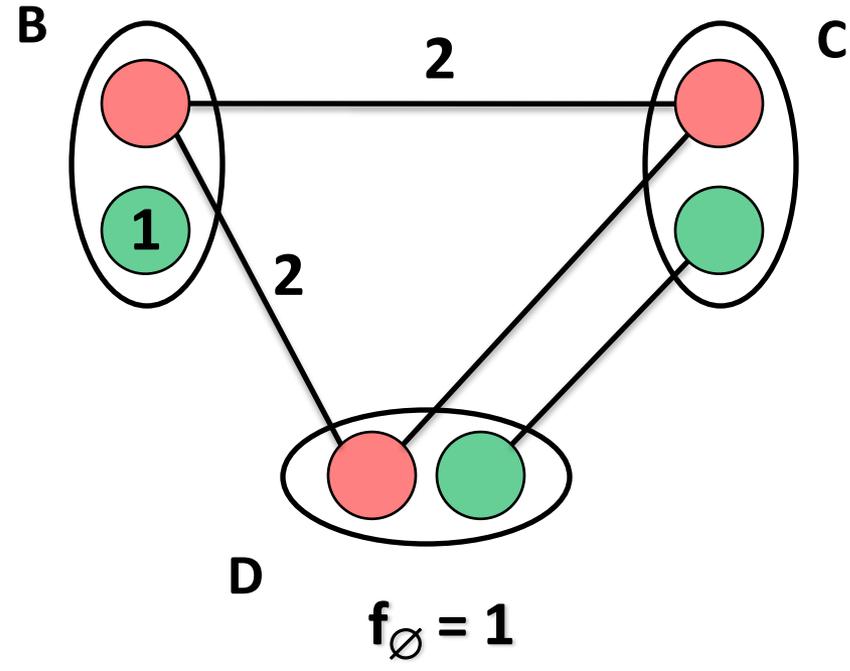
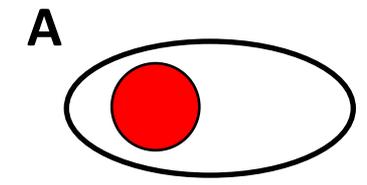
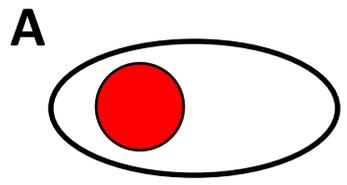
Reformulation optimale



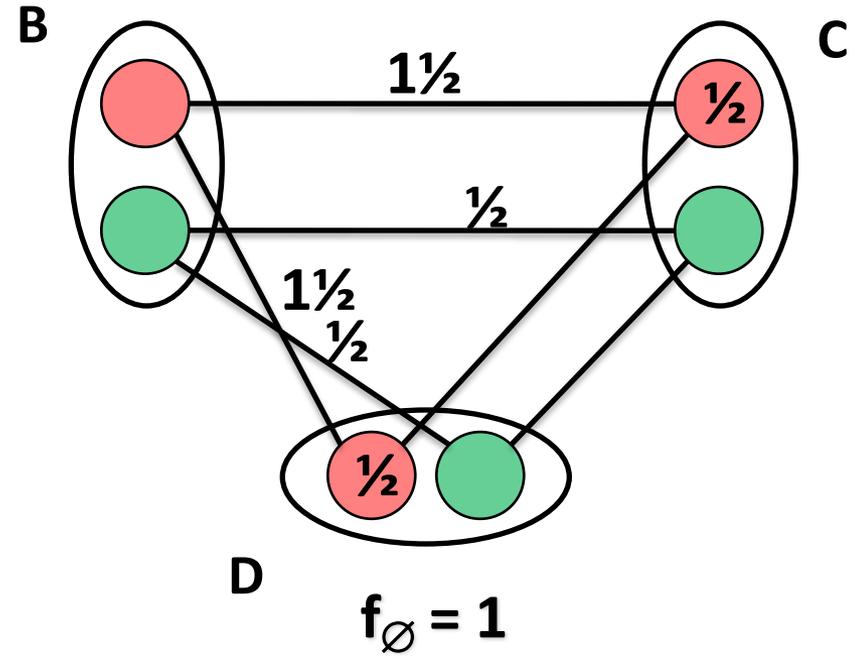
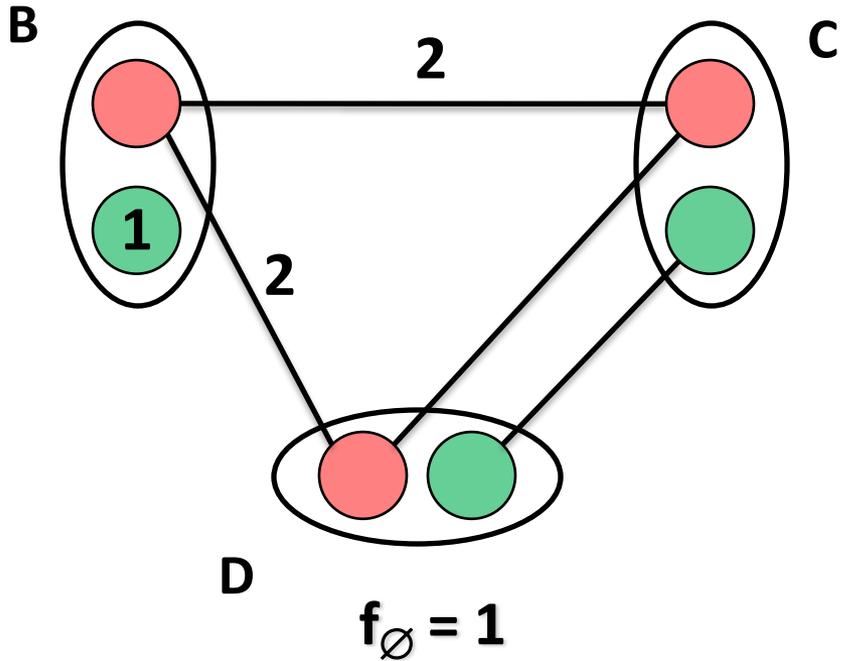
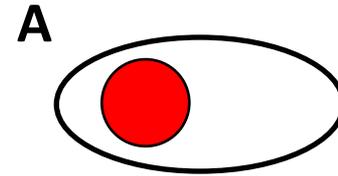
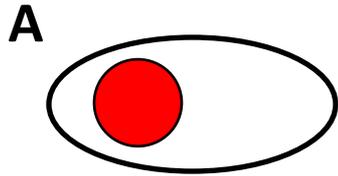
Reformulation optimale



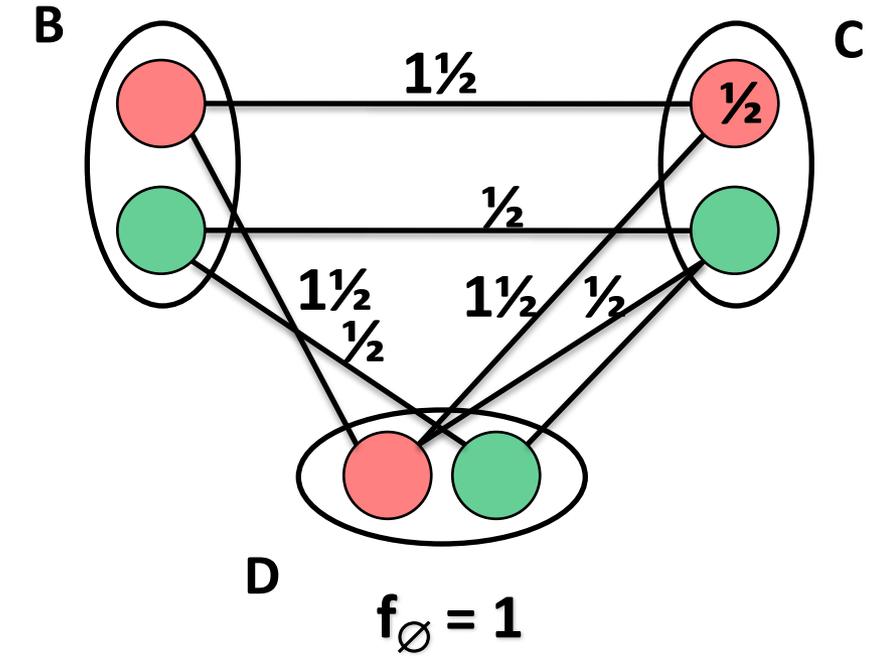
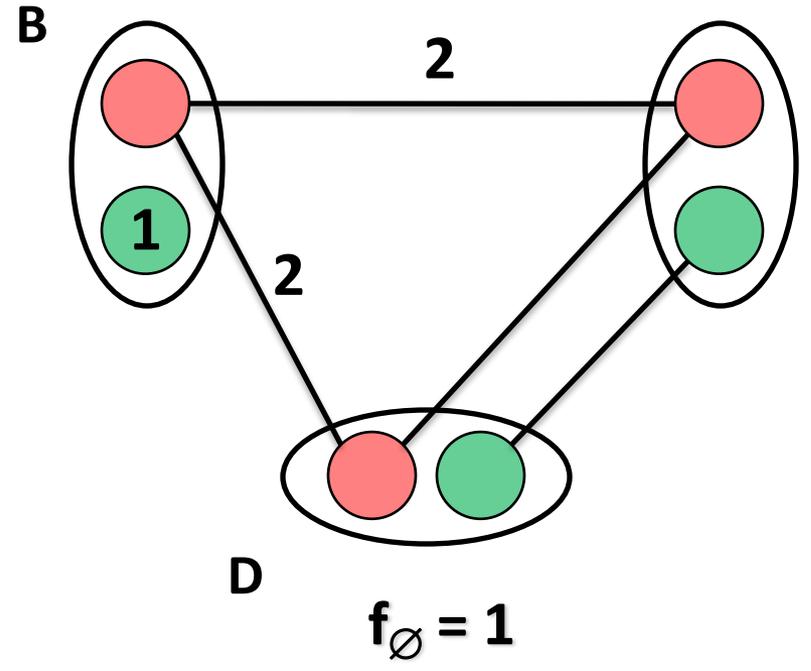
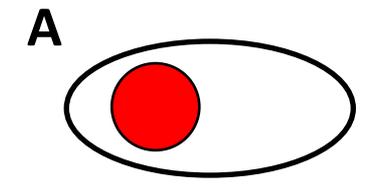
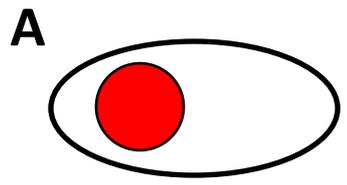
Reformulation optimale



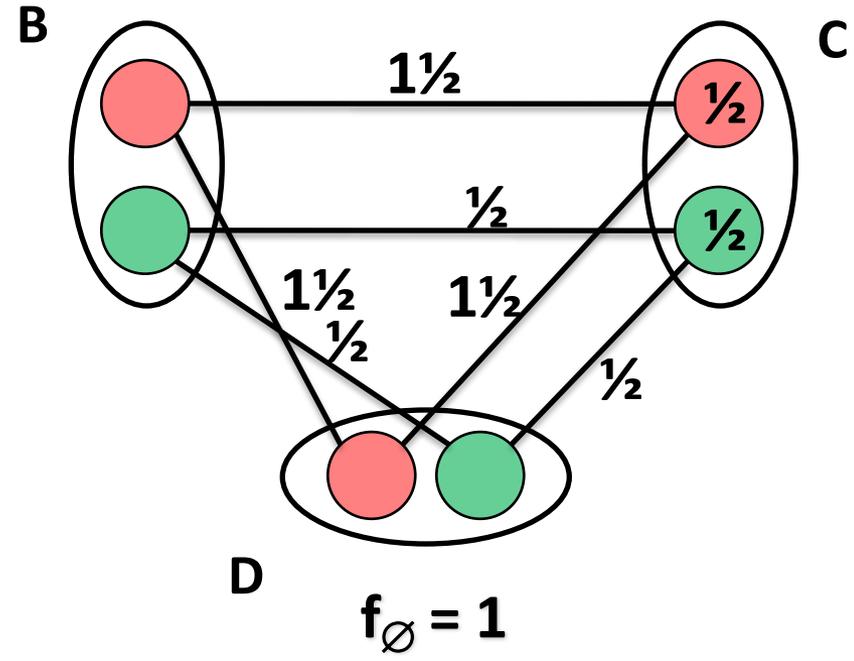
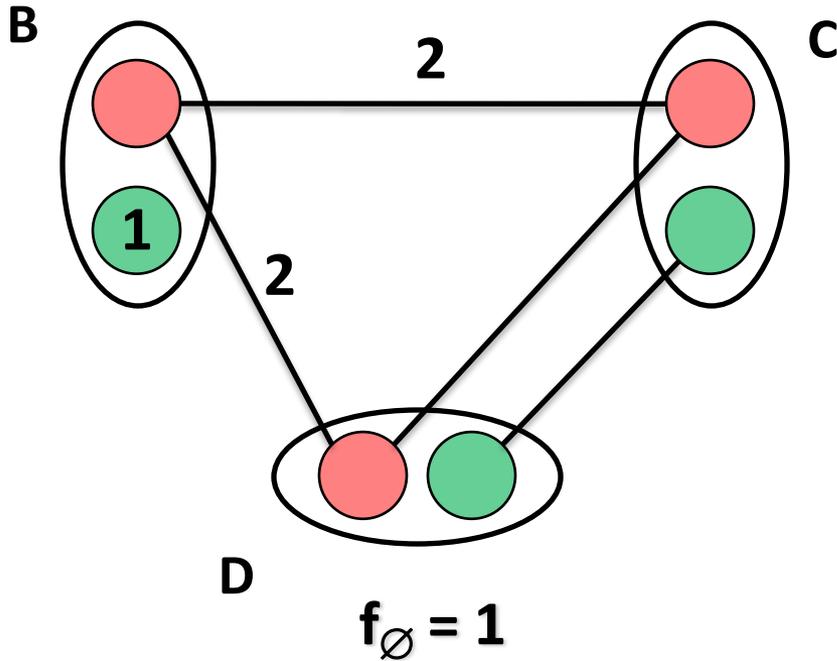
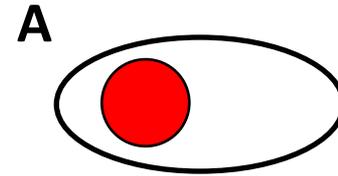
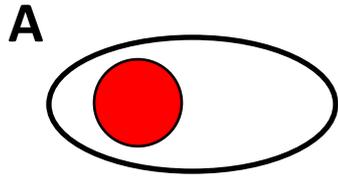
Reformulation optimale



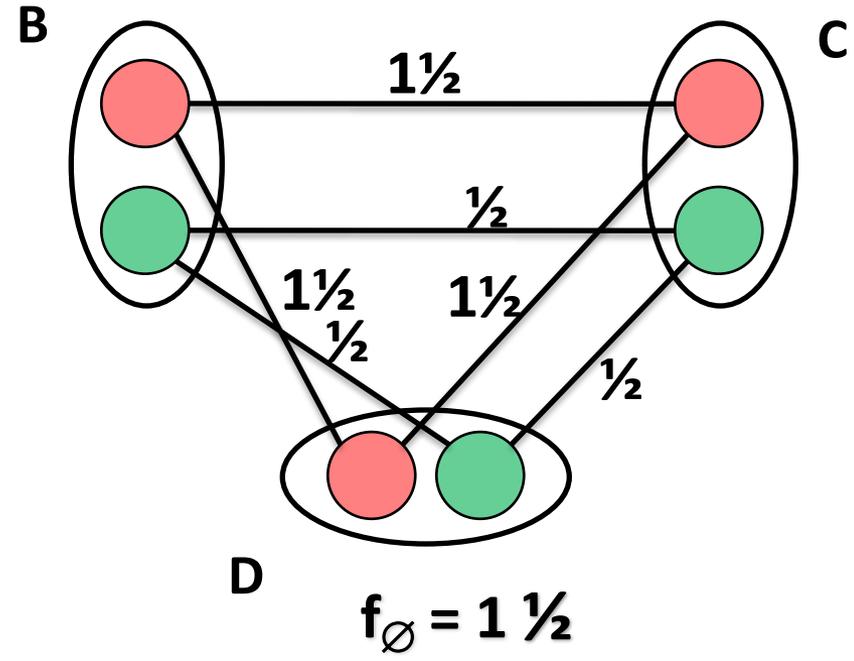
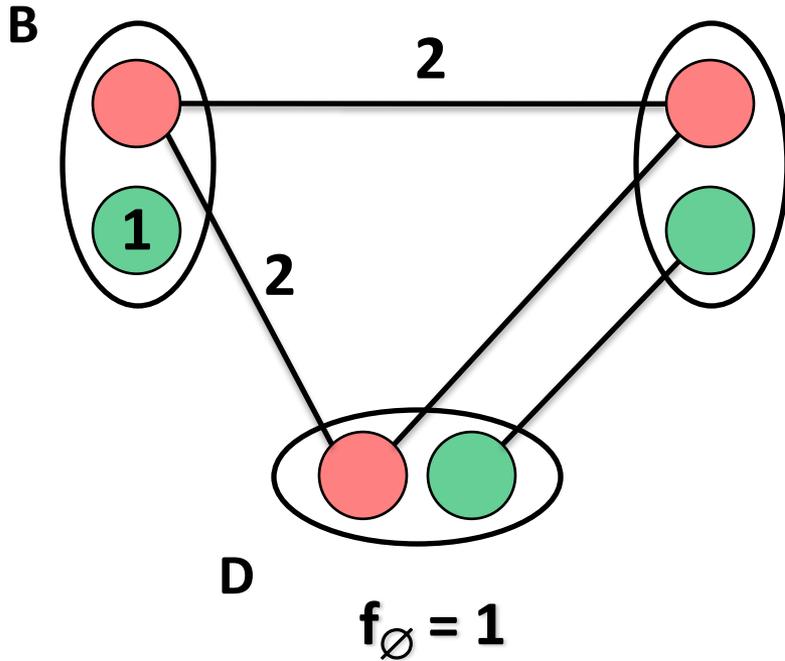
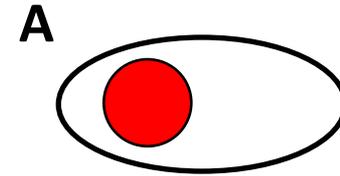
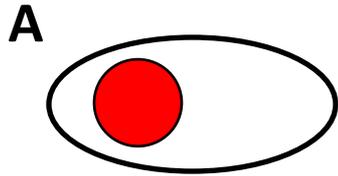
Reformulation optimale



Reformulation optimale



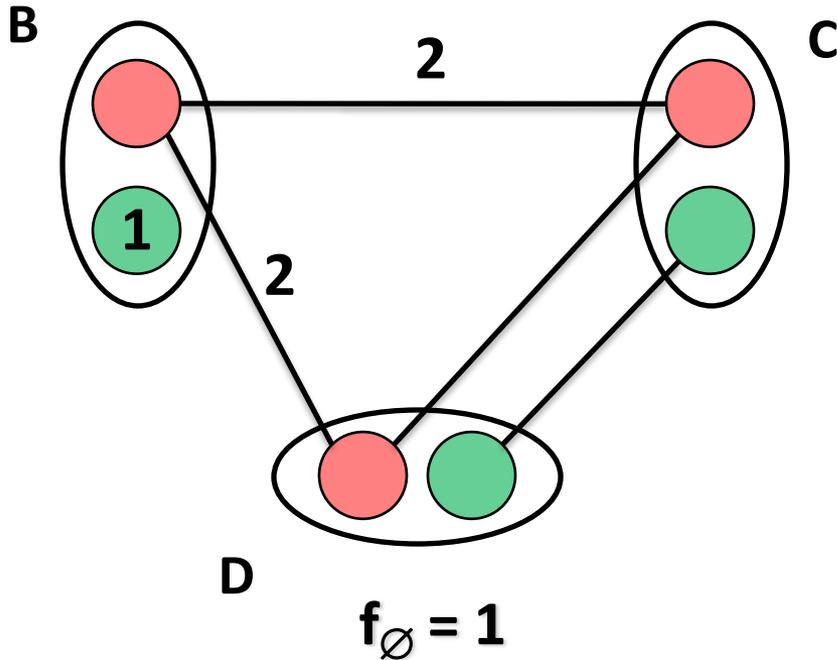
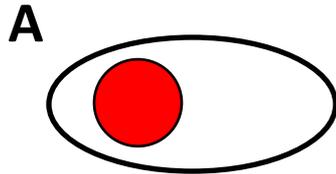
Reformulation optimale



Problème arc-cohérent optimal (OSAC)

➔ CELAR (Cooper, de Givry, Schiex, IJCAI 07) (Cooper et al, AIJ 10)

Reformulation planifiée



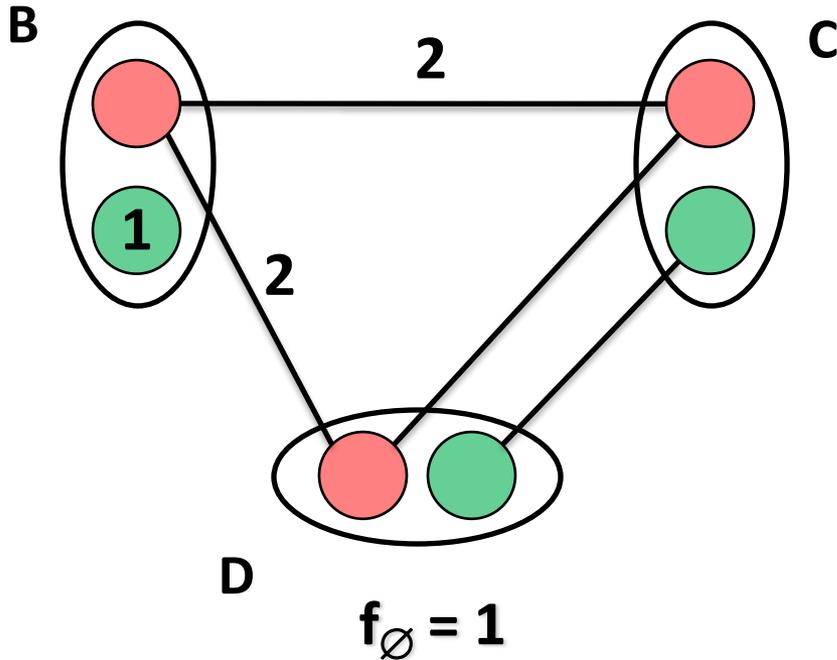
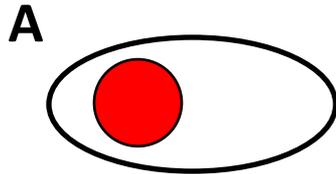
Hypothèse : **coûts rationnels**

Définition

$\text{Bool}(X, D, F) = (X, D, C)$

$\forall f_s, C_s$ telque $t \in C_s$ si $f_s(t) = 0$

Reformulation planifiée



Hypothèse : **coûts rationnels**

Définition

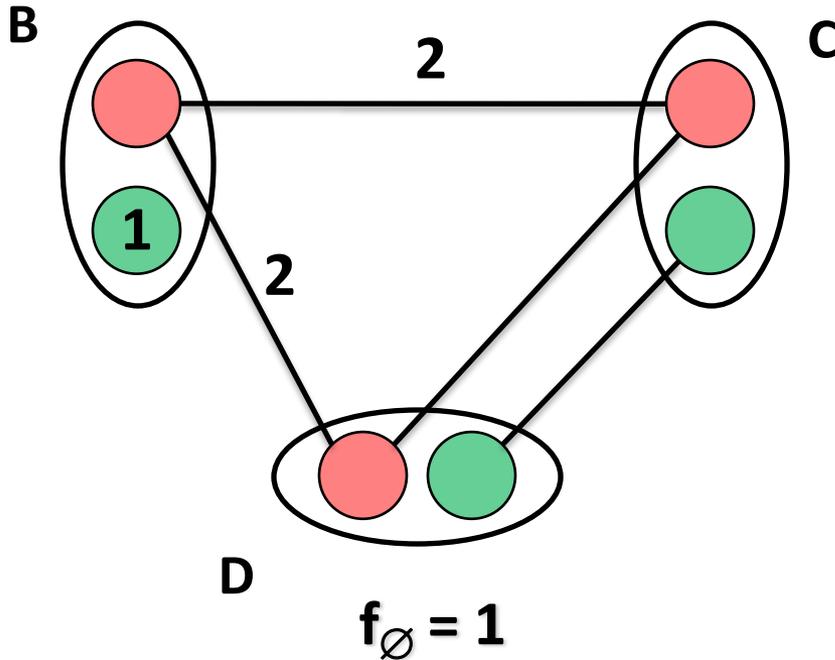
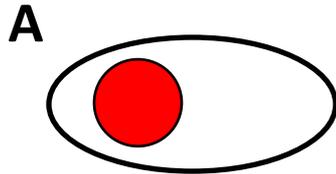
$\text{Bool}(X,D,F) = (X,D,C)$

$\forall f_s, C_s$ telque $t \in C_s$ si $f_s(t)=0$

Théorème

\exists séquence d'opérations AC/DAC augmentant f_{\emptyset} ssi $\text{Bool}(X,D,F)$ est arc-incohérent

Reformulation planifiée



Hypothèse : **coûts rationnels**

Définition

$\text{Bool}(X,D,F) = (X,D,C)$

$\forall f_s, C_s$ telque $t \in C_s$ si $f_s(t)=0$

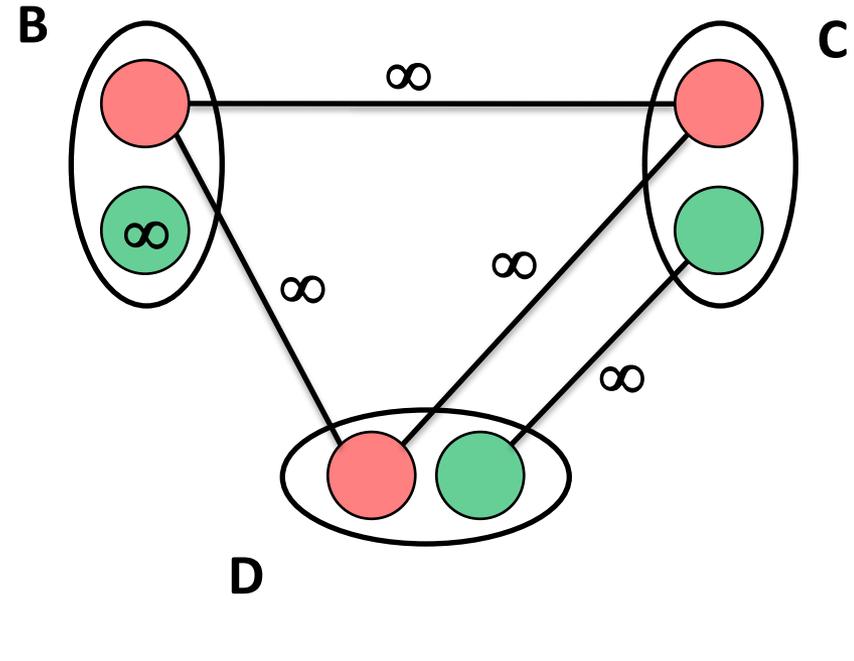
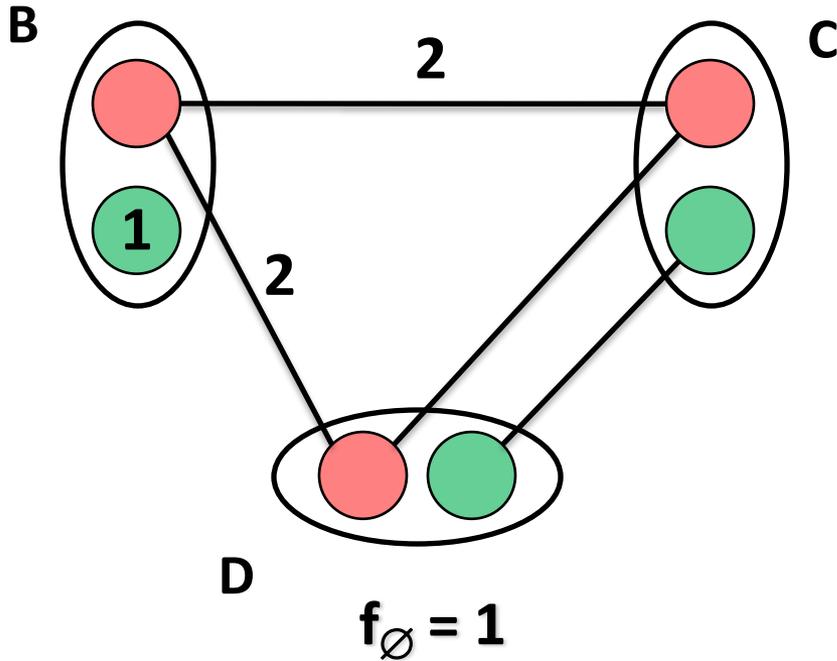
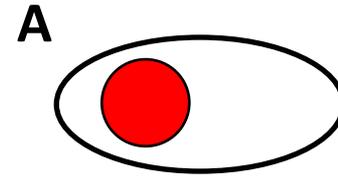
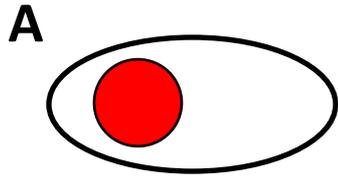
Théorème

\exists séquence d'opérations AC/DAC augmentant f_{\emptyset} ssi $\text{Bool}(X,D,F)$ est arc-incohérent

Propriété

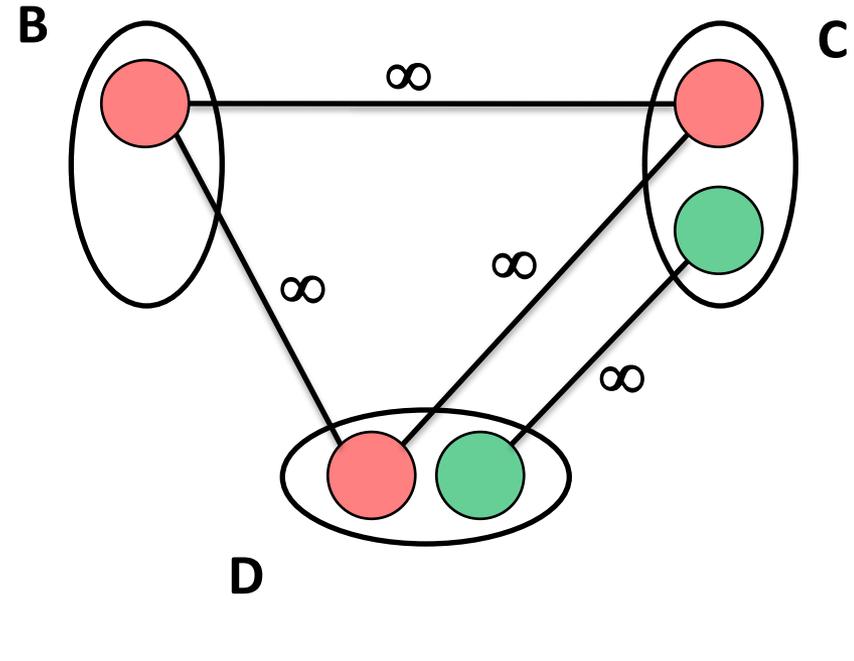
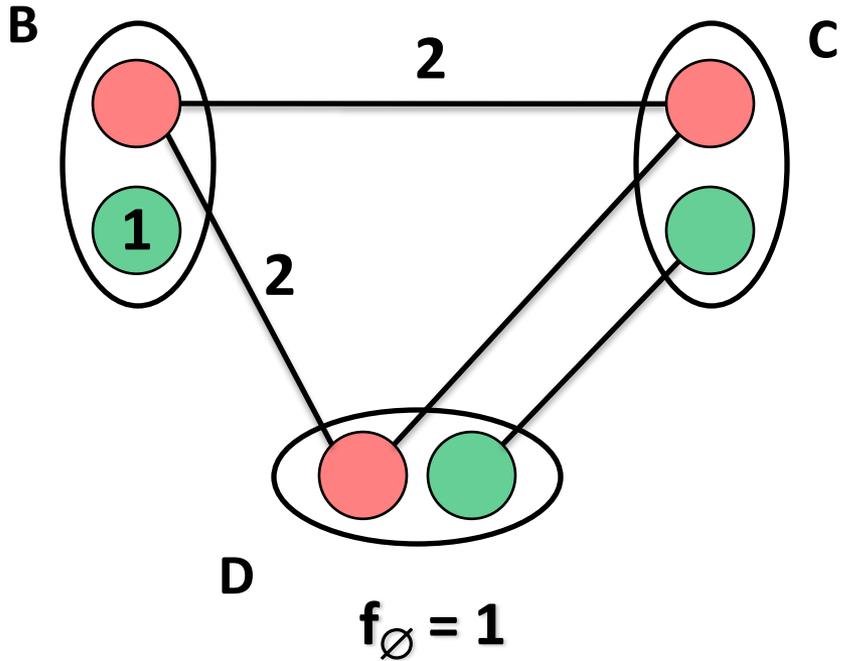
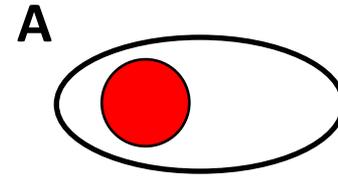
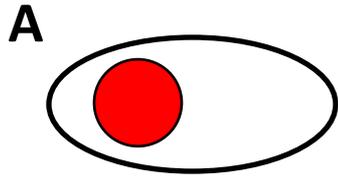
Problème arc-cohérent virtuel ssi $\text{Bool}(X,D,F)$ est arc-cohérent

Reformulation planifiée



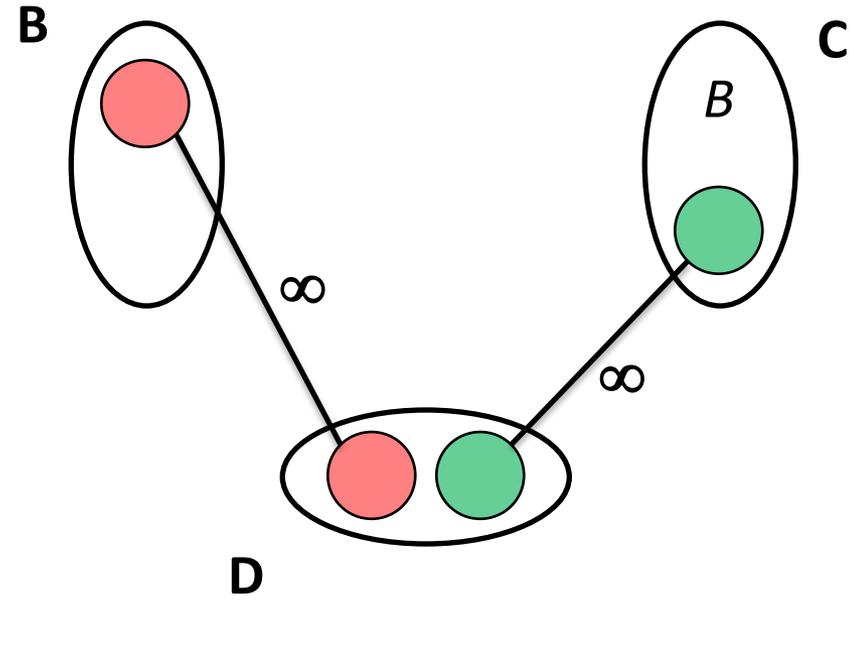
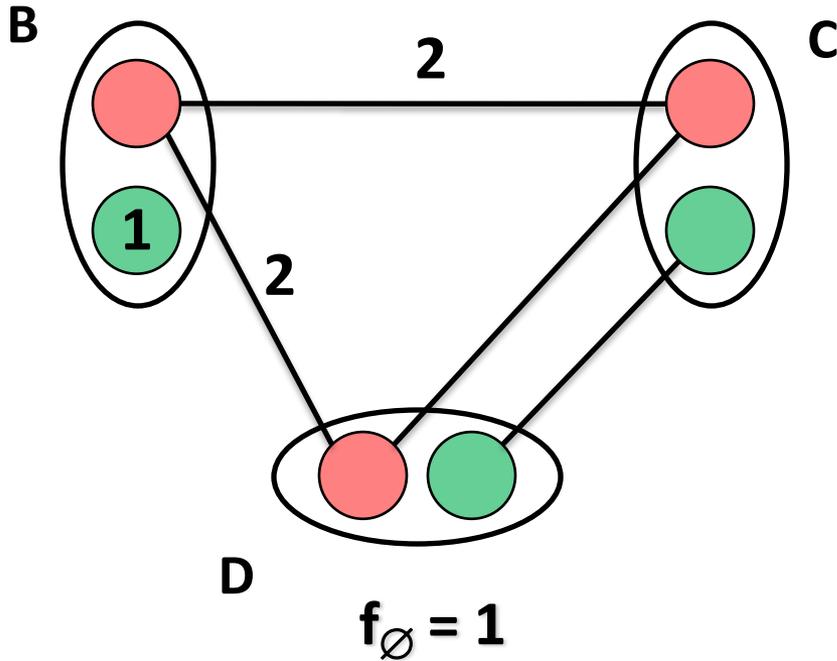
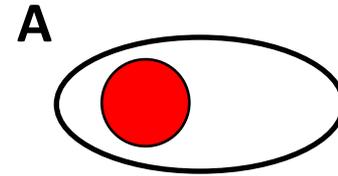
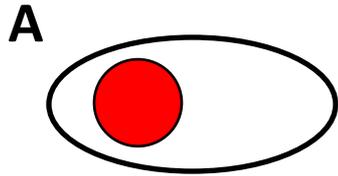
$\text{Bool}(X, D, F)$

Reformulation planifiée



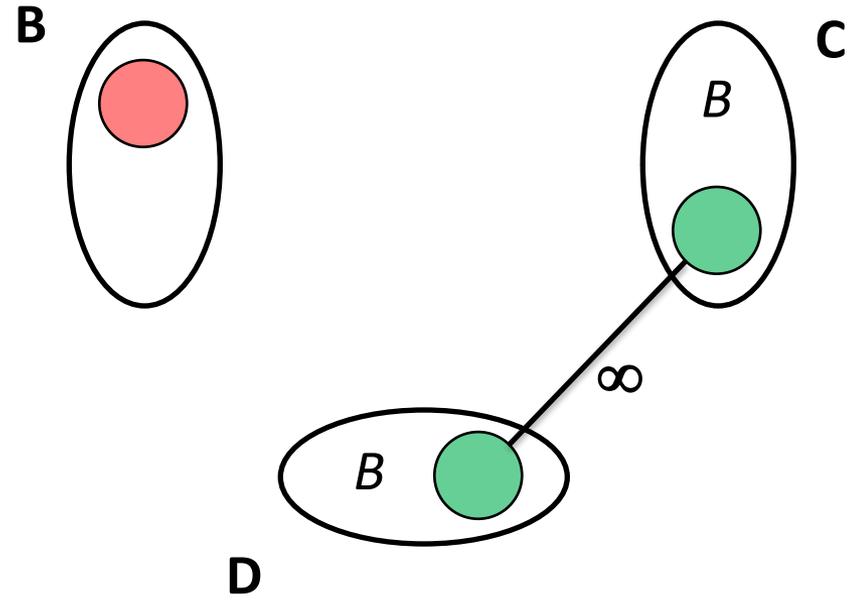
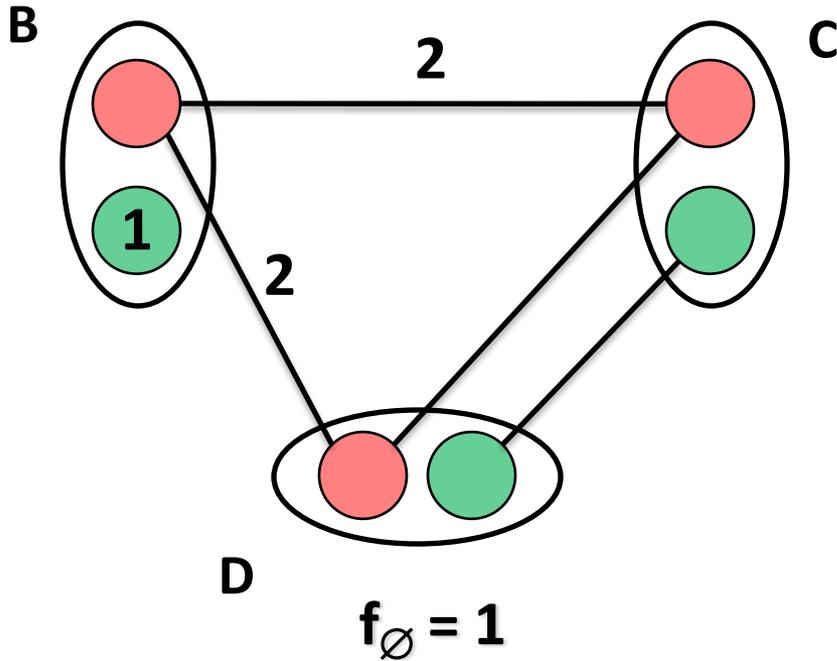
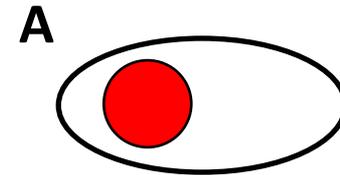
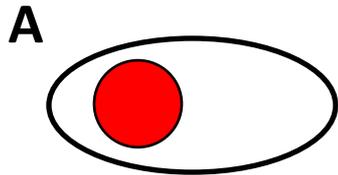
Bool(X,D,F)

Reformulation planifiée



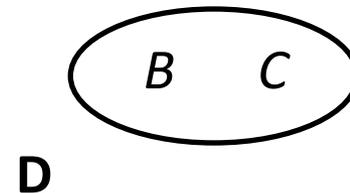
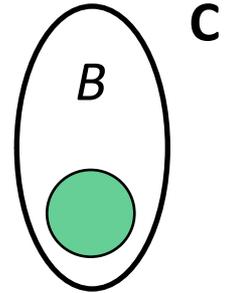
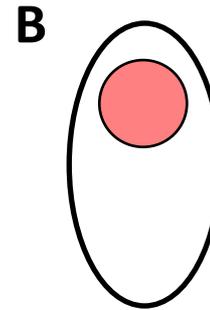
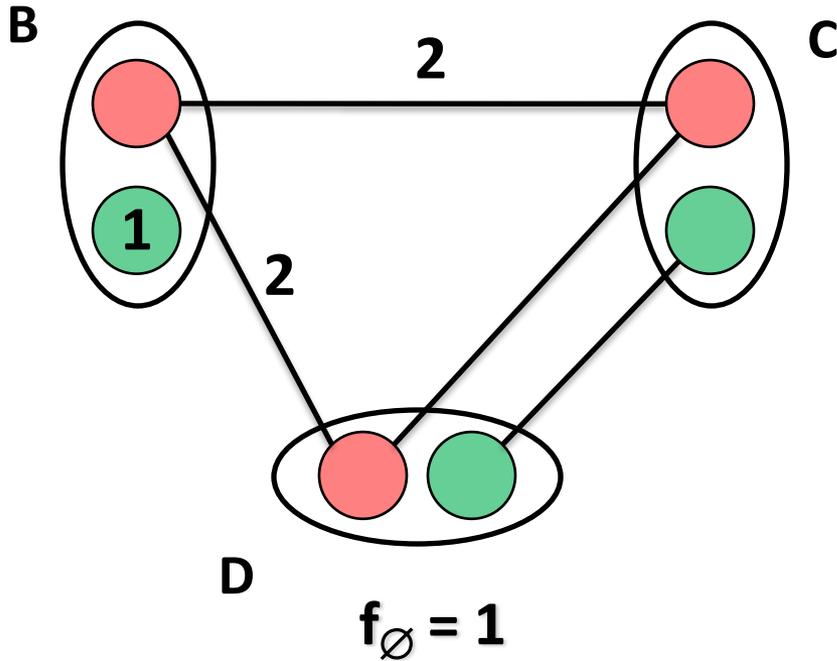
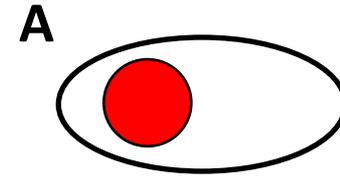
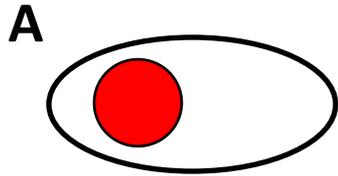
$\text{Bool}(X, D, F)$

Reformulation planifiée



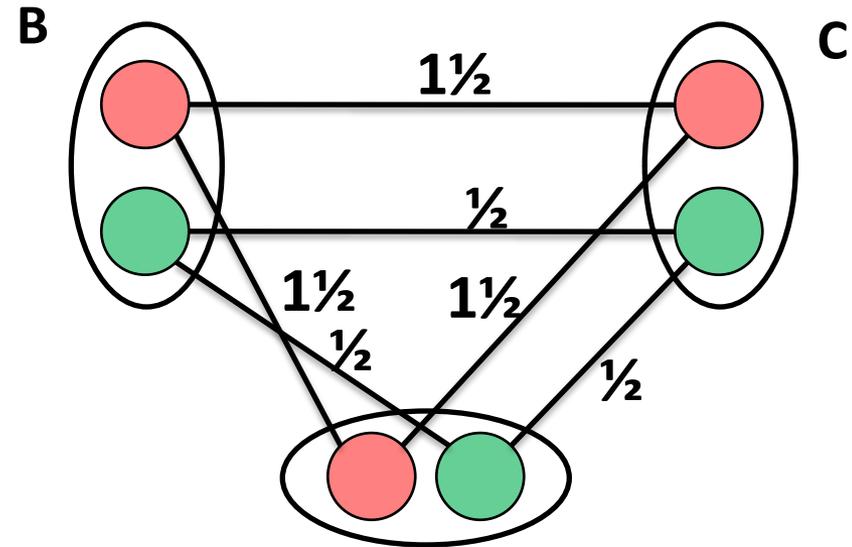
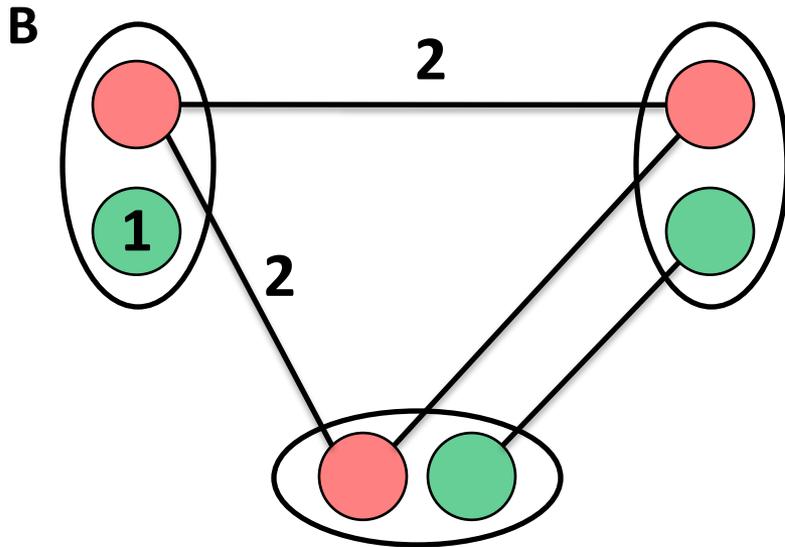
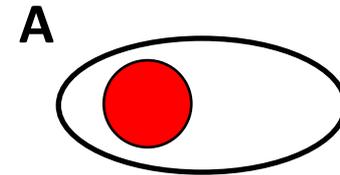
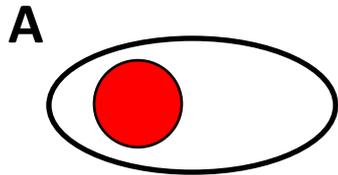
Bool(X,D,F)

Reformulation planifiée



Bool(X,D,F)

Reformulation planifiée



D $f_{\emptyset} = 1$

D $f_{\emptyset} = 1 \frac{1}{2}$

Problème EDAC

Problème arc-cohérent virtuel (VAC)

→ CELAR (Cooper, de Givry, Sanchez, Schiex, Zytnicki, AAAI 08) (Cooper et al, AIJ 10)

Hiérarchie des cohérences locales

Cohérence locale	Complexité		Classes polynomiales
	en temps	en espace	
NC*	$O(nd)$	$O(nd)$	-
BAC [∅]	$O(n^2 d^3)$	$O(e)$	-
AC*	$O(n^2 d^2 + ed^3)$	$O(ed)$	-
DAC	$O(ed^2)$	$O(ed)$	Arbre
FDAC*	$O(end^3)$	$O(ed)$	Arbre
EDAC*	$O(ed^2 \max(nd, k))$	$O(ed)$	Arbre
VAC _ε	$O(ed^2 k/\epsilon)$	$O(ed)$	Arbre, fct. sous-modulaires
OSAC	$poly(ed + n)$	$poly(ed^2 + nd)$	Arbre, fct. sous-modulaires

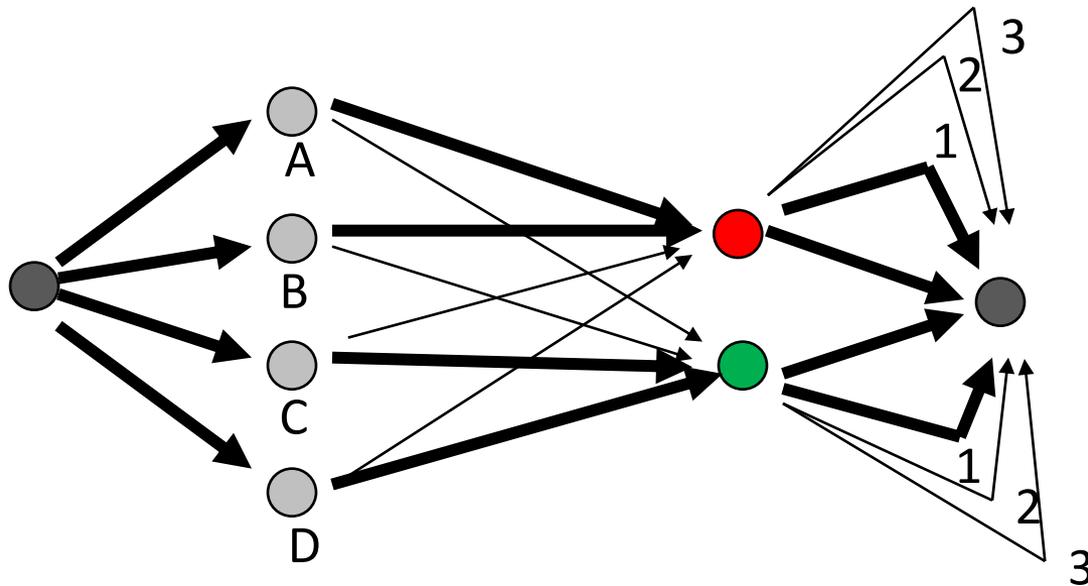
Remarques

- Cadre CSP étendu via fonctions de coût d'arité 0 et 1
BAC[∅] (Zytnicki, Gaspin, de Givry, Schiex, JAIR 09)
- Lien entre OSAC et la relaxation linéaire
 (Werner, IEEE Pat. Rec. 07)

Fonction de coût globale représentée par un flot. Exemple de softAllDiff

≡ Nombre min de paires de variable avec la même valeur
decomposition-based costs (Beldiceanu & Petit, CPAIOR'04),

(Métivier, 2010)



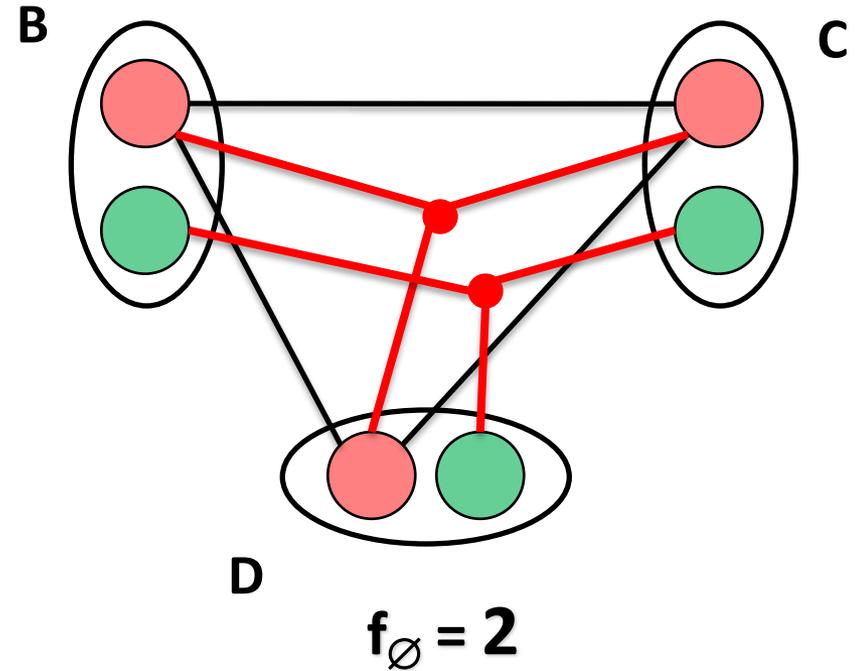
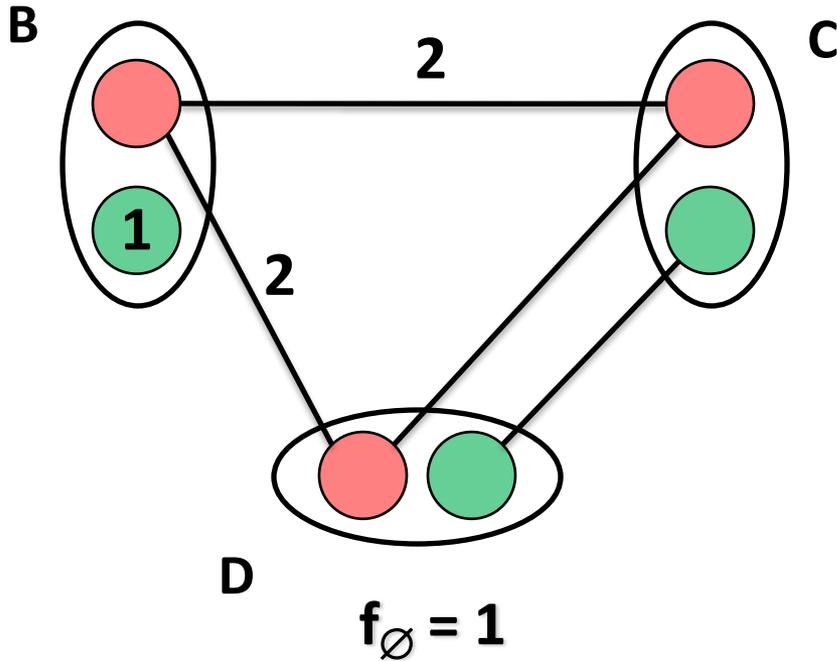
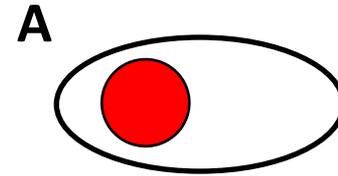
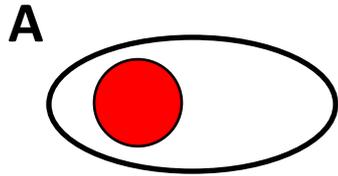
Tous les arcs
ont une capacité
égale à 1

Flot max de coût min égal à 2

(Hoeve, Pesant, Rousseau, J. Heuristics 06)

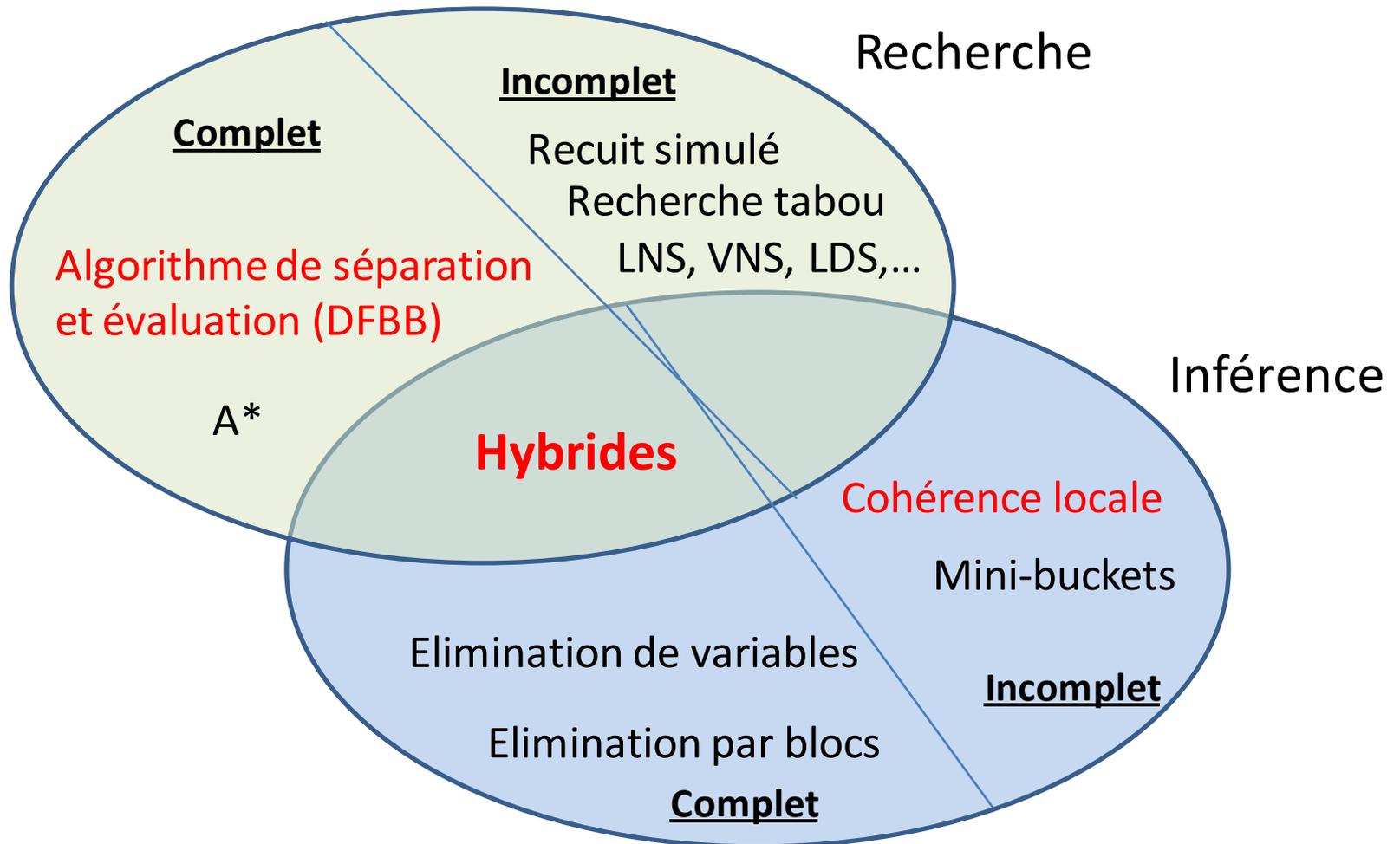
*EDGAC** (Lee, Leung, IJCAI 09, AAAI 10) intégré dans toulbar2

Reformulation d'arité supérieure

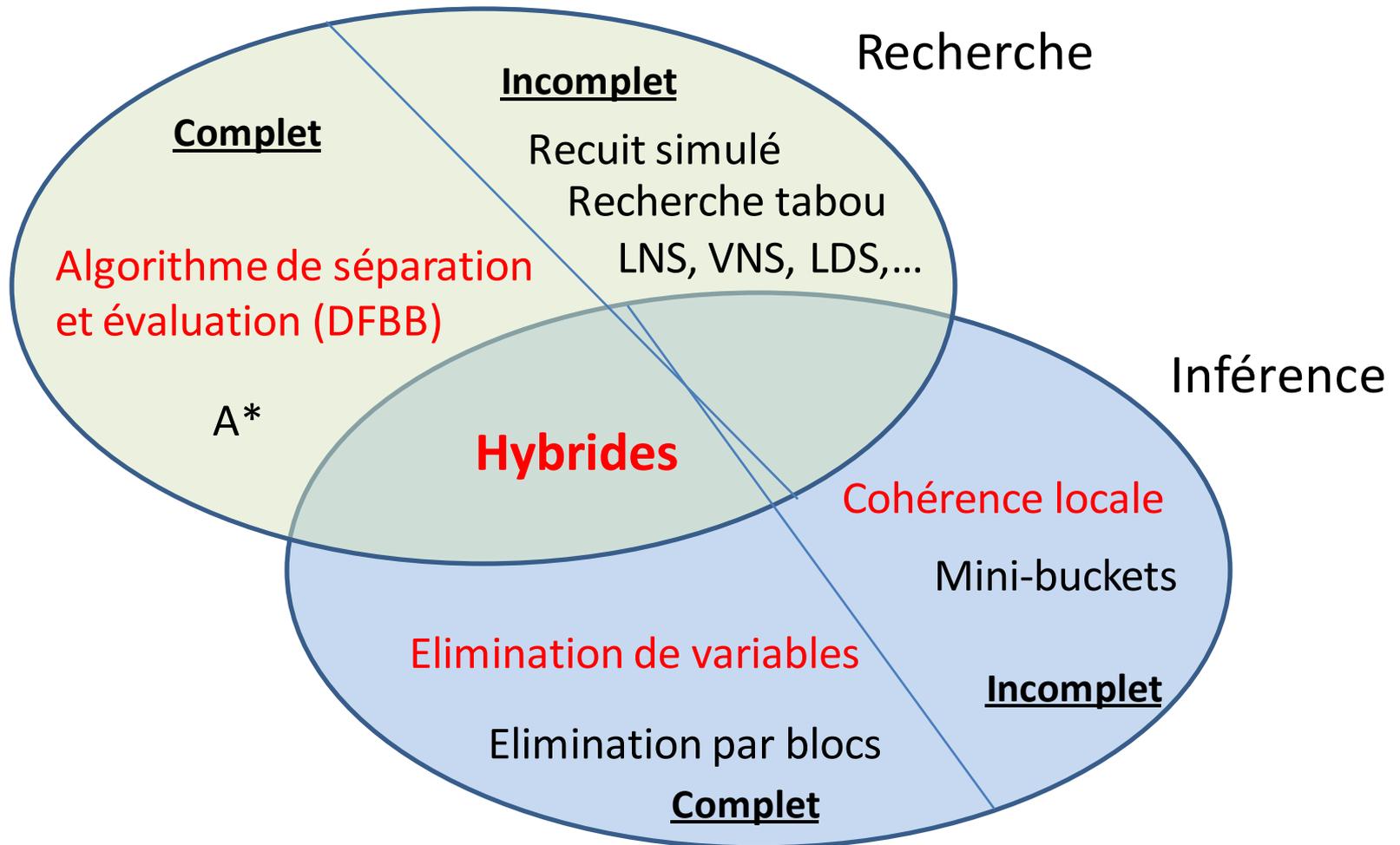


Problème chemin-inverse-cohérent virtuel (VPIC)

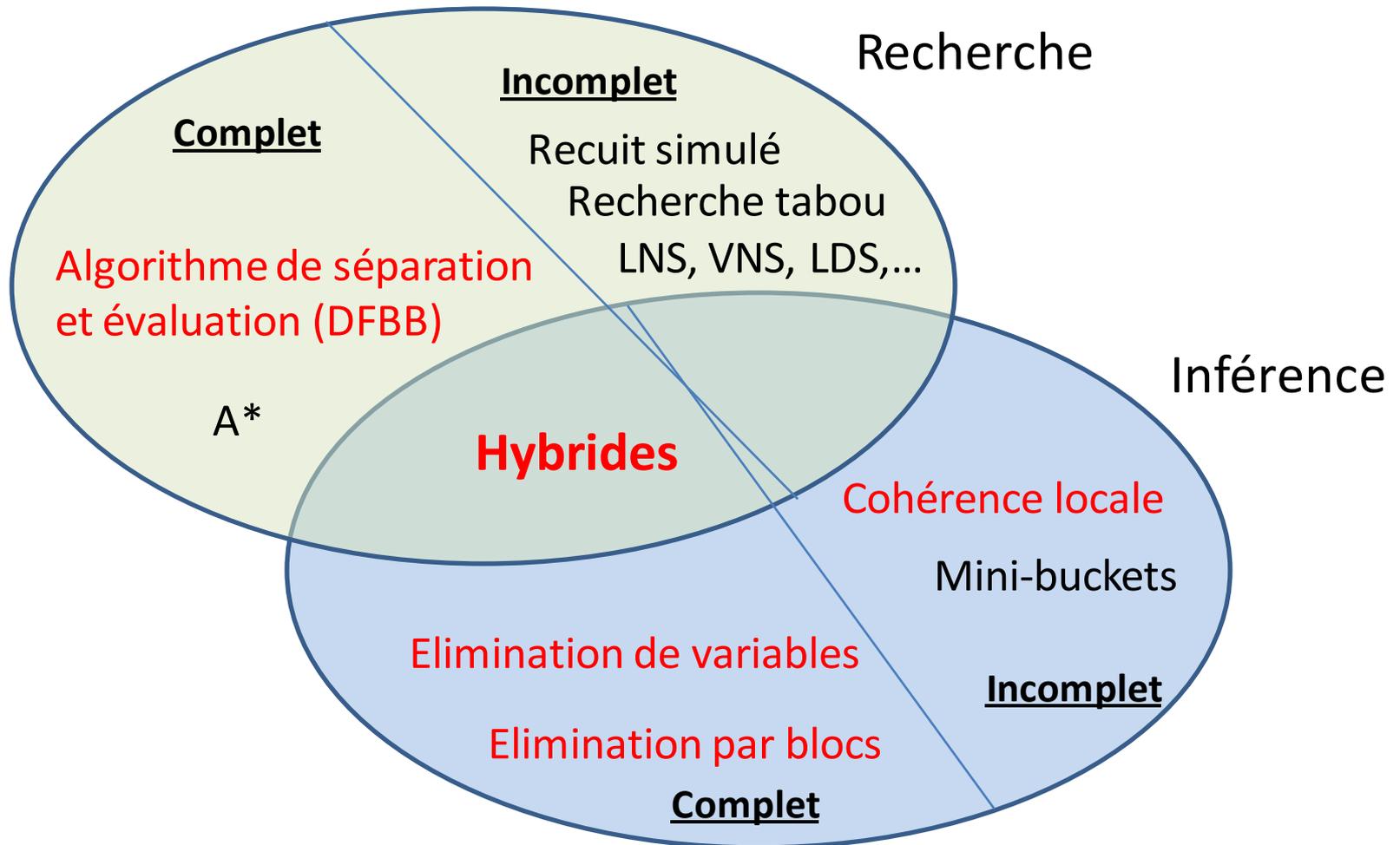
Panorama des méthodes de résolution



Panorama des méthodes de résolution

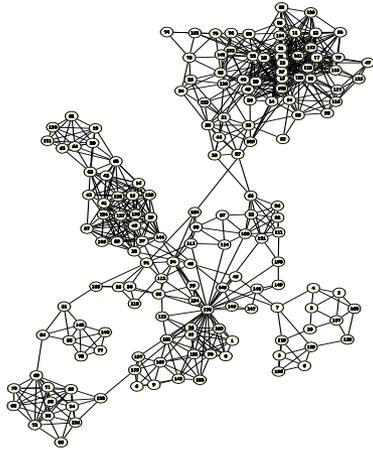


Panorama des méthodes de résolution



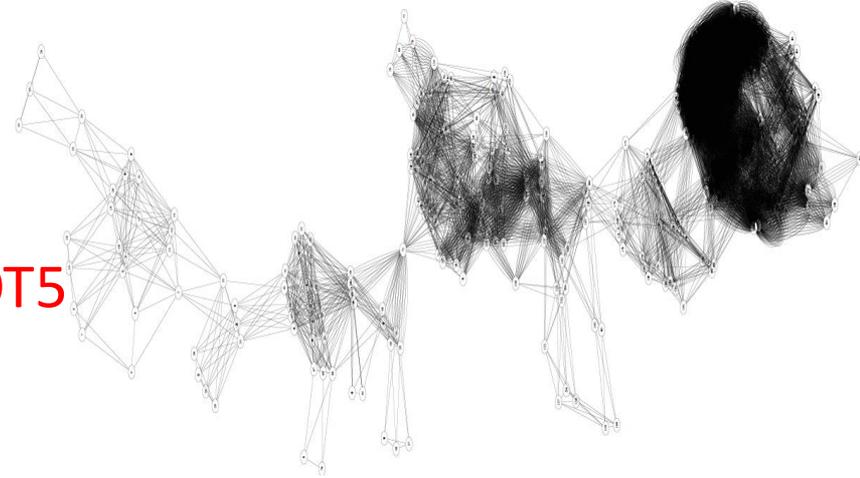
Problèmes réels structurés

CELAR



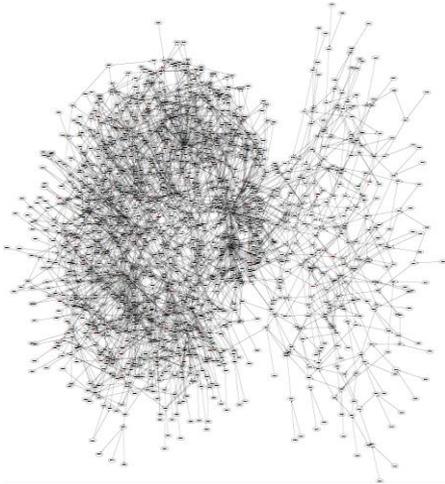
CELAR SCEN-07r
(Constraints 4(1), 1999)

SPOT5



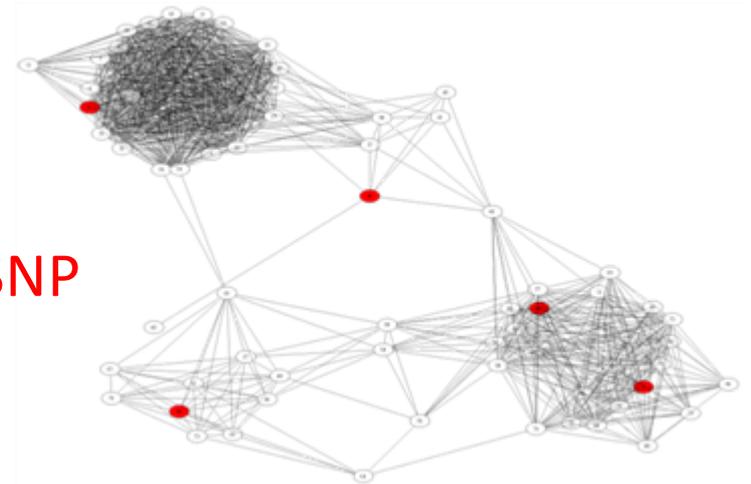
SPOT5 #509 (Constraints 4(3), 1999)

MENDEL



langladeM7 sheep pedigree
(Constraints 13(1), 2008)

TAGSNP



HapMap chr01 $r^2 \geq 0.8$ #14481
(Bioinformatics 22(2), 2006)

Résultats sur CELAR 6 depuis 1993

(n=100, d=44, e=1222)

Temps preuve d'optimalité	Méthodes	Publication
26 jours (SUN UltraSparc 167 MHz)	Ad-hoc problem decomposition & Russian Doll Search (<i>22 vars only</i>)	(de Givry, Verfaillie, Schiex, CP 97)
3 jours (SUN Sparc 2)	Ad-hoc problem decomposition & PFC-MRDAC (<i>22 vars only</i>)	(Larrosa, Mesequer, Schiex, AIJ 98)
8 heures (DEC Alpha 500MP)	Preprocessing rules & Cluster Tree Elimination	(Koster PhD thesis, 99)
3 heures (PC 2.4 GHz)	DFBB with EDAC & tree decomposition (BTD)	(de Givry, Schiex, Verfaillie, AAAI 06)
1' 26" (PC 2.5 GHz)	BTD & variable ordering heuristics & dichotomous branching	(Sanchez, Allouche, de Givry, Schiex, IJCAI 09)

CELAR 7 (n=200) résolu en 4.5 jours (Sanchez et al, IJCAI 09)

CELAR 8 (n=458) résolu en 127 jours (Allouche, de Givry, Schiex, CP 10)

Toutes les instances CELAR et GRAPH ont été fermées !

Résultats sur CELAR 6 depuis 1993

(n=100, d=44, e=1222)

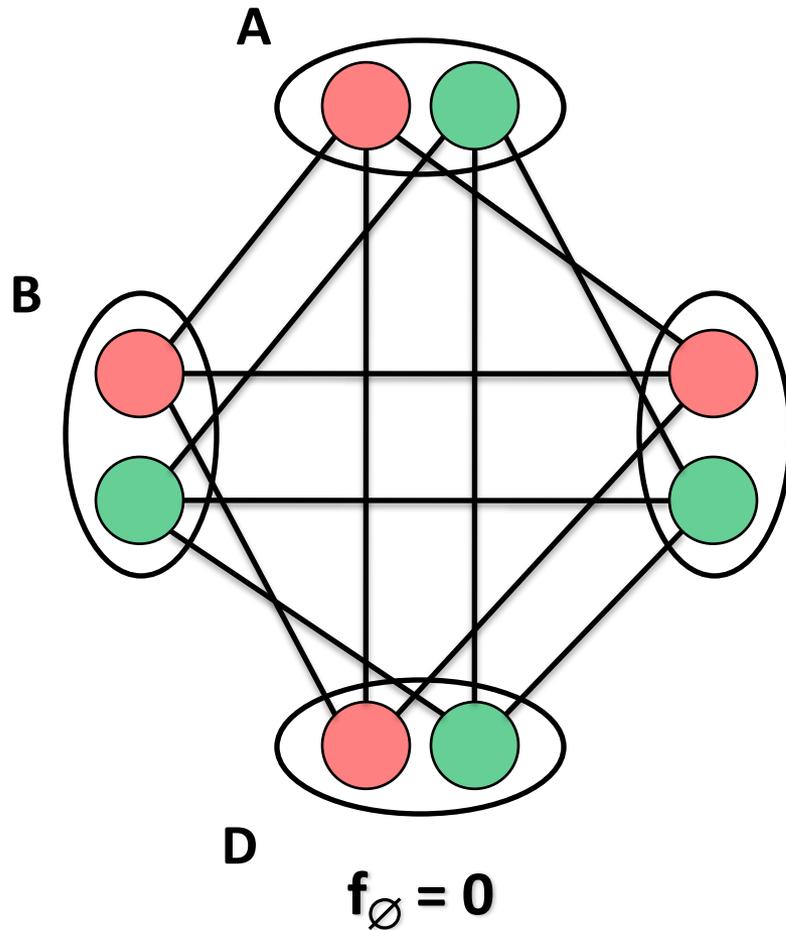
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1' 26" (PC 2.5 GHz)	26000x 15 x BTD & variable ordering heuristisc & dichot. branching	(Sanchez, Allouche, de Givry, Schiex, IJCAI 09)

CELAR 7 (n=200) résolu en 4.5 jours (Sanchez et al, IJCAI 09)

CELAR 8 (n=458) résolu en 127 jours (Allouche, de Givry, Schiex, CP 10)

Toutes les instances CELAR et GRAPH ont été fermées !

Inférence : élimination de variables



Principe

- Jointure (somme) des fonctions portant sur la variable à éliminer
- Projection (min) du résultat sur les variables voisines

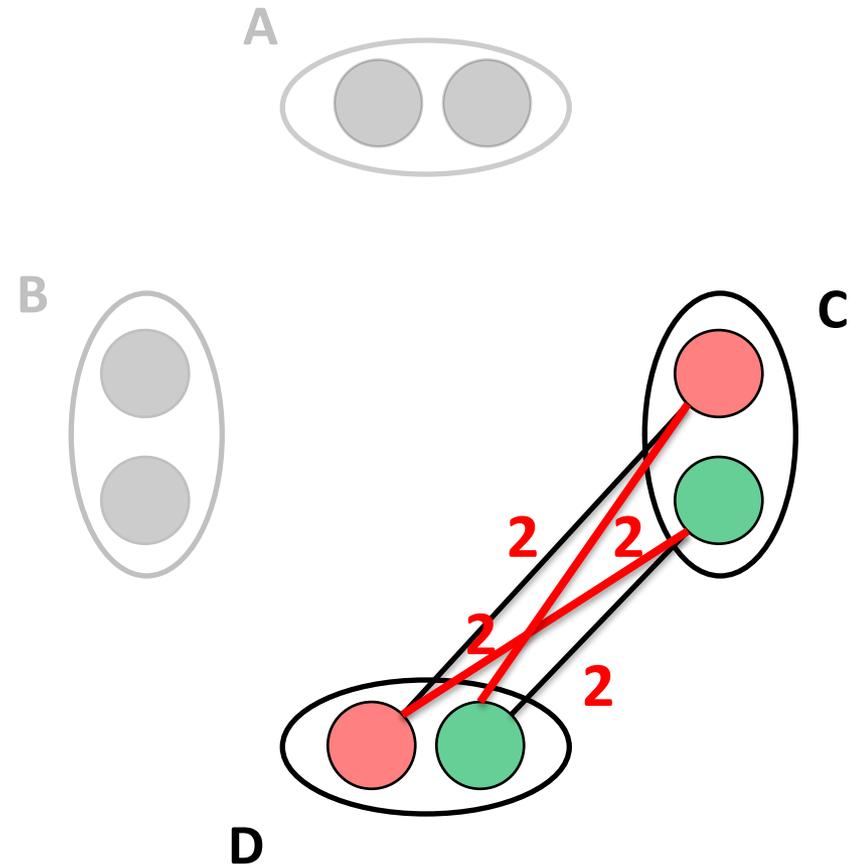
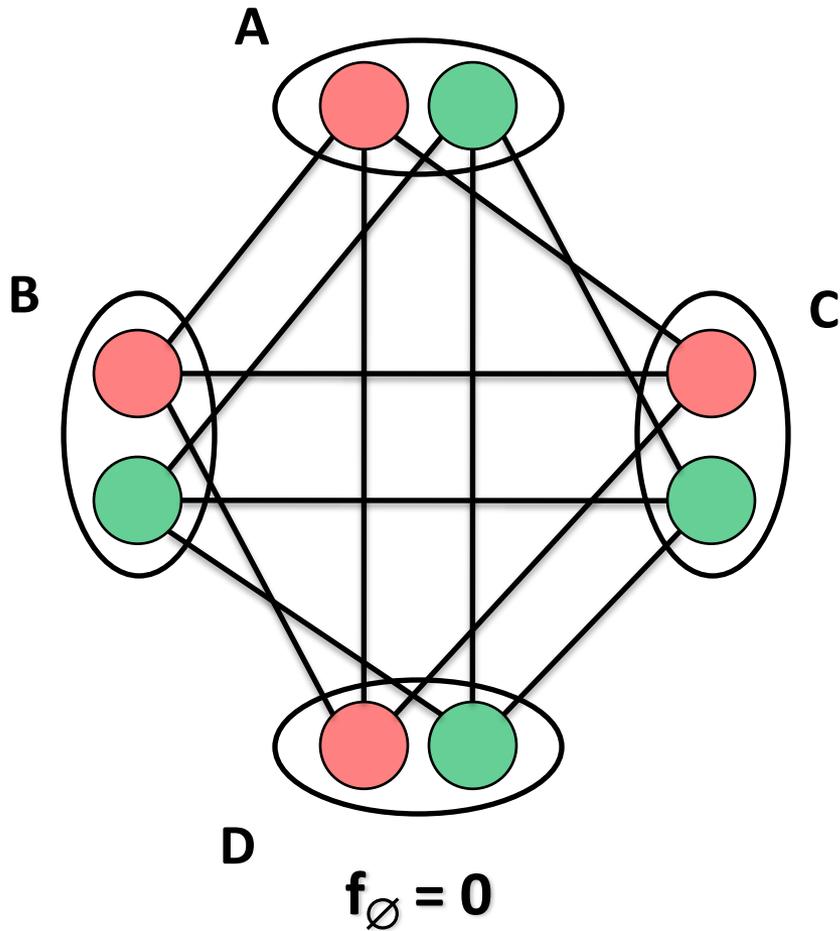
Élimination de toutes les variables jusqu'à obtention de l'optimum

Complexité (temps & espace)

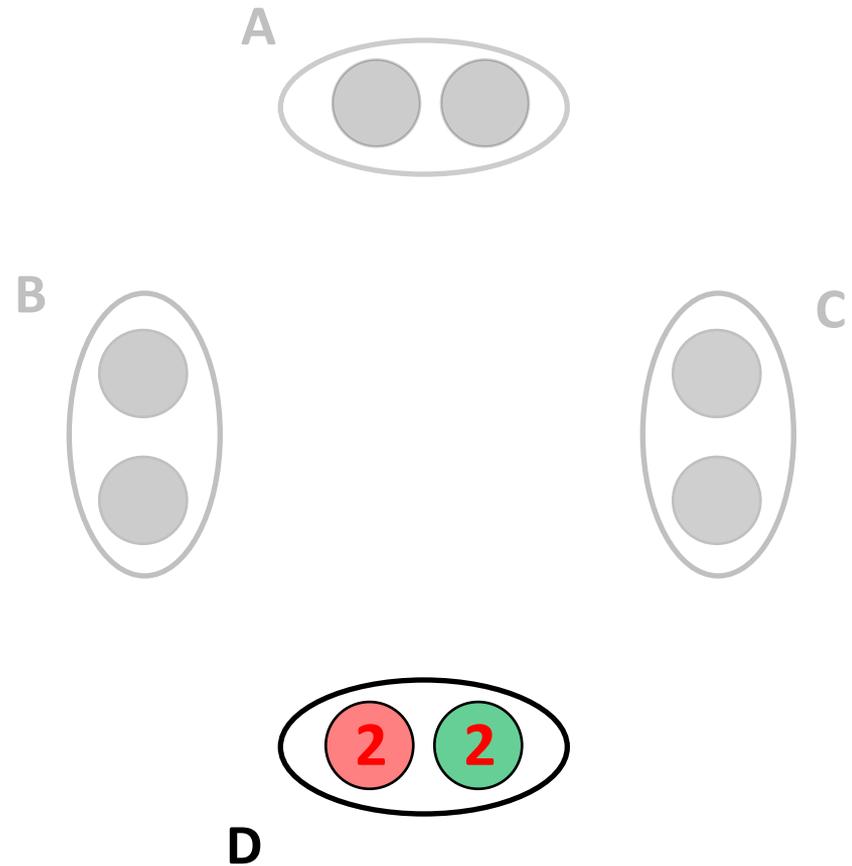
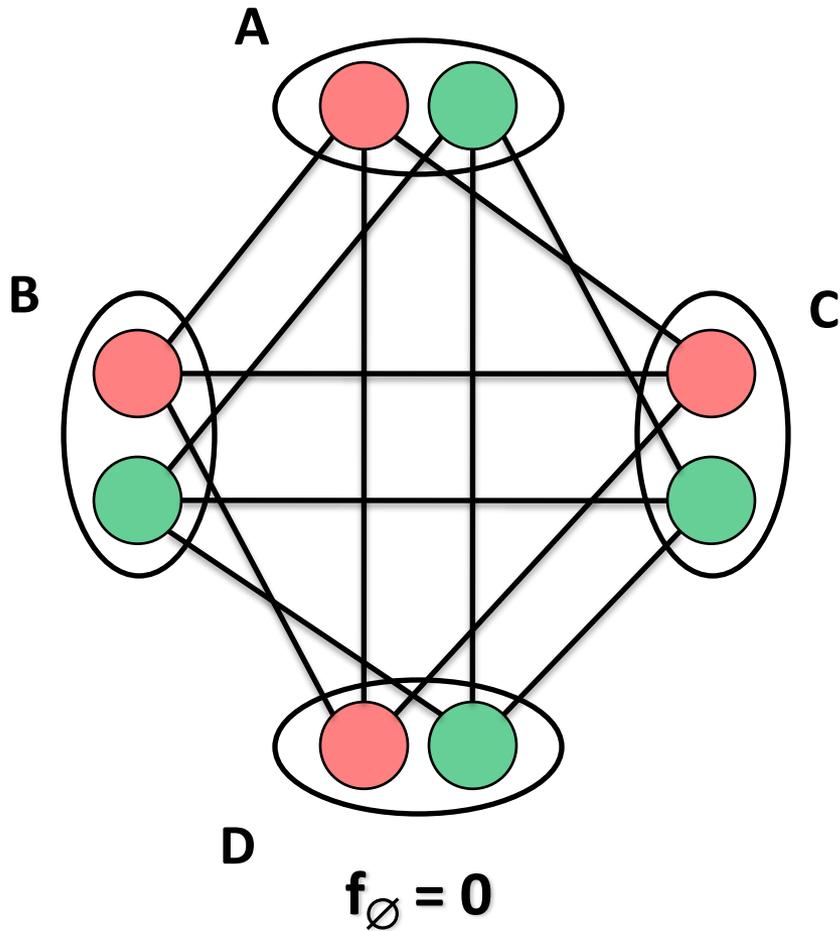
$O(n \exp(w))$

avec $w = \#$ voisins max dans le graphe induit par l'ordre d'élim.

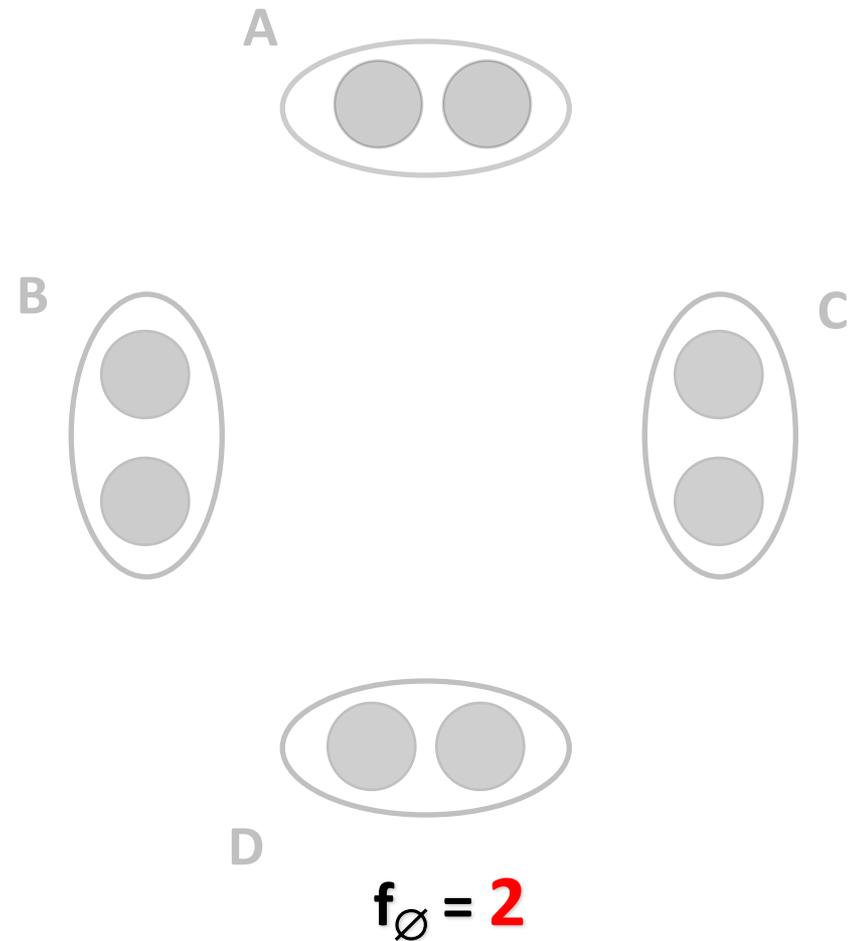
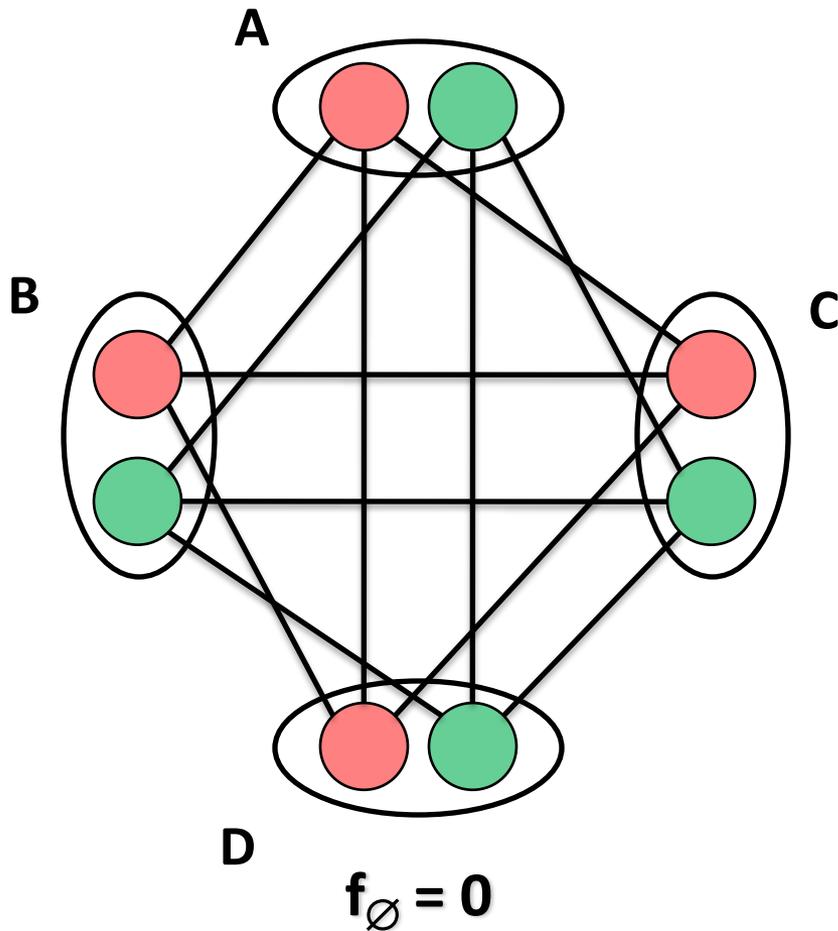
Inférence : élimination de variable



Inférence : élimination de variable



Inférence : élimination de variable



Arité maximum des fonctions générées durant l'élimination = 3
(en général, cela dépend de l'ordre d'élimination)

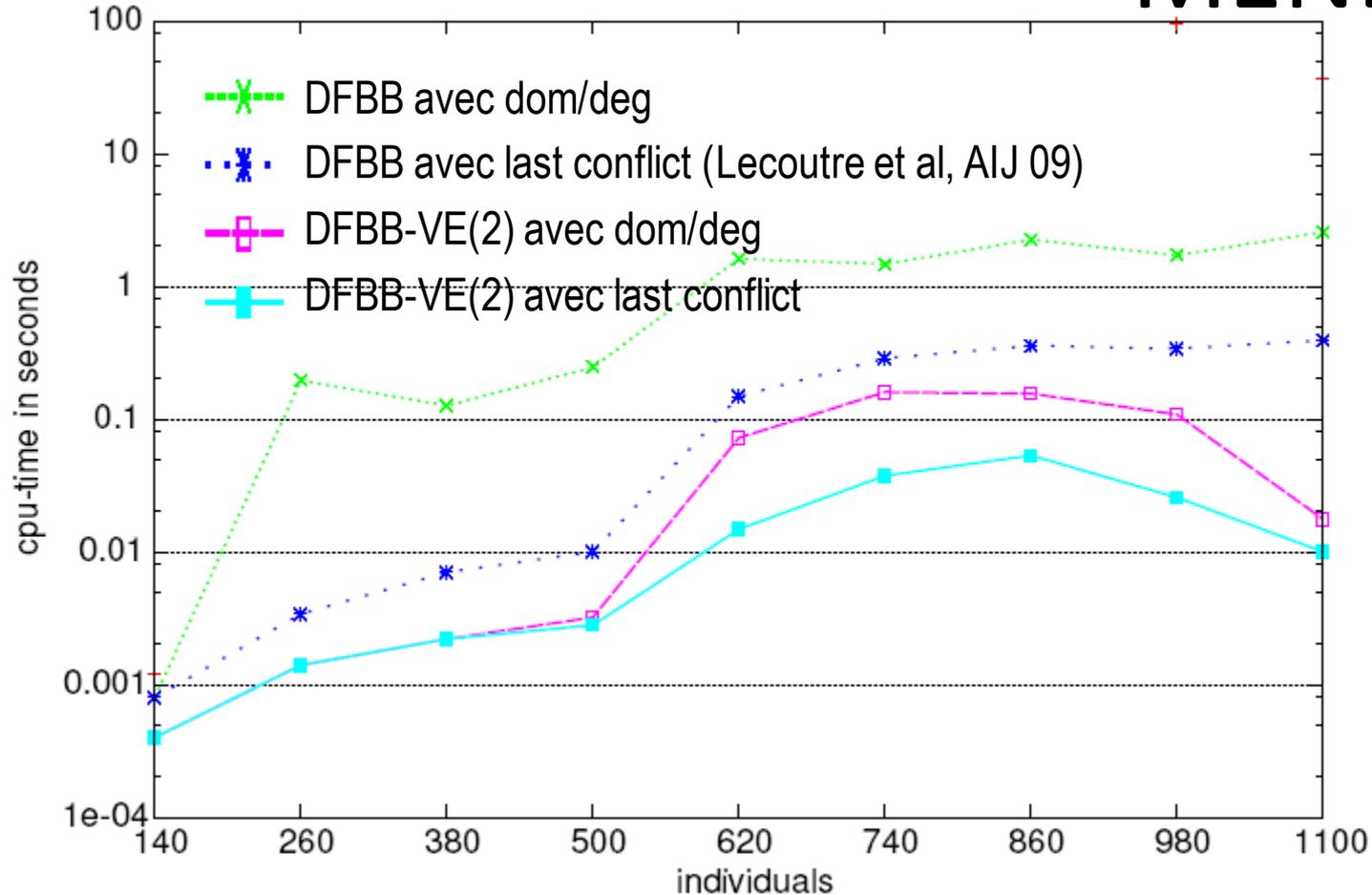
DFBB-VE(i) : élimination d'arité maximum i à chaque nœud de l'arbre de recherche

Temps $O(\exp(n))$
Espace $O(n \exp(i))$

(Larrosa, CP 00)

pedigree_class_C

MENDEL



- toulbar2 v0.5 avec EDAC3 et branchement binaire
- Temps CPU en secondes pour trouver et prouver l'optimum sur PC linux 3 GHz avec 16 GB

(Sanchez, de Givry, Schiex, Constraints 08)

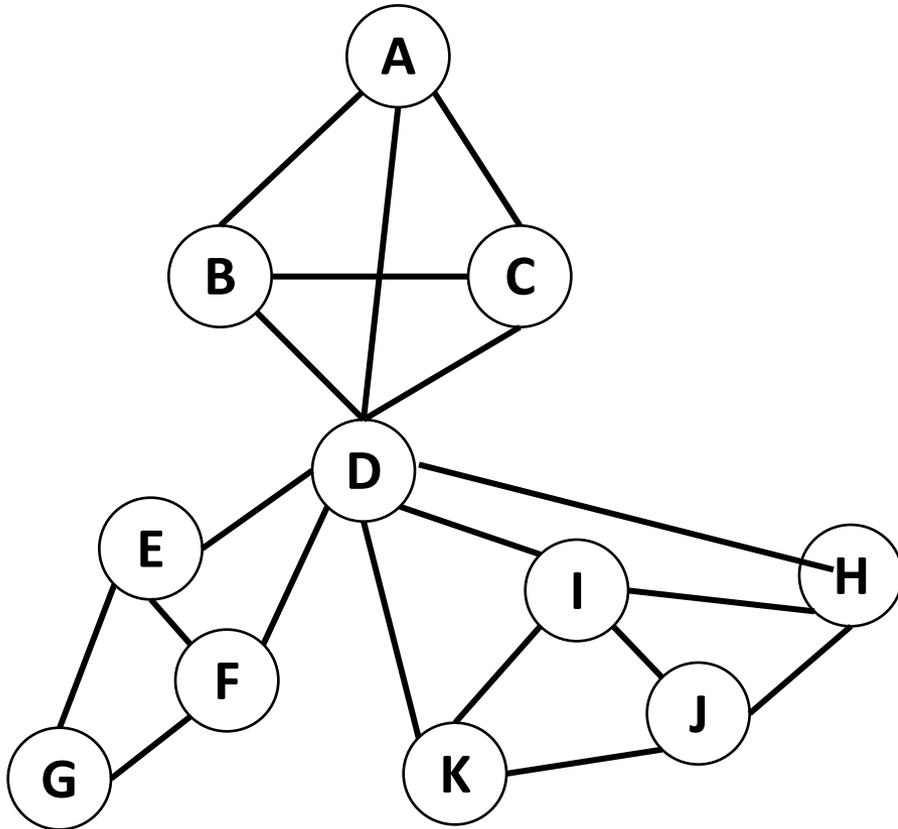
- toulbar2 v0.5 avec last conflict
- Temps CPU en secondes pour trouver et prouver l'optimum

MENDEL

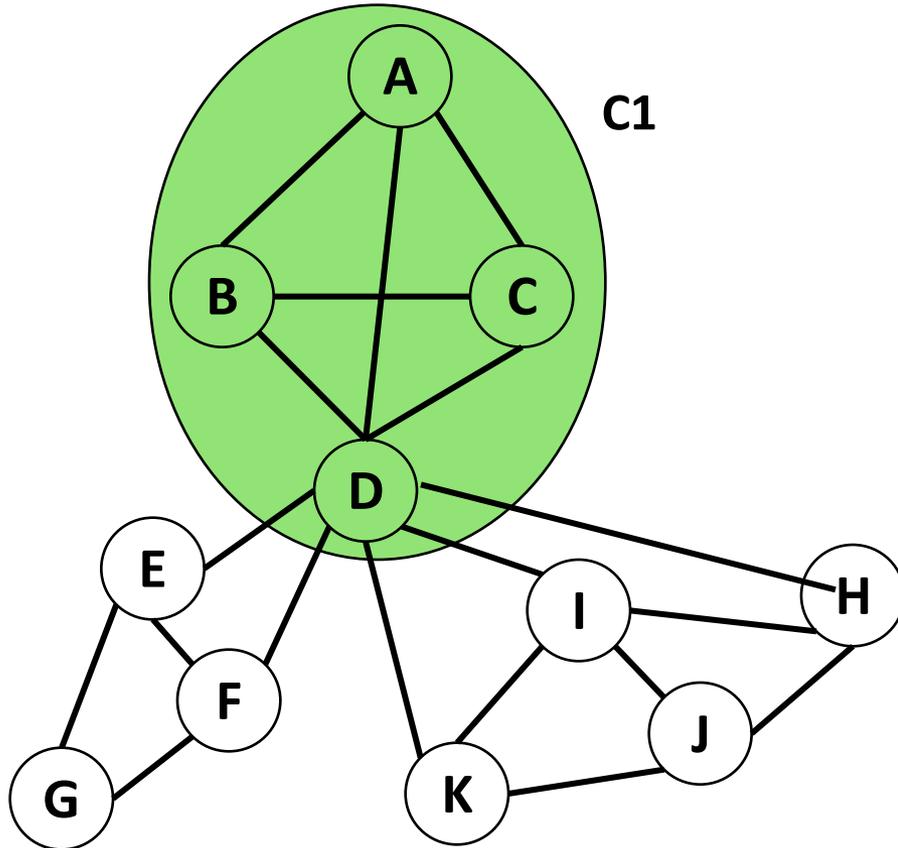
DFBB-VE(2)

	ind	vars	genotyped	alleles	nf	ngen	treewidth ub	errors	time	nodes
<i>eye</i>	36	36	28	6	11	4	2	1	0.02	0
<i>cancer</i>	49	48	37	8	18	5	2	1	0.21	0
<i>parkinson</i>	37	34	13	4	7	7	5	0	0	6
<i>berrichon_{1nc}</i>	129516	9947	2448	4	8821	17	262	2	4.73	8805
<i>berrichon₁</i>	129516	10017	2483	4	8786	17	330	23	5.81	8384
<i>berrichon_{2nc}</i>	27255	19337	10215	4	4719	19	-	41	5.89	6170
<i>berrichon₂</i>	27255	19562	10215	4	2381	19	-	106	17.23	15445
<i>langlade₁</i>	1355	1209	711	9	298	13	84	38	12.28	391
<i>langlade₂</i>	1355	1223	715	7	298	13	82	89	60.56	17857
<i>langlade₃</i>	1355	1258	787	5	298	13	85	39	14.19	6731
<i>langlade₄</i>	1355	1186	672	8	298	13	83	43	59.7	3520
<i>moissac₁</i>	283	260	183	2	81	5	6	0	0	5
<i>moissac₂</i>	283	244	167	7	81	5	6	0	0.51	6
<i>moissac₃</i>	283	225	151	3	81	5	6	0	0	4
<i>moissac₄</i>	283	256	179	2	81	5	6	0	0	5
<i>moissac₅</i>	283	237	161	8	81	5	6	0	1.02	5
<i>moissac₆</i>	283	201	131	11	81	5	5	0	5.64	6

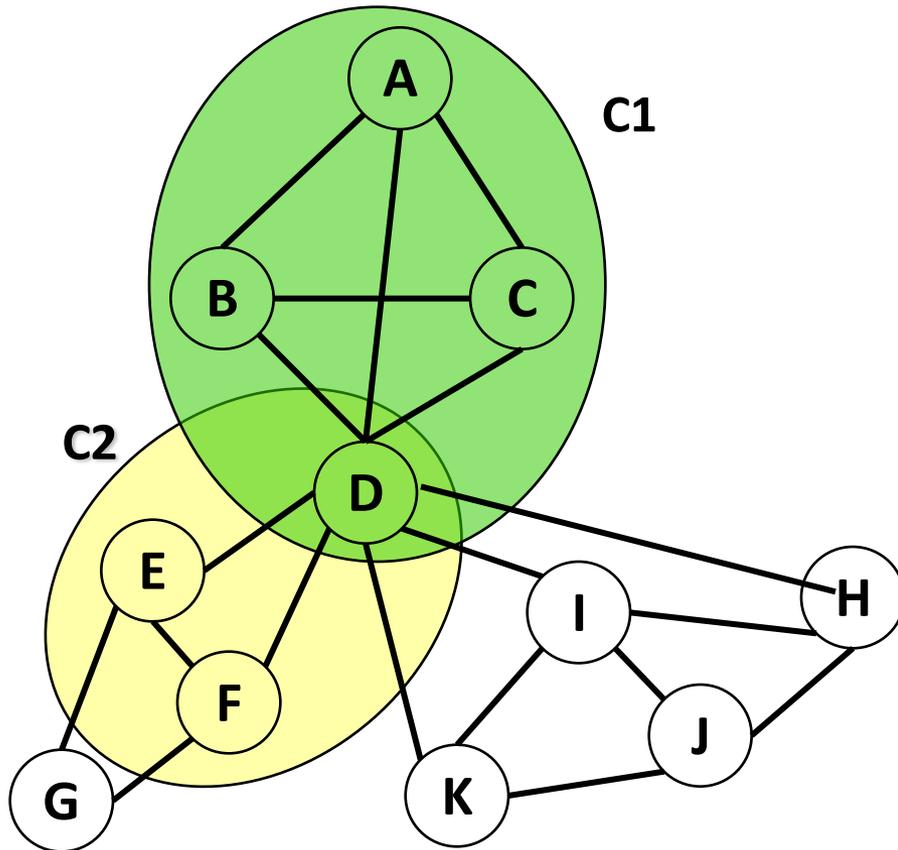
Décomposition arborescente



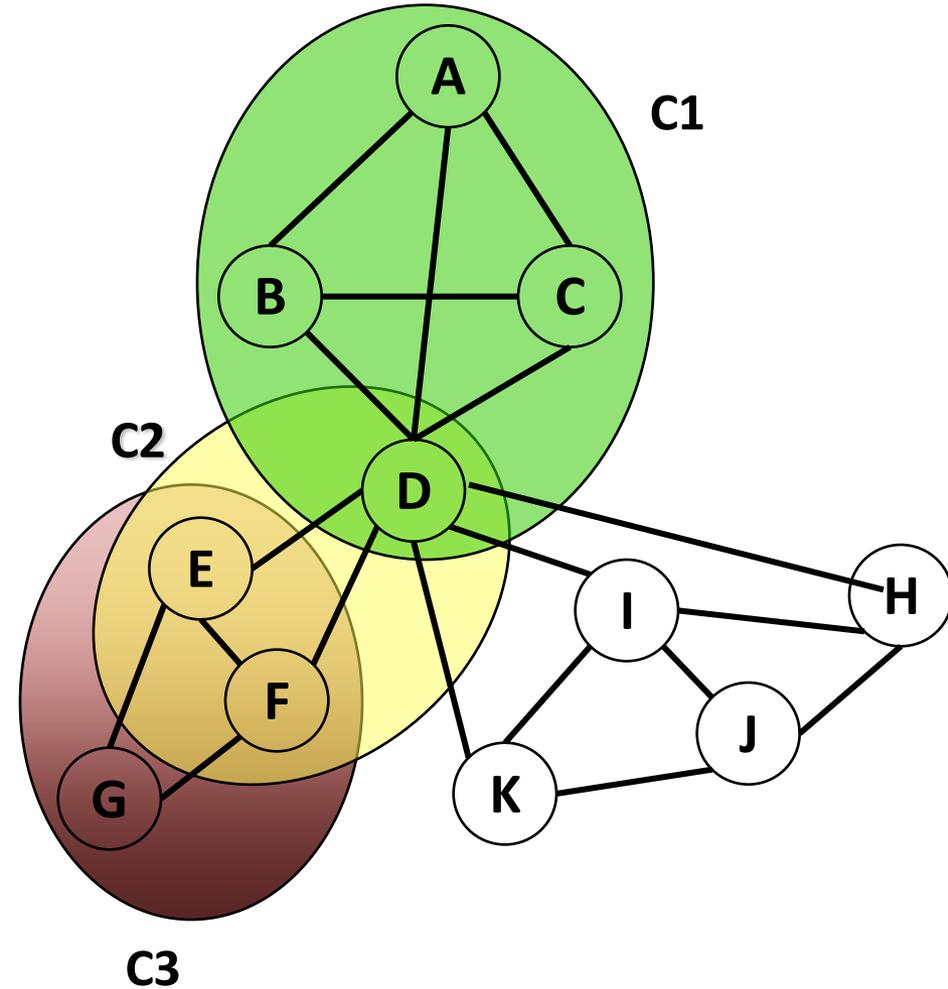
Décomposition arborescente



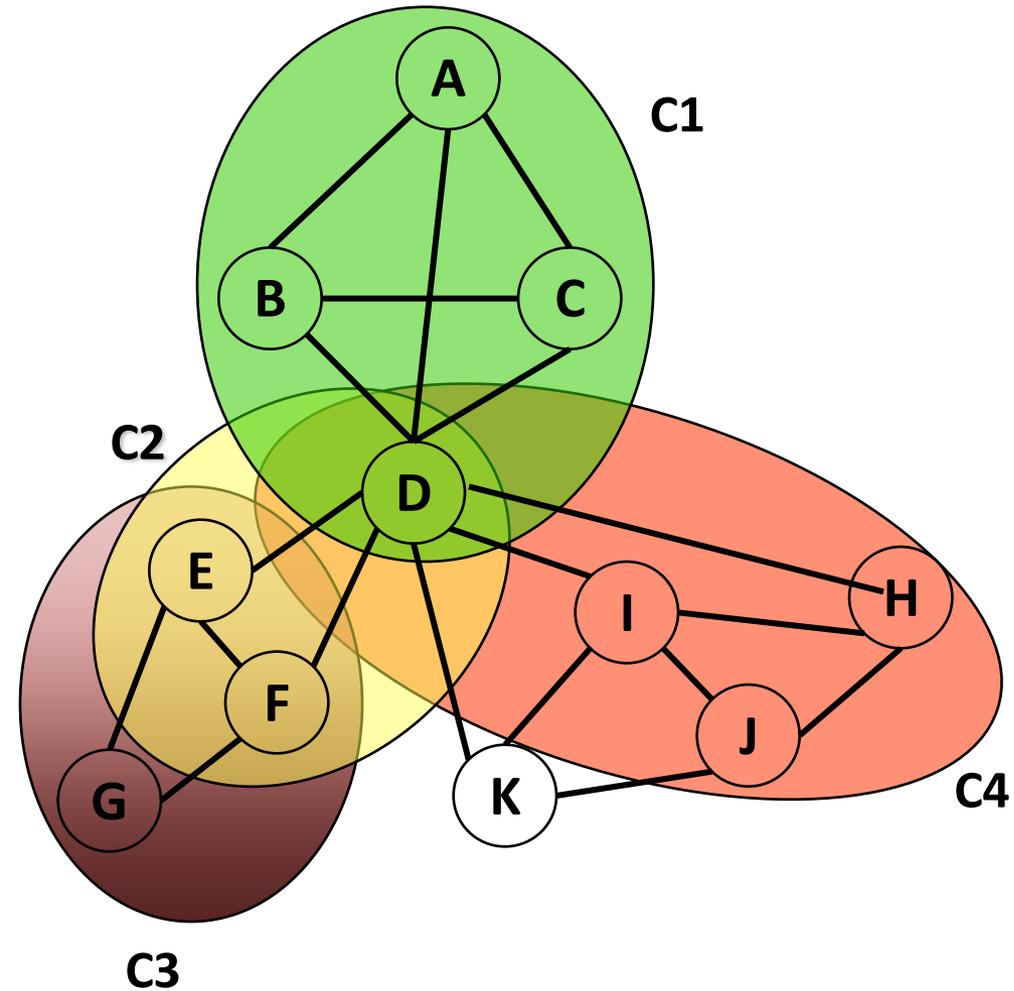
Décomposition arborescente



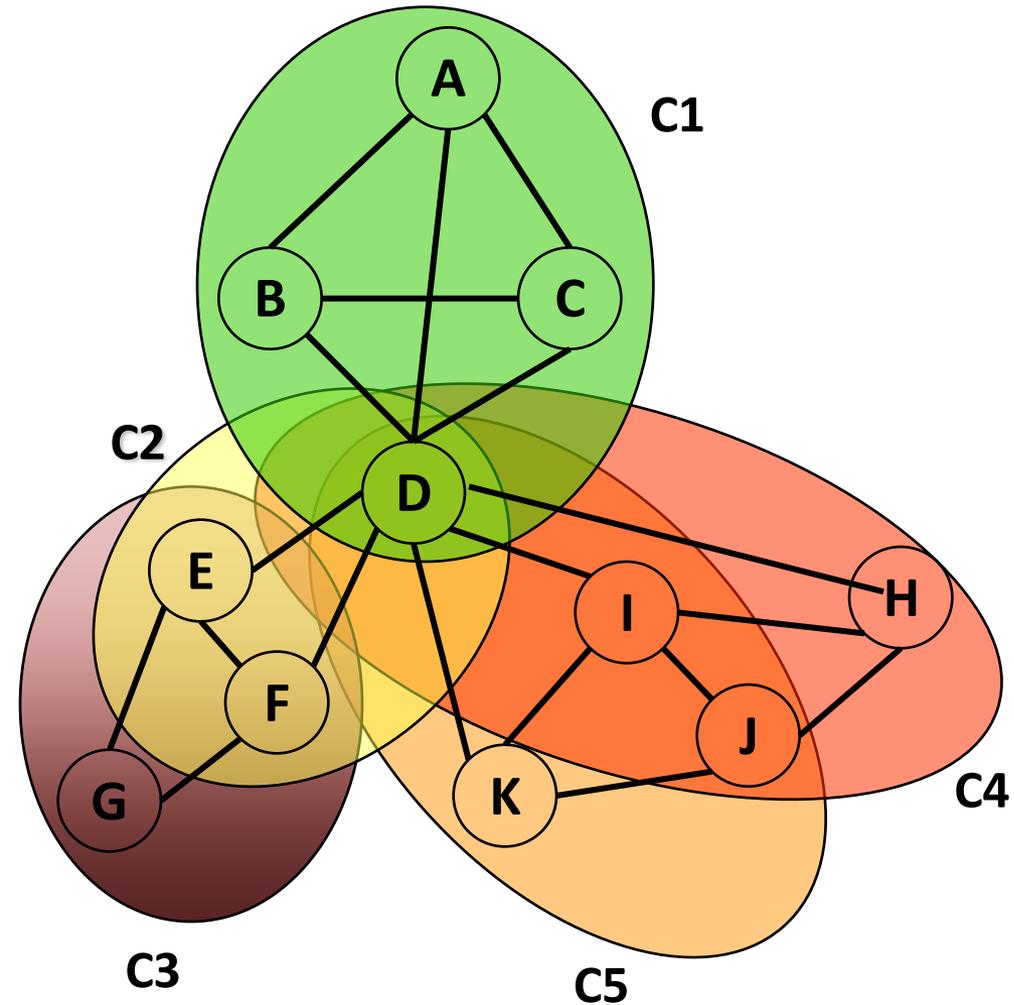
Décomposition arborescente



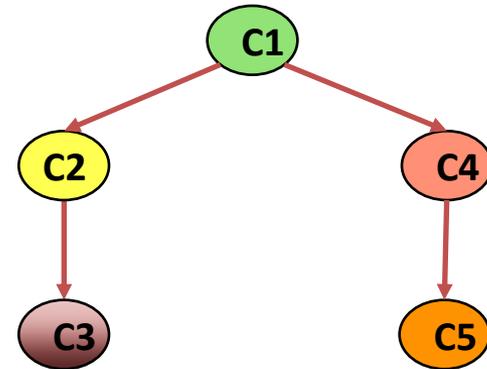
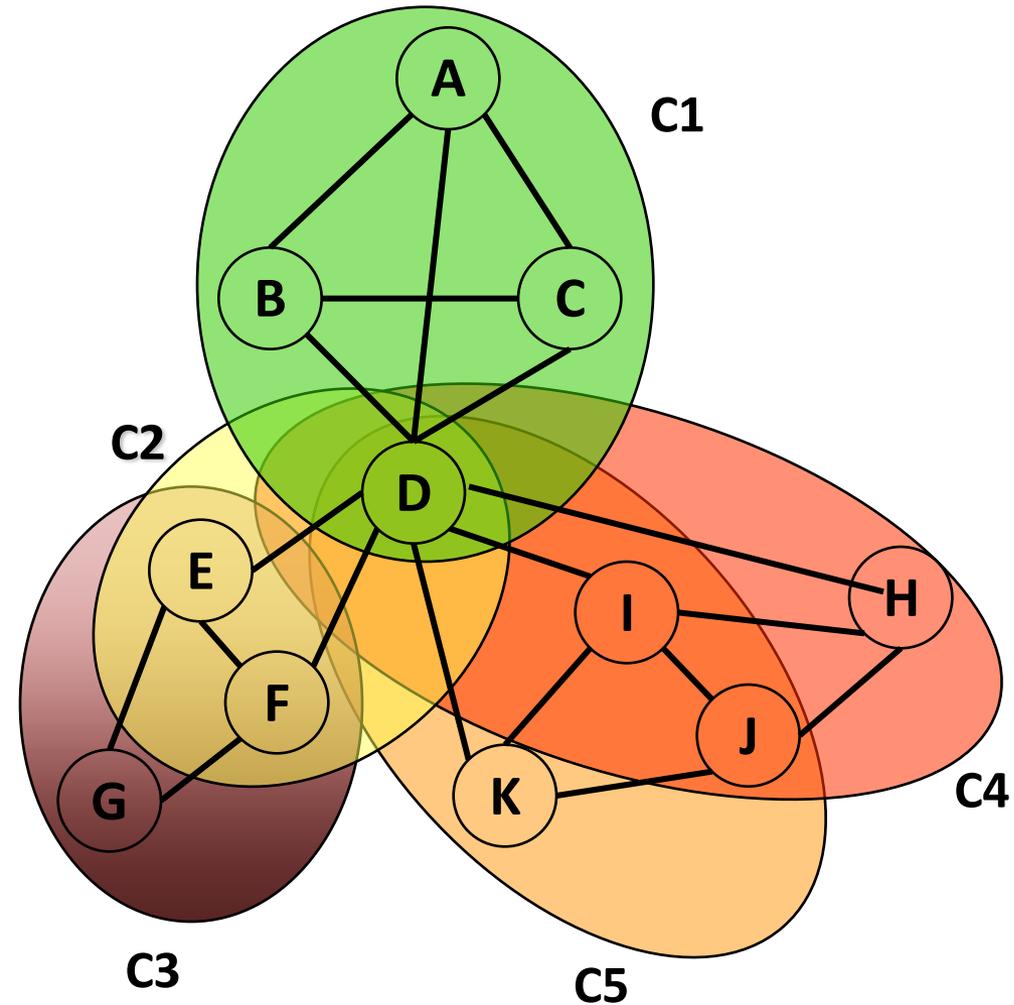
Décomposition arborescente



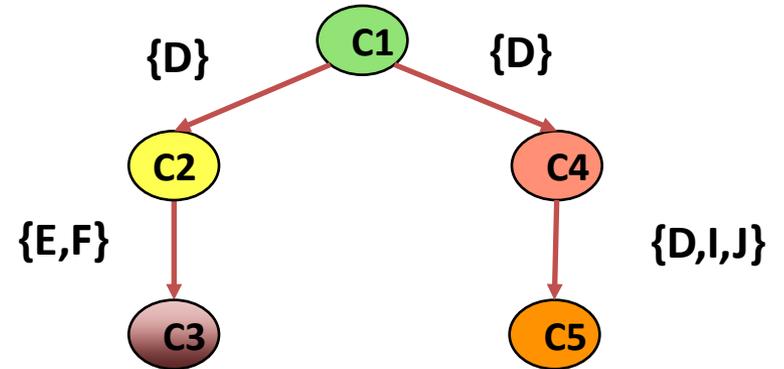
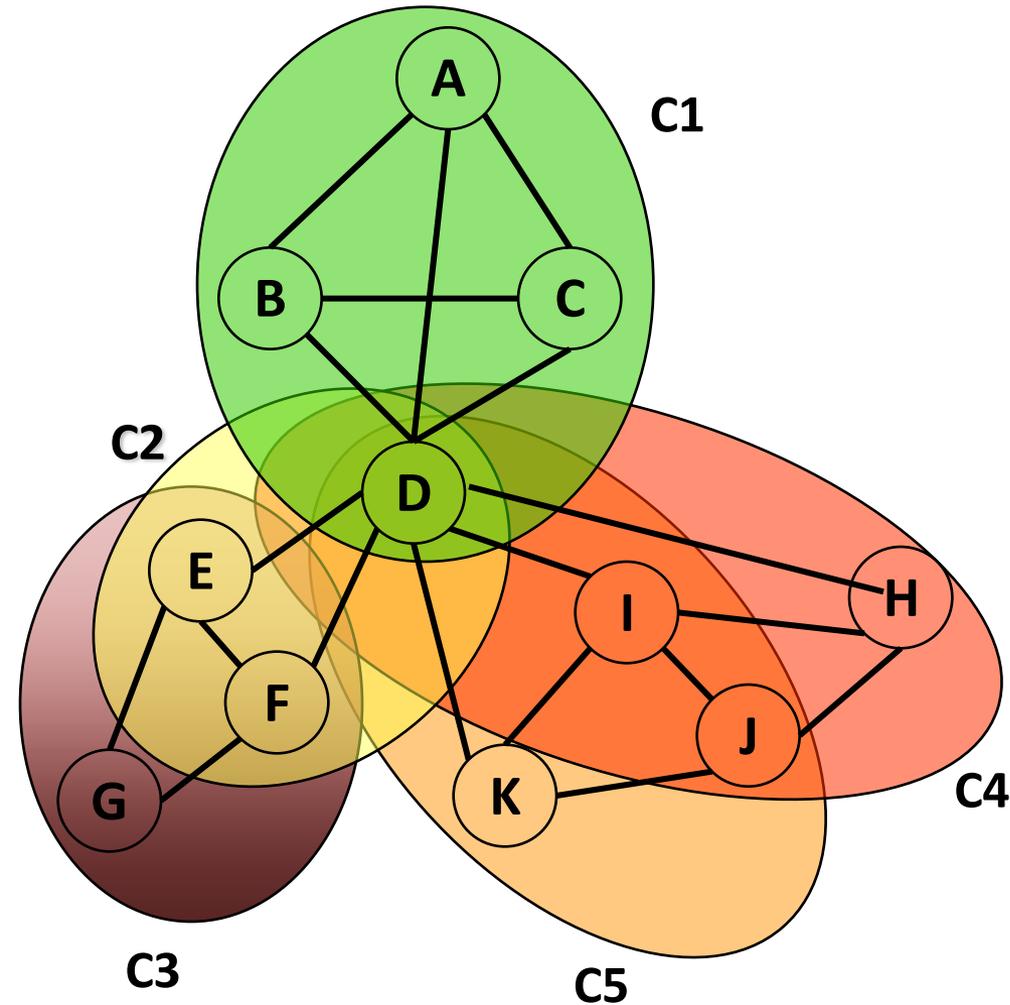
Décomposition arborescente



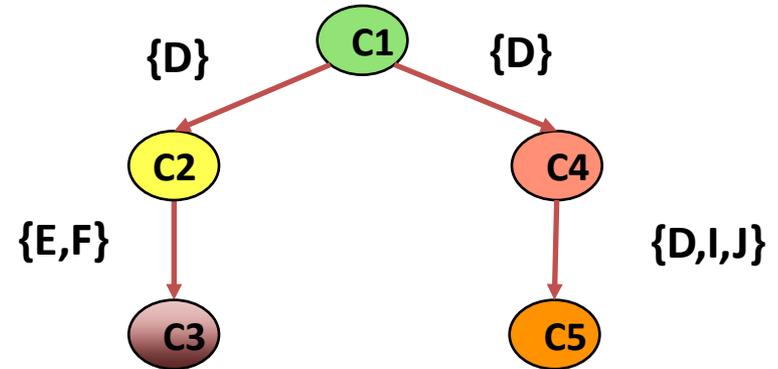
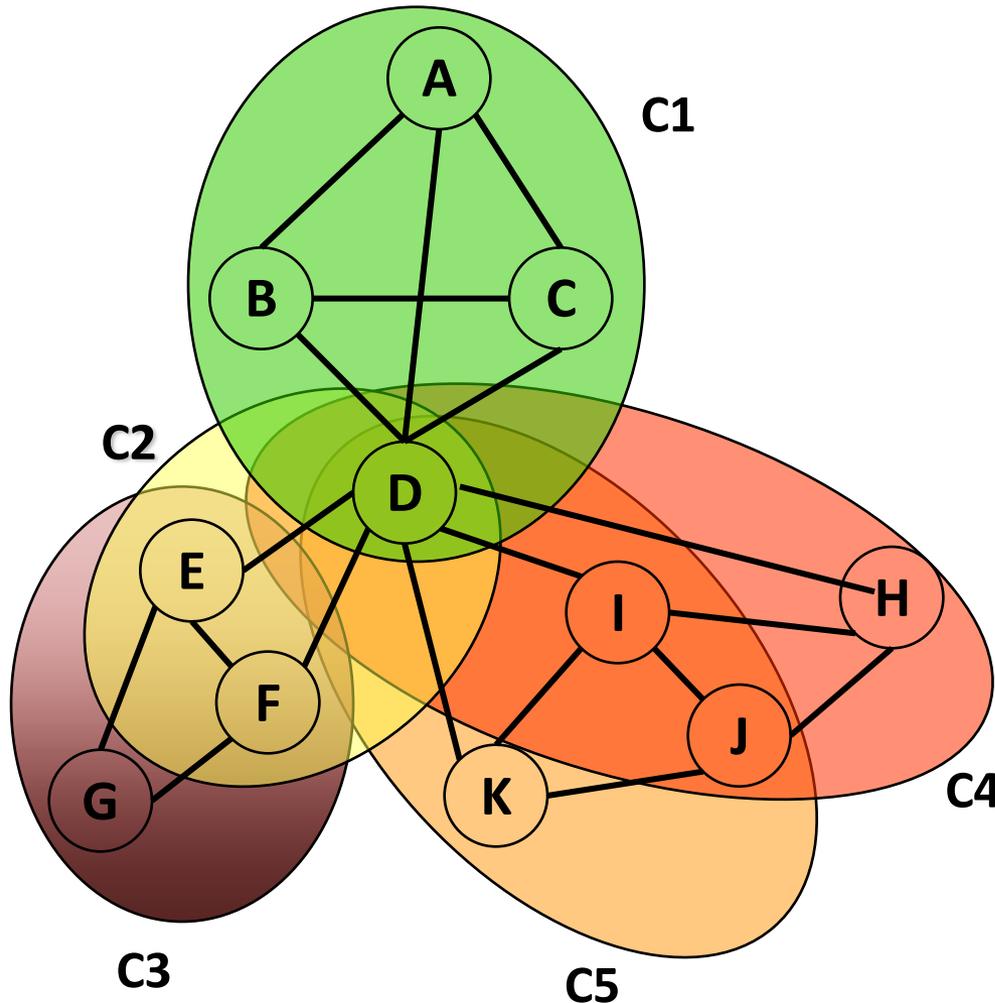
Décomposition arborescente



Décomposition arborescente



Décomposition arborescente



Cluster Tree Elimination (CTE)

(Dechter, Pearl, AIJ 89)

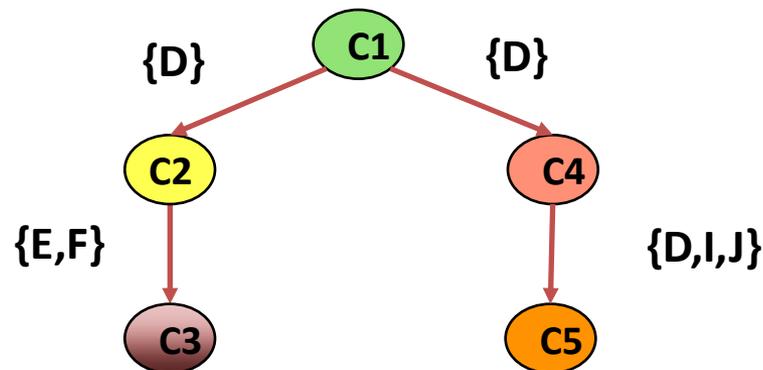
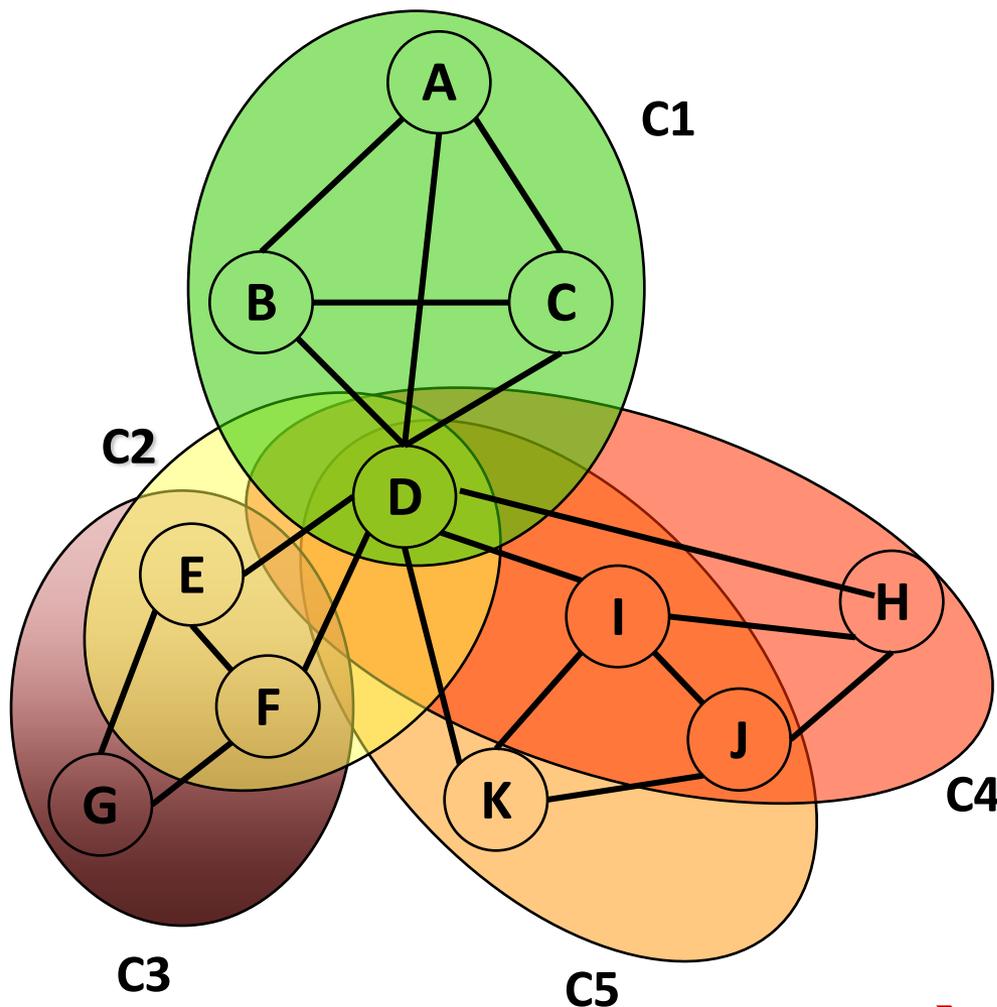
élimination par blocs par le bas

Temps $O(n \exp(w))$

Espace $O(n \exp(s))$

$s = \#$ variables max dans les séparateurs

Décomposition arborescente



Cluster Tree Elimination (CTE)

(Dechter, Pearl, AIJ 89)

élimination par blocs par le bas

Temps $O(n \exp(w))$

Espace $O(n \exp(s))$

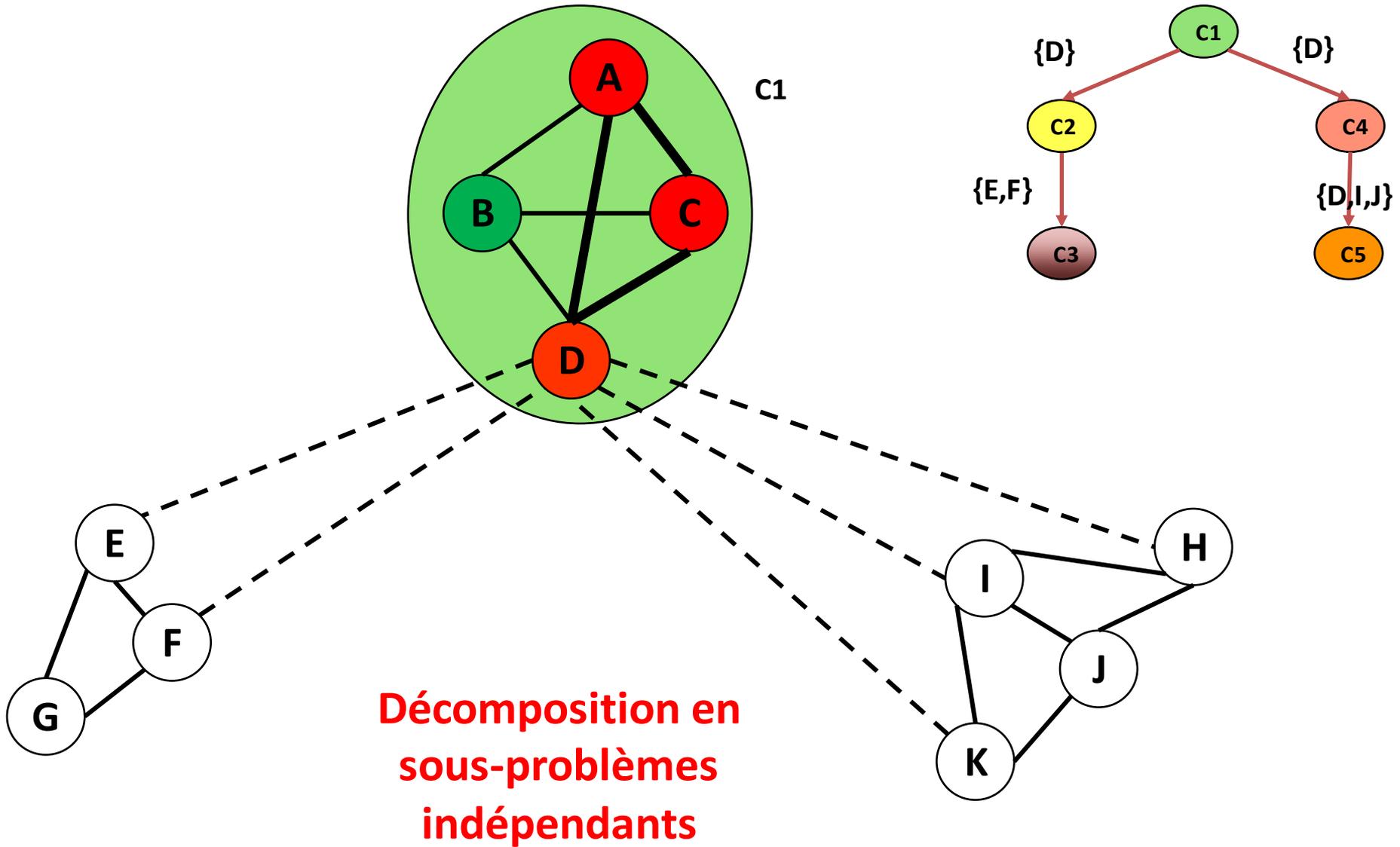
$s = \#$ variables max dans les séparateurs

Parallélisation des calculs :

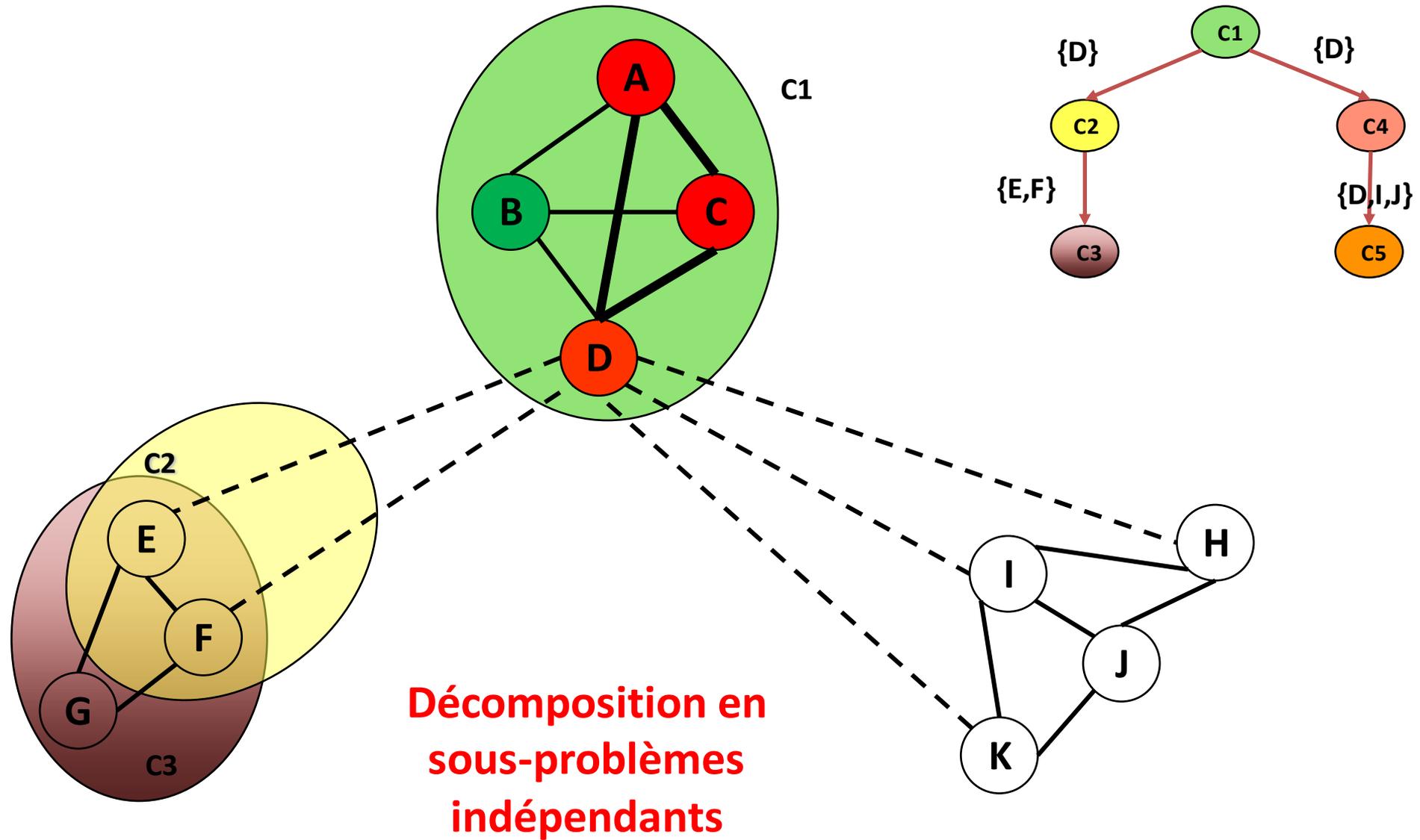
blocs résolus avec DFBB-VE(2)

→ CELAR 8 (Allouche, de Givry, Schiex, CP 10)

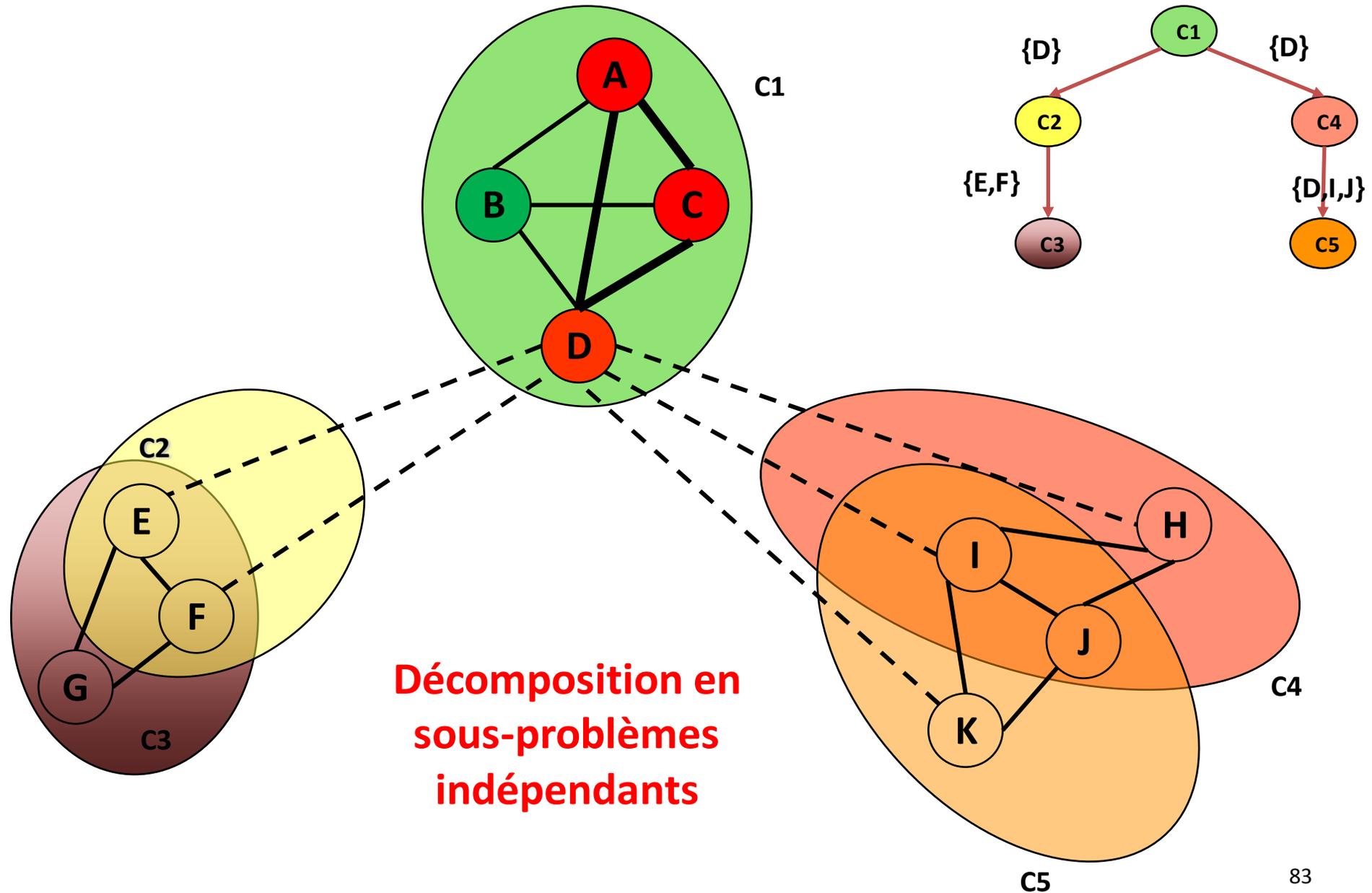
Recherche avec décomposition arborescente



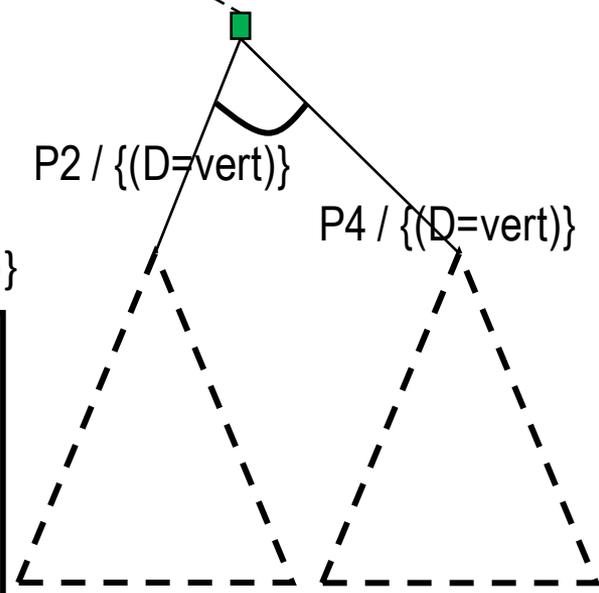
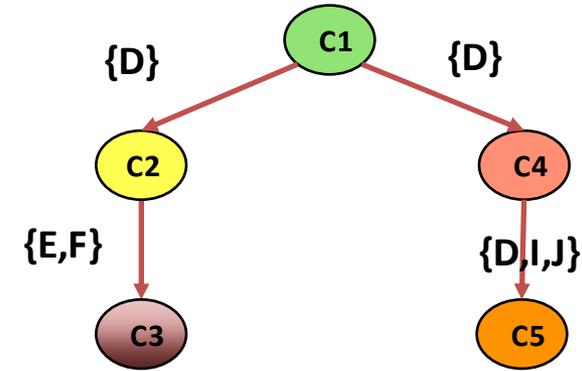
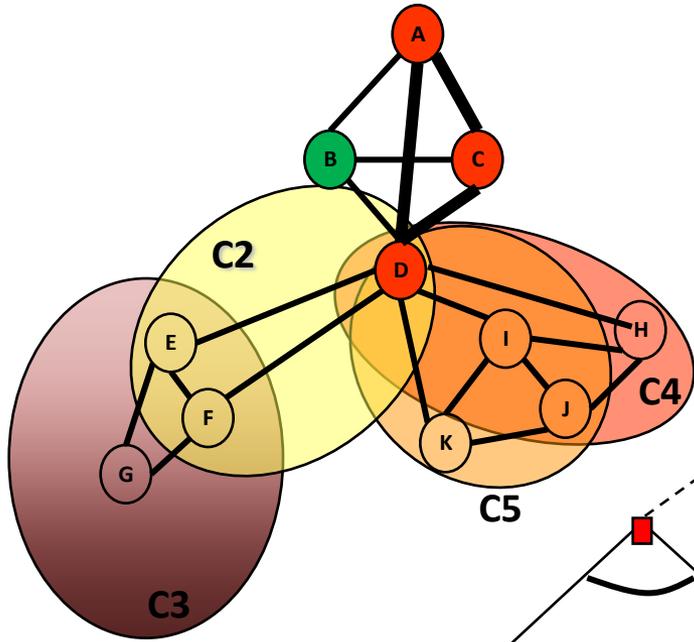
Recherche avec décomposition arborescente



Recherche avec décomposition arborescente

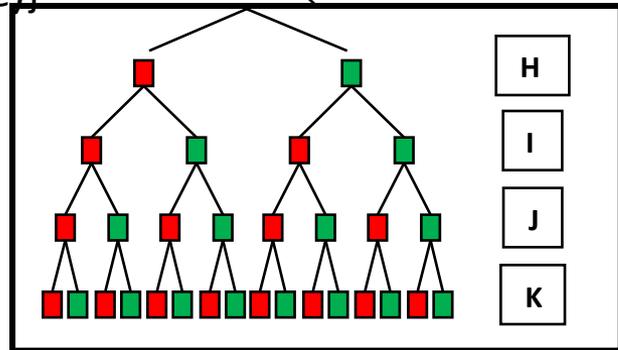
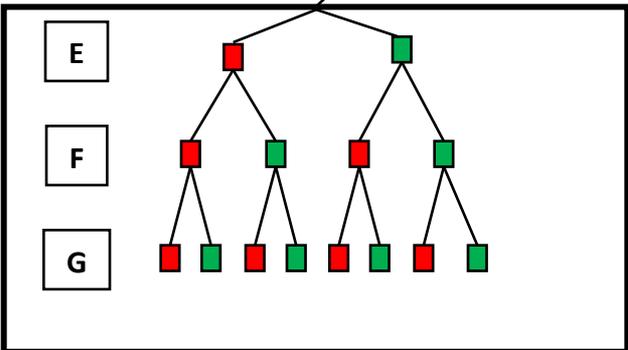


Recherche avec décomposition arborescente

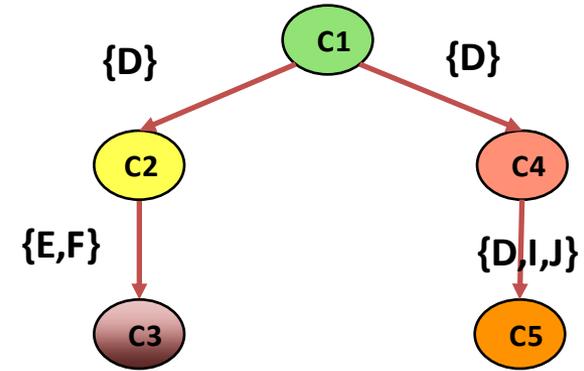
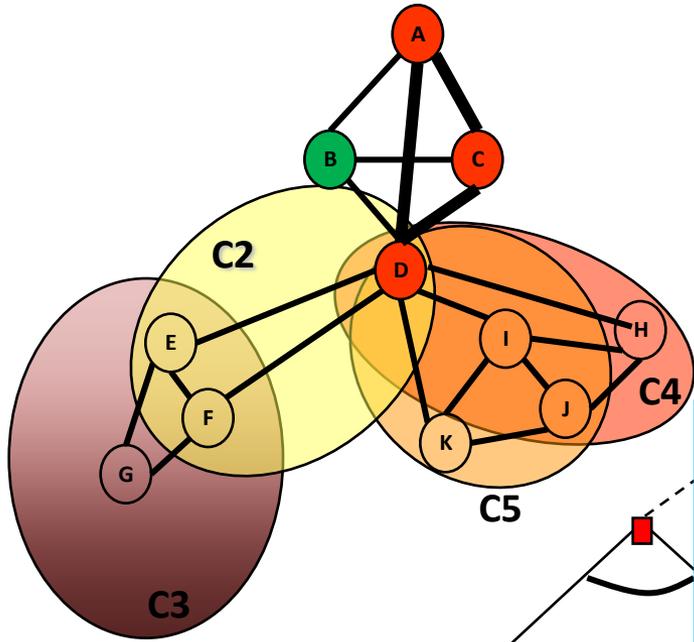


P2 / {(D=rouge)}

P4 / {(D=rouge)}



Recherche avec décomposition arborescente



AND/OR tree search

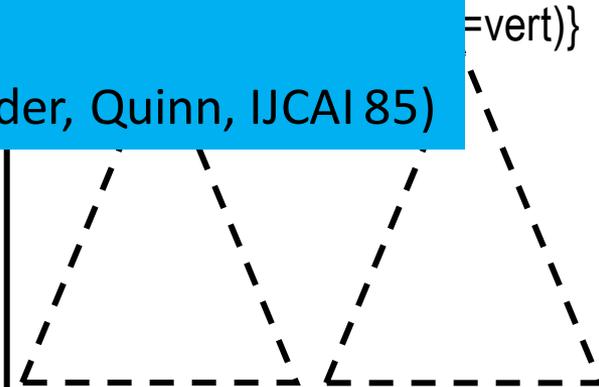
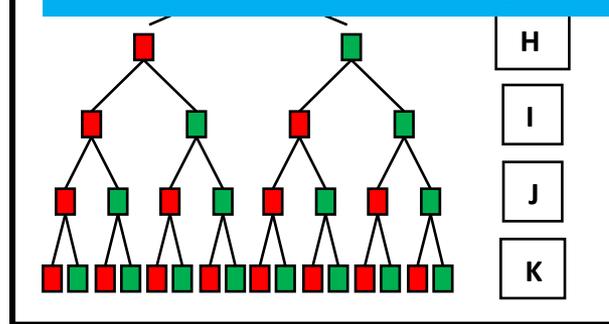
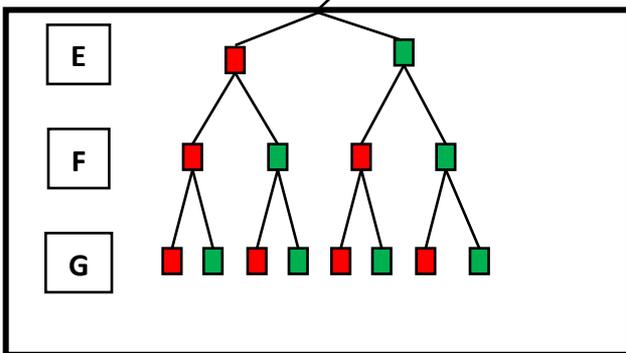
(Marinescu, Dechter, IJCAI 05)

Temps $O(n \exp(w \log(n)))$

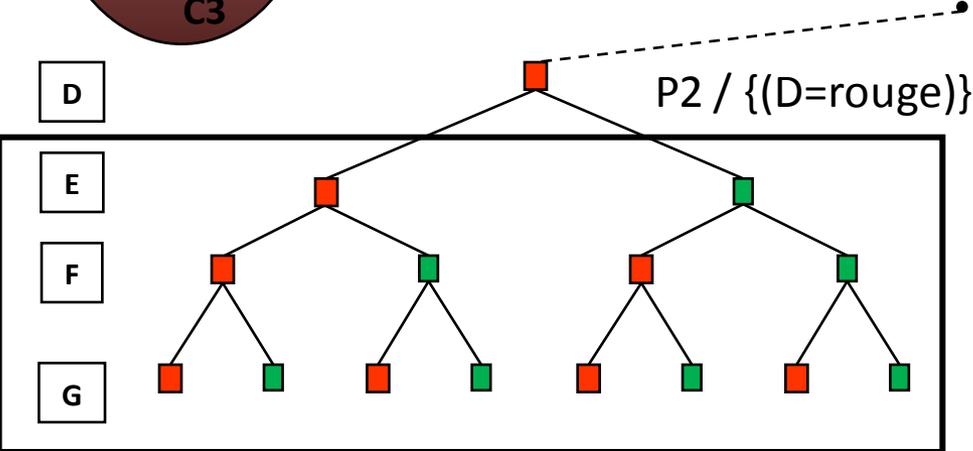
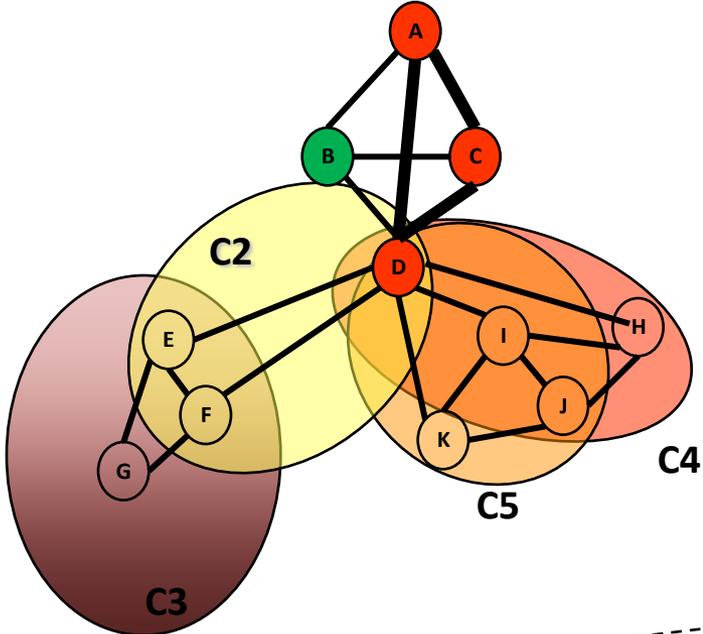
Espace $O(n)$

Pseudo-tree search (Freuder, Quinn, IJCAI 85)

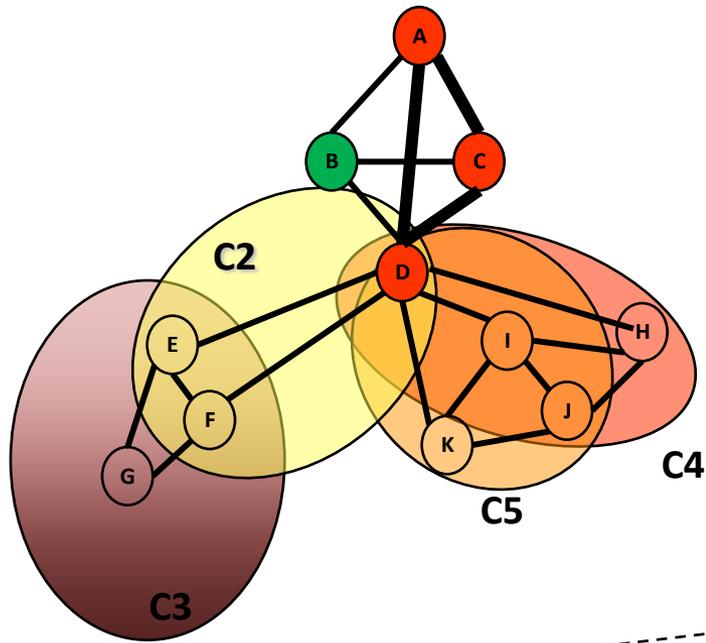
P2 / {(D=rouge)}



Recherche avec décomposition arborescente



Recherche avec décomposition arborescente



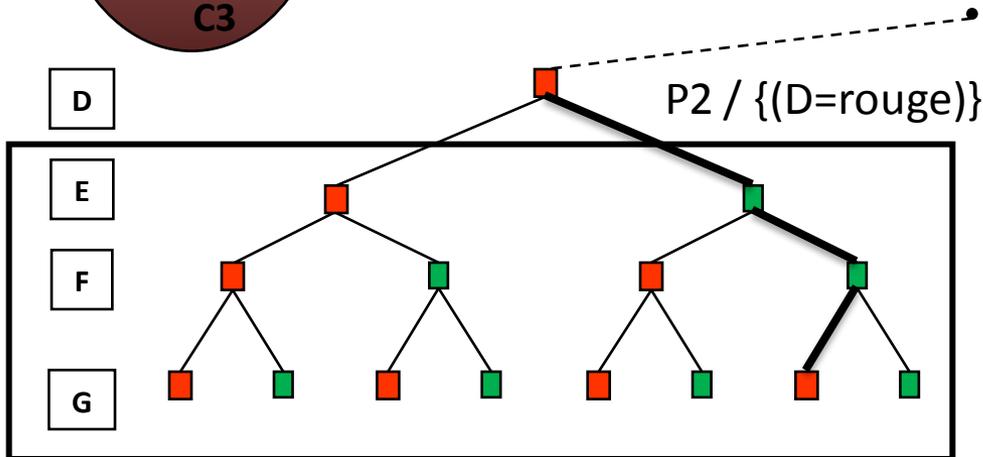
$$\text{OPT}_{P2 / \{(D=\text{rouge})\}} = 1$$

AND/OR graph search

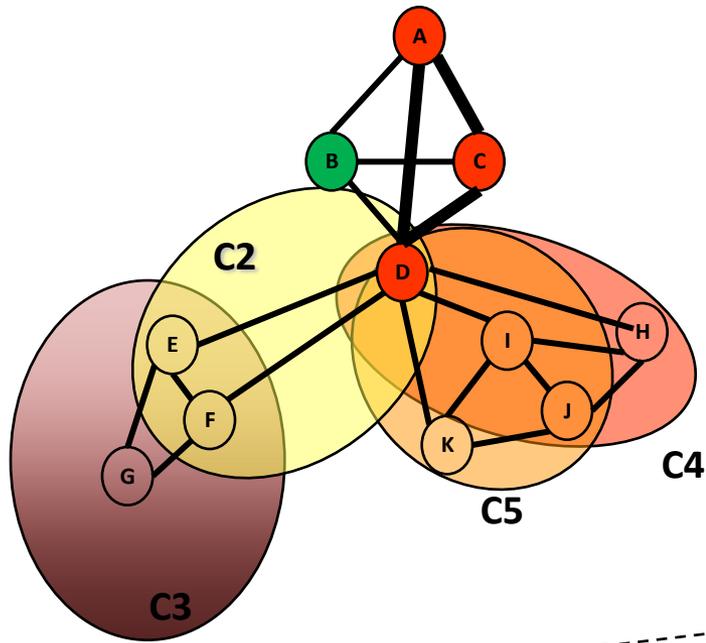
(Marinescu, Dechter, AAAI 06)

Temps $O(n \exp(w))$

Espace $O(n \exp(s))$

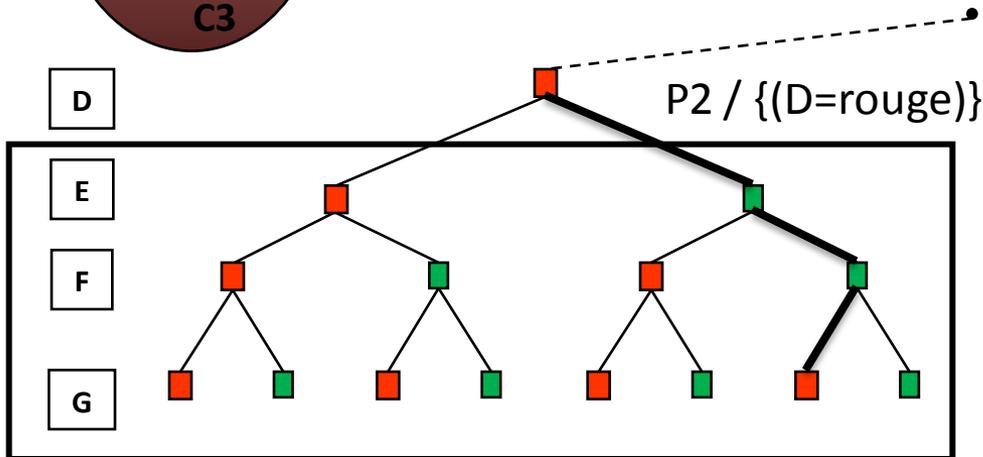


Recherche avec décomposition arborescente



$$OPT_{P2 / \{(D=rouge)\}} = 1$$

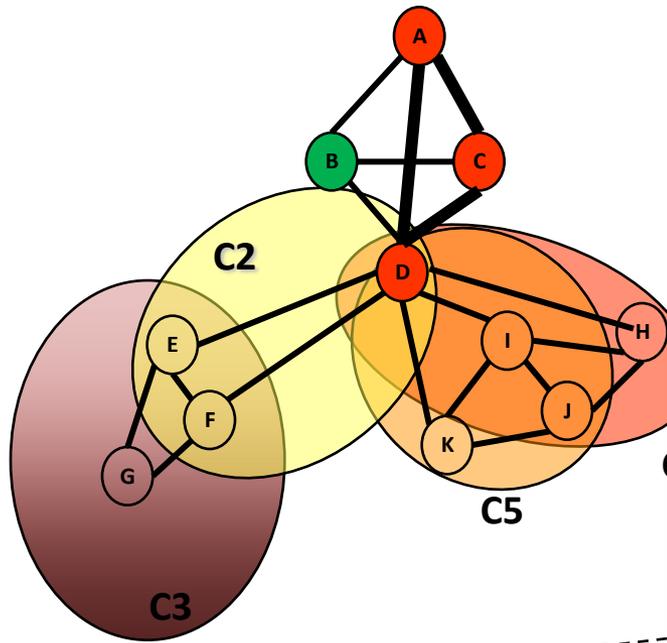
Cela ne prend pas en compte
ni les coupes locales
ni les cohérences locales !



$$LB_{P2 / \{(D=rouge)\}} = 1$$

$$OPT_{P2 / \{(D=rouge)\}} = \text{false}$$

Recherche avec décomposition arborescente



$$OPT_{P2 / \{(D=rouge)\}} = 1$$

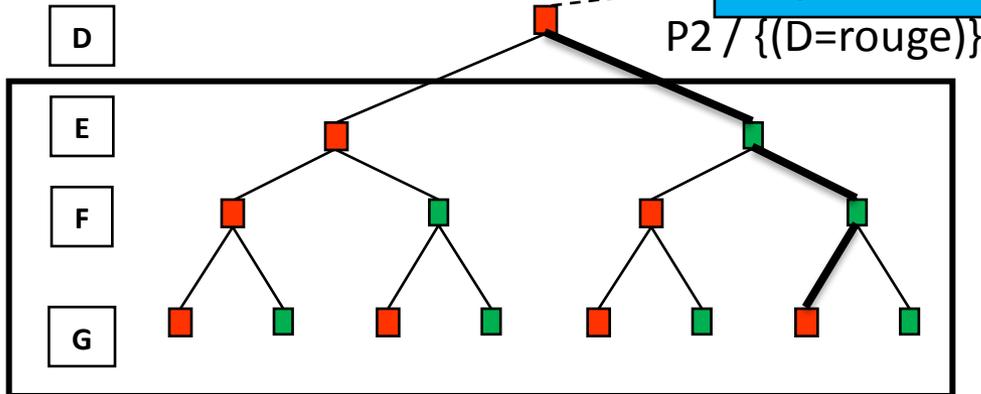
B backtrack with Tree Decomposition (BTD)

(Jégou, Terrioux, ECAI 04)

→ CELAR (de Givry, Schiex, Verfaillie, AAAI 06)

Temps $O(k n \exp(w))$

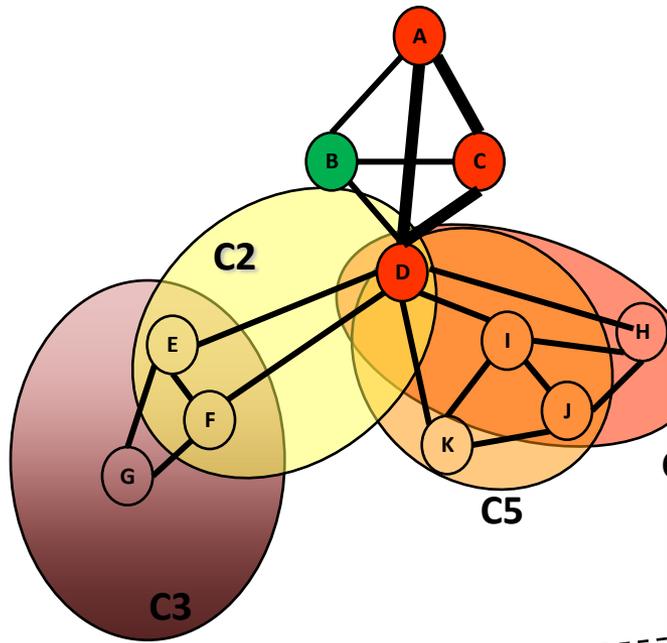
Espace $O(n \exp(s))$



$$LB_{P2 / \{(D=rouge)\}} = 1$$

$$OPT_{P2 / \{(D=rouge)\}} = \text{false}$$

Recherche avec décomposition arborescente



$$\text{OPT}_{P2 / \{(D=\text{rouge})\}} = 1$$

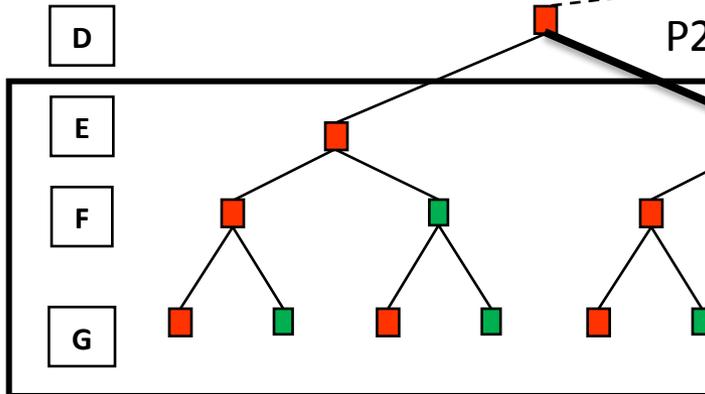
B backtrack with Tree Decomposition (BTD)

(Jégou, Terrioux, ECAI 04)

→ CELAR (de Givry, Schiex, Verfaillie, AAAI 06)

Temps $O(k n \exp(w))$

Espace $O(n \exp(s))$



Russian Doll Search with BTD (RDS-BTD)

→ CELAR, SPOT5, TAGSNP (Sanchez et al, IJCAI 2009)

Temps $O(k n \exp(w))$

Espace $O(n \exp(s))$

Russian Doll Search (Verfaillie et al, AAAI 96)

Pseudo-Tree RDS (Larrosa et al, ECAI 02)

Applications & benchmarks

<http://carlit.toulouse.inra.fr/cgi-bin/awki.cgi/SoftCSP>

- Resource Allocation
 - Frequency assignment (Allouche et al, CP 10) CTE, BTD, VAC
 $n \leq 458$, $d=44$, $e(2) \leq 5,000$
 - Satellite management (Verfaillie et al, AAI 96) RDS, RDS-BTD
 $n \leq 364$, $d=4$, $e(2-3) \leq 10,108$
 - Uncapacitated warehouse location (Zytnicki et al, IJCAI 05) EDAC, VAC, PLNE
 $n \leq 1,100$, $d \leq 300$, $e(2) 100,000$
- Bioinformatics
 - Genetic linkage analysis (Marinescu & Dechter, AAI 06) DFBB-VE-dec-ps
 $n \leq 1,200$, $d \leq 7$, $e(2-5) \leq 2,000$
 - Mendelian error detection (Sanchez et al, Constraints 08) EDAC3, DFBB-VE
 $n \leq 20,000$, $d \leq 66$, $e(3) \leq 30,000$
 - RNA gene finding (Zytnicki et al, Constraints 08) BAC
 $n \approx 20$, $d > 100$ million, $e(4) \approx 10$
 - Tag SNP selection (Sanchez et al, IJCAI 09) RDS-BTD, PLNE
 $n \leq 1,500$, $d \leq 266$, $e(2) \leq 150,000$

<https://mulcyber.toulouse.inra.fr/projects/toulbar2/>

2010 Approximate Inference Evaluation (UAI'10)

Seconds	PR	MAR	MPE
20	Arthur Choi (UCLA)	Arthur Choi (UCLA)	Joris Mooij (Max Planck)
1200	Vibhav Gogate (UW+UCI)	Vibhav Gogate (UW+UCI)	Thomas Schiex (INRA)
3600	Vibhav Gogate (UW+UCI)	Vibhav Gogate (UW+UCI)	Joris Mooij (Max Planck)

73%
instances
résolues
à l'optimum
par toulbar2

2010 Approximate Inference Evaluation (UAI'10)

Seconds	PR	MAR	MPE
20	Arthur Choi	Arthur Choi	Joris Mooij

Travaux en direction de l'inférence approchée

- **#BTD** – comptage de solutions dans les CSPs (*Favier, de Givry, Jégou, CP 09*)
- Décomposition fonctionnelle par paire (*Favier, de Givry, Legarra, Schiex, IJCAI 11*)

(Larkin, UAI 03), (Wexler, Meek, NIPS 08), (Rollon, Dechter, AAAI 10),...

3600	Vibhav Gogate (UW+UCI)	Vibhav Gogate (UW+UCI)	Joris Mooij (Max Planck)
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Apprentissage de la structure d'un réseau de gènes

Modèle graphique discret

→ Réseau Bayésien sur données discrètes

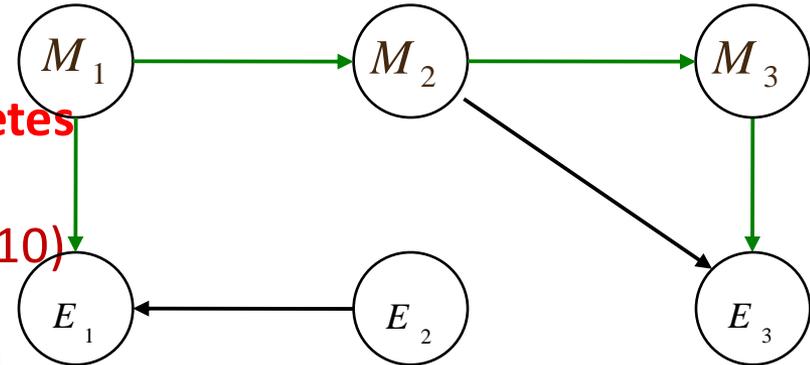
(Friedman, Science 00)

(Vandel, de Givry, Mangin, Vignes, CAP 10)

Prog. Dyn. (Silander, Myllymäki, UAI 06)

DFBB (de Campos, Zeng, Ji, ML 09)

Max-SAT (Cussens, UAI 08)



Modèle linéaire

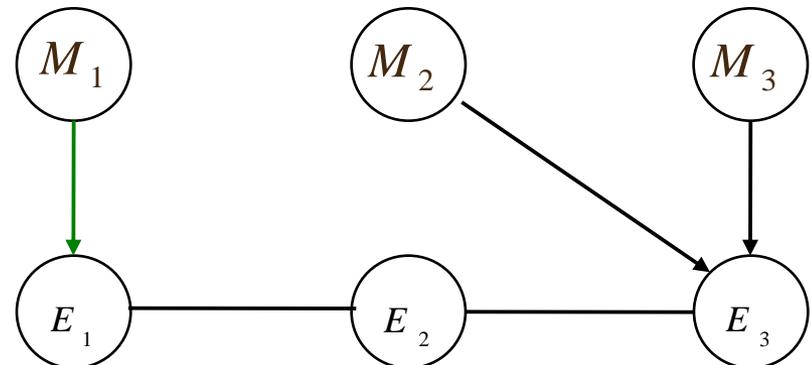
Modèle graphique gaussien

→ régression locale

Lasso (Tibshirani 1996),

ElasticNet (Zou and Hastie 2004)

Dantzig (Candès and Tao 2007)



Conclusion et perspectives

- Nouvelle génération d'outils de **programmation par contraintes** exploitant des cohérences locales souples efficaces
- Intégration et poursuite des travaux sur **Max-SAT** et WCSP
- Choix automatique d'une **bonne décomposition** arborescente
- **Inférence approchée** exploitant les cohérences locales
- Optimisation exacte ou approchée d'un score pour **l'apprentissage de structure**

MERCI !

MERCI !

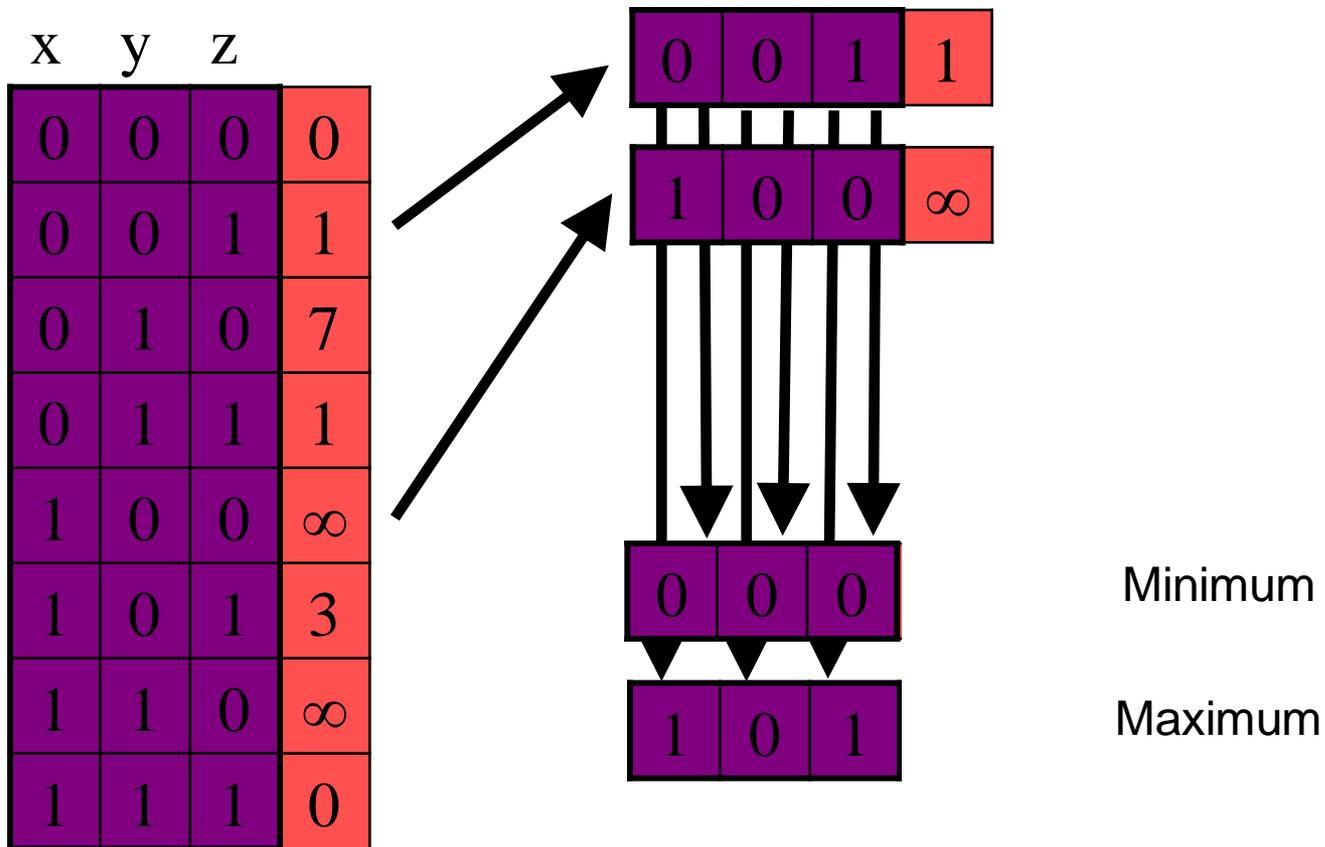


Liens vers d'autres formalismes

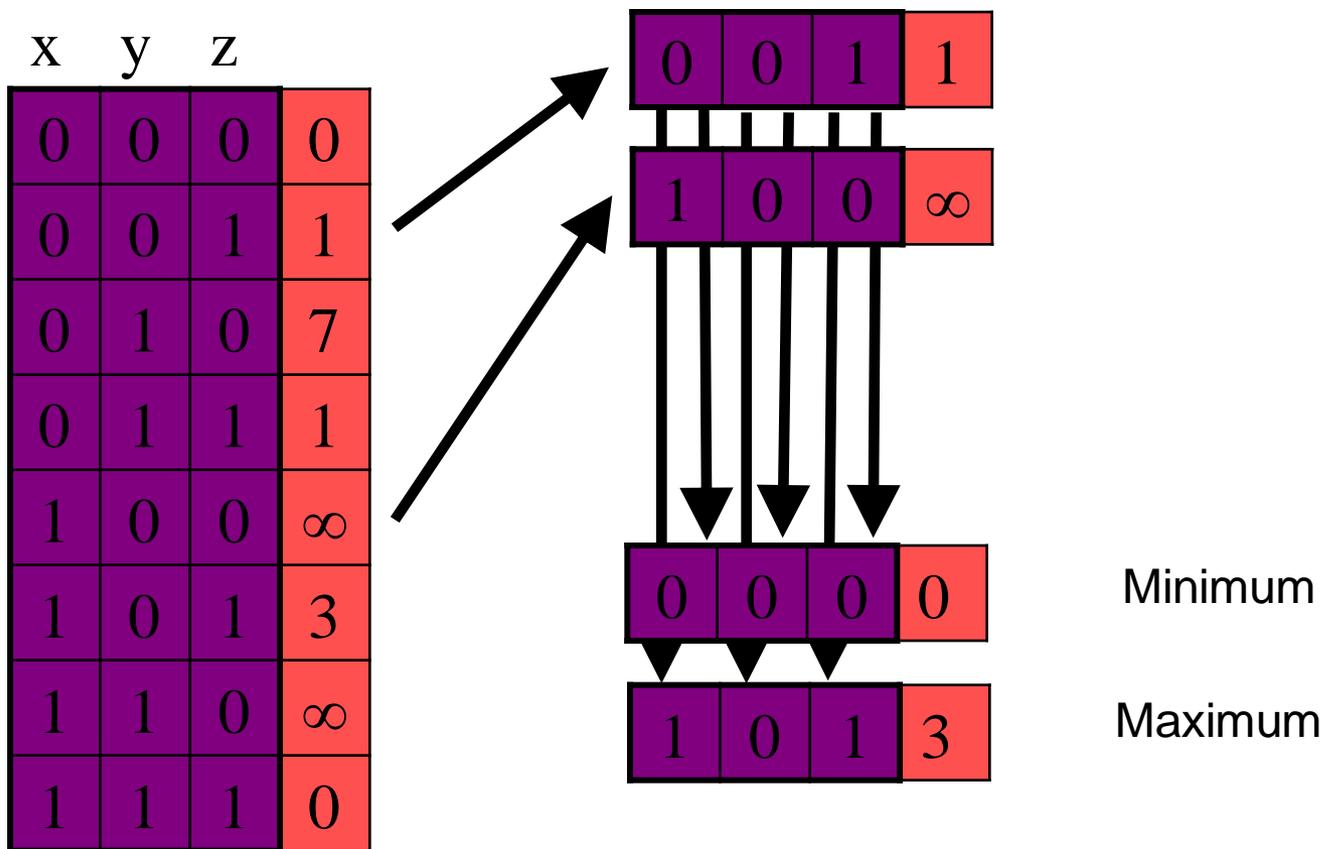
- **Problème de satisfaction de contraintes valuées (VCSP)** (Schiex, Fargier, Verfaillie, IJCAI 95) $(X, D, F, (E, \oplus, \leq))$
Structure de valuation (E, \oplus, \leq)
 - E : ensemble de valuations avec un ordre total \leq
 - \oplus : opérateur binaire d'agrégation des valuations

« Tous les VCSP avec un ensemble discret de valuations et un opérateur \ominus se décomposent en une séquence min/max de WCSP » (Cooper, Constraints 05)

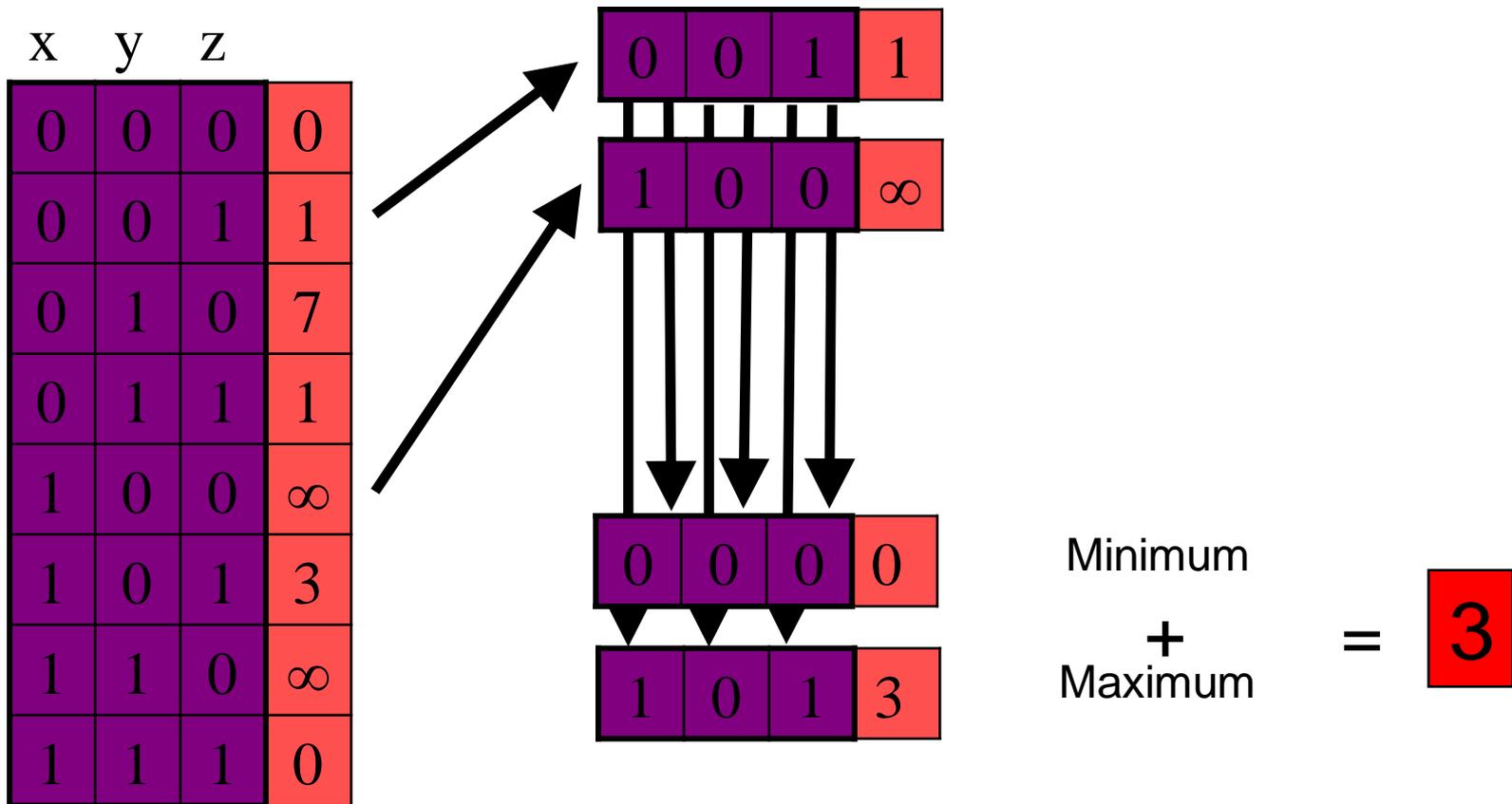
Examples of submodular functions



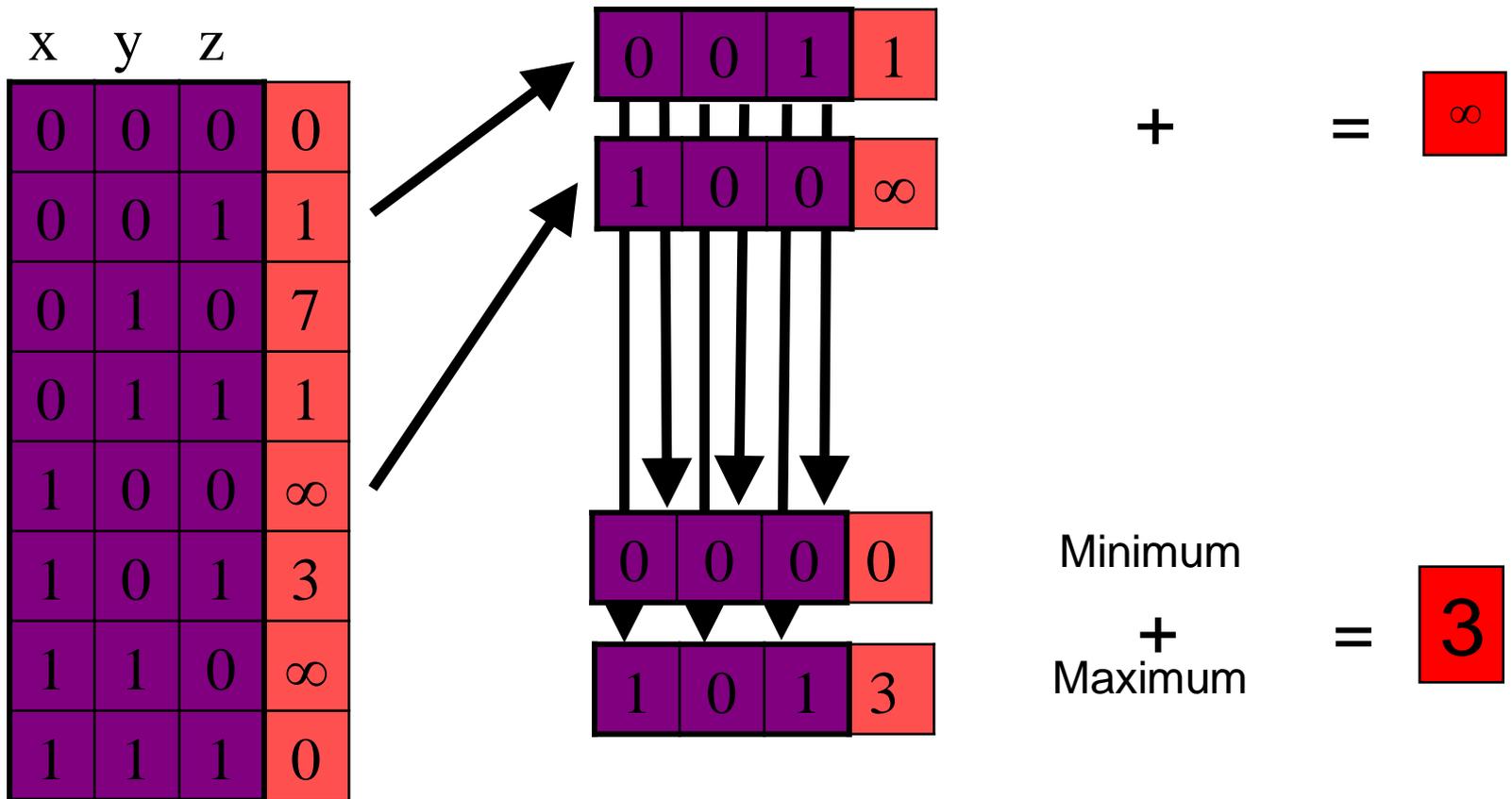
Examples of submodular functions



Examples of submodular functions

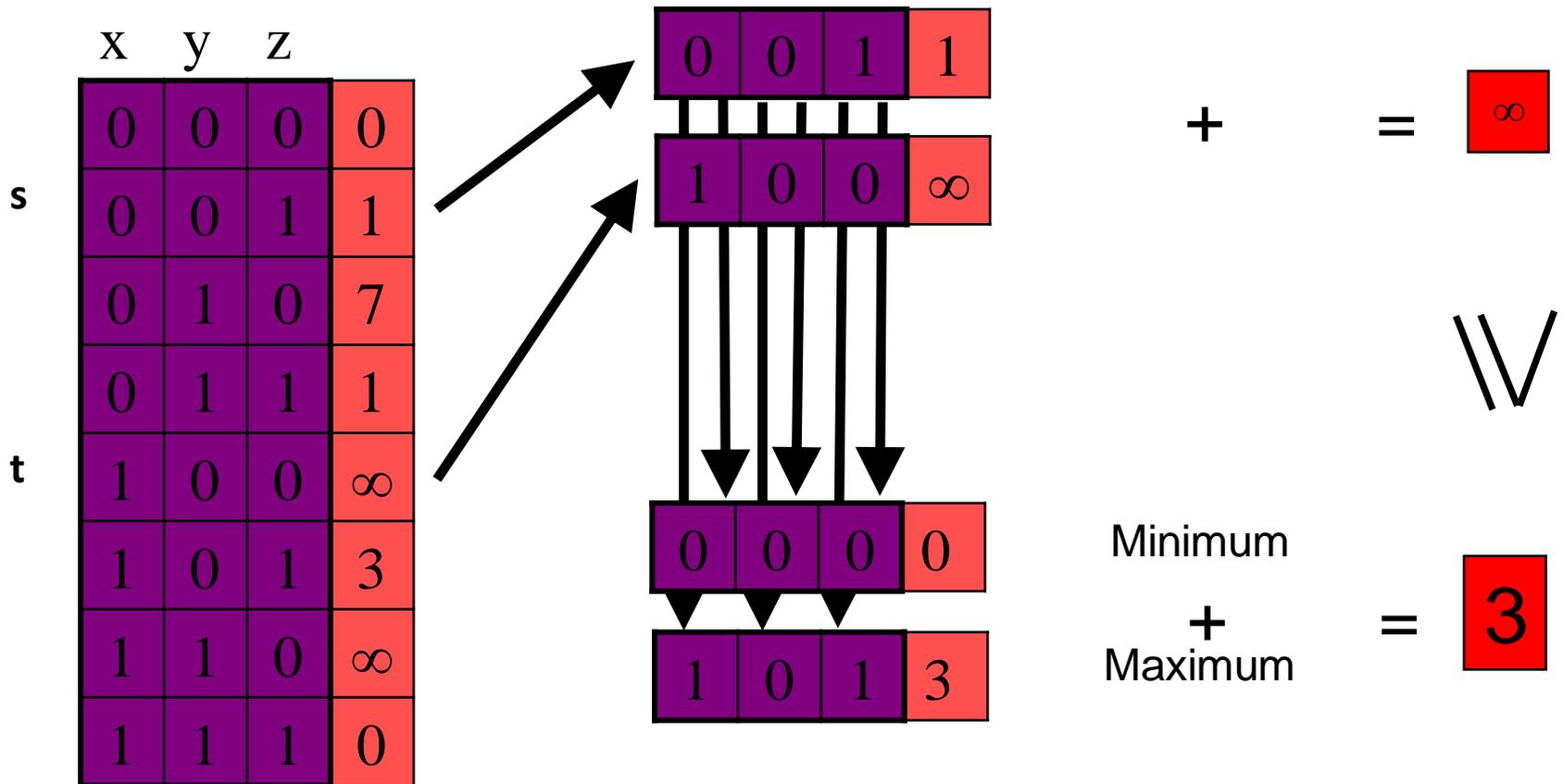


Examples of submodular functions



Examples of submodular functions

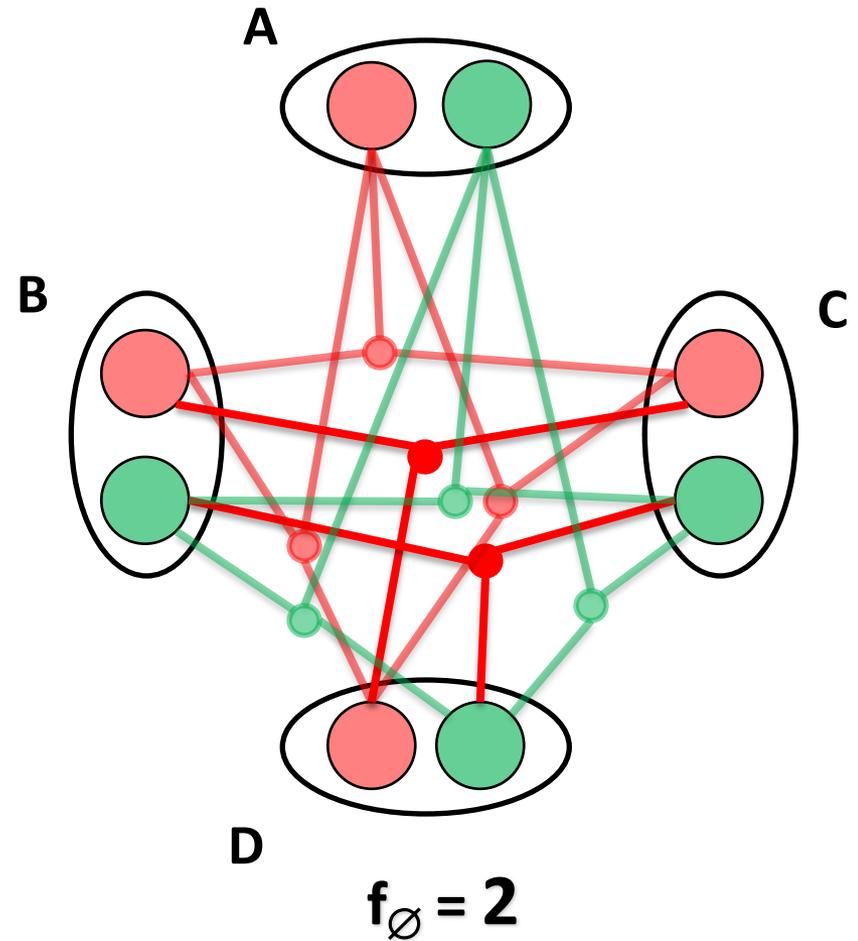
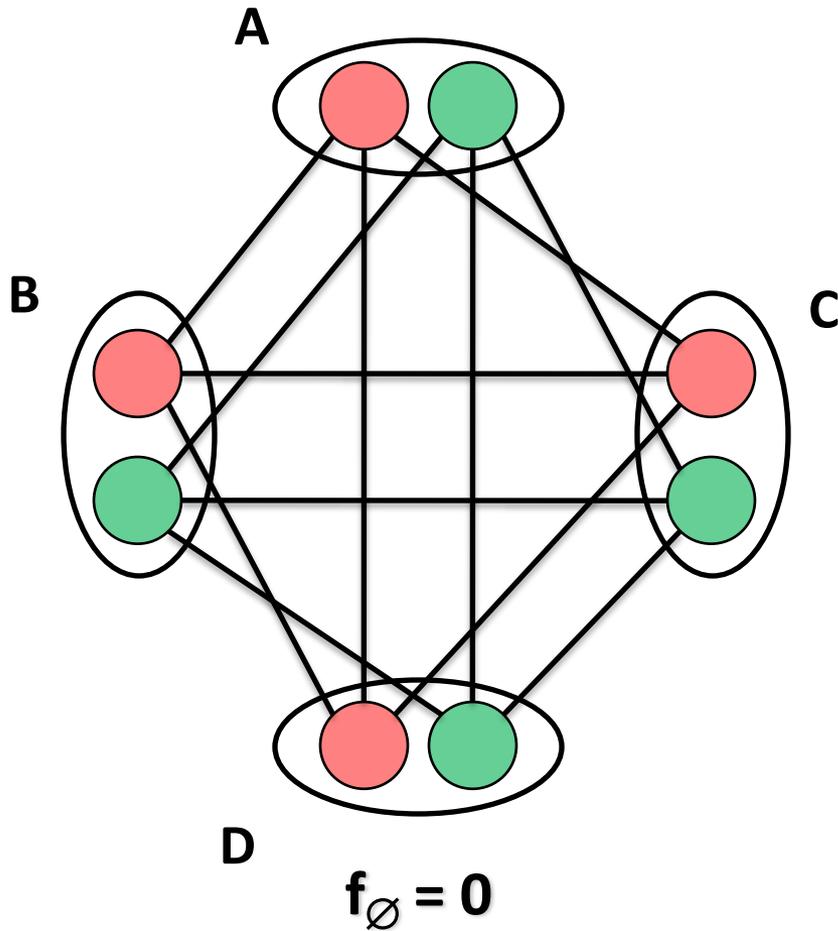
$$\forall s, t \quad \text{Cost}(\text{Min}(s, t)) + \text{Cost}(\text{Max}(s, t)) \leq \text{Cost}(s) + \text{Cost}(t)$$



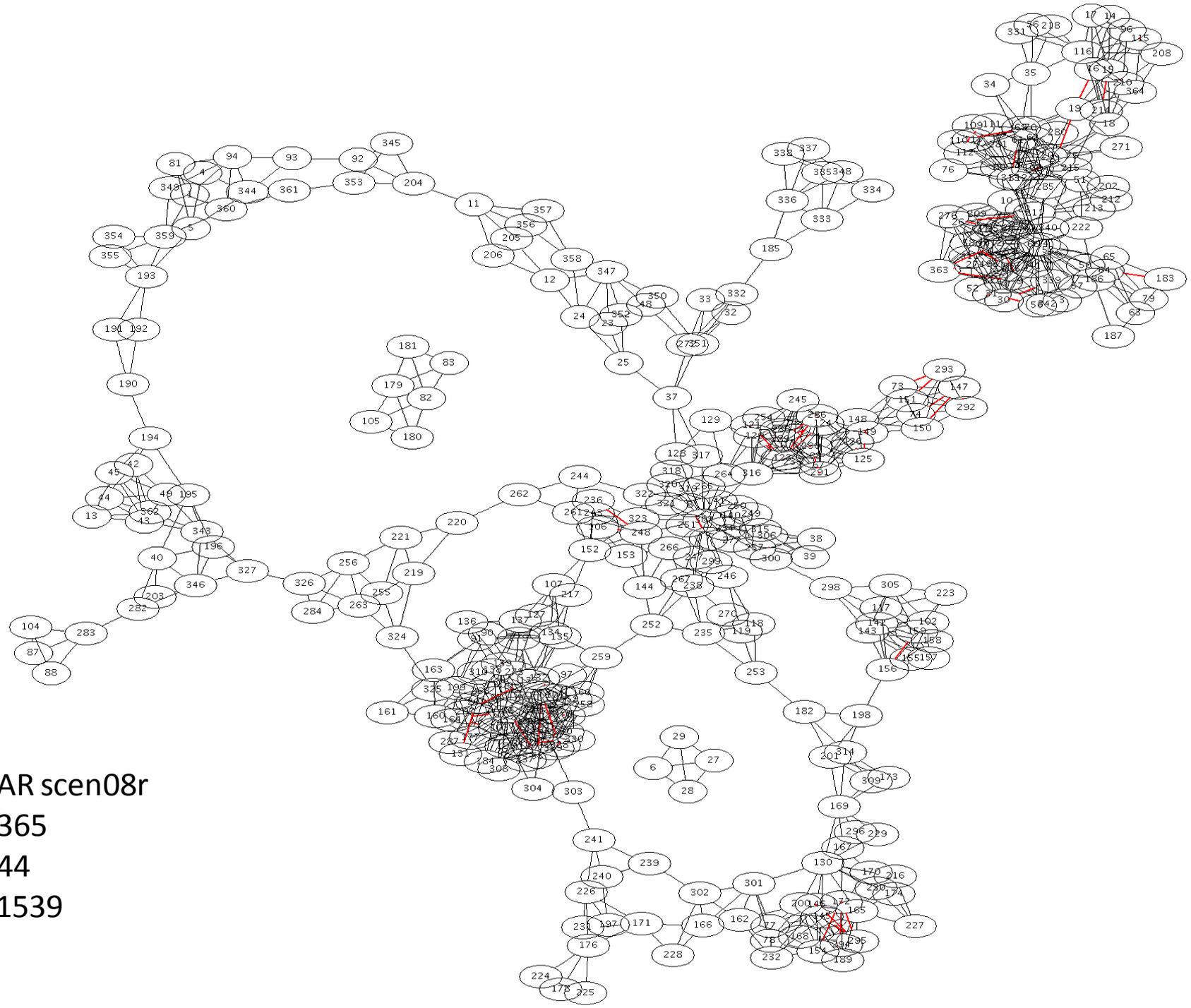
Examples of submodular functions

- all unary functions
- all linear functions (of any arity)
- the binary function ϕ_{cut}
where $\phi_{\text{cut}}(a,b)=1$ if $(a,b)=(0,1)$ (0 otherwise)
- the rank function of a matroid
- the Euclidean distance function between two points $(x_1, x_2), (x_3, x_4)$ in the plane
- $\phi(x,y)=(x-y)^r$ if $x \geq y$ (∞ otherwise) for $r \geq 1$ (compare
“Simple Temporal CSPs with strictly monotone preferences”
Khatib et al, IJCAI 2001)

Reformulation d'arité supérieure



Problème chemin-inverse-cohérent singleton virtuel (VSPIC)

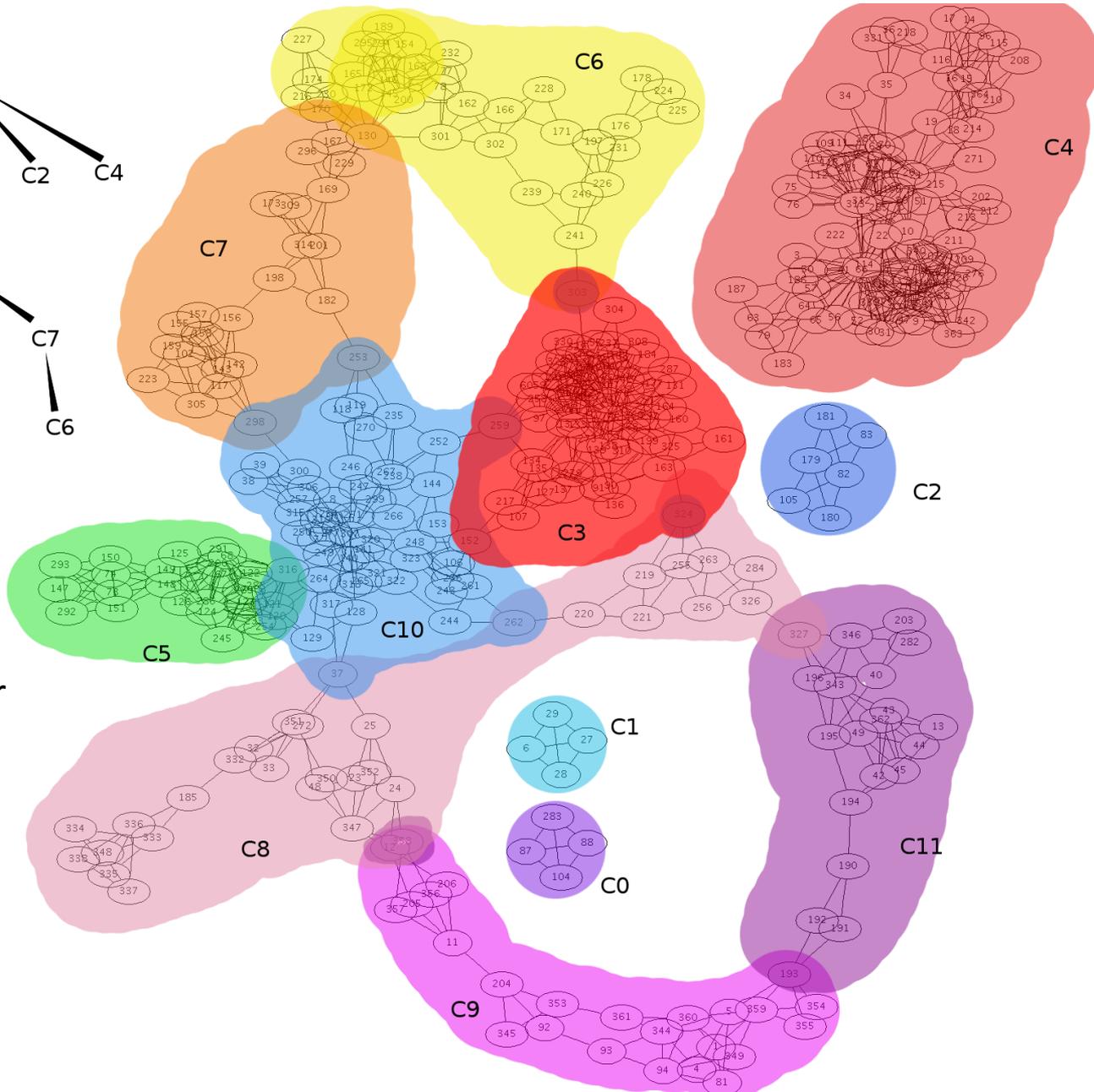
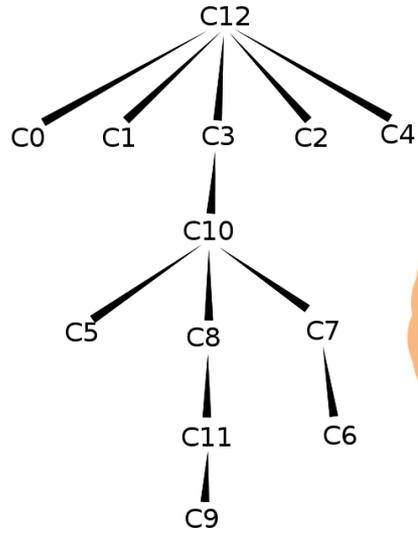


CELAR scen08r

$n = 365$

$d = 44$

$e = 1539$



CELAR scen08r

$n = 365$

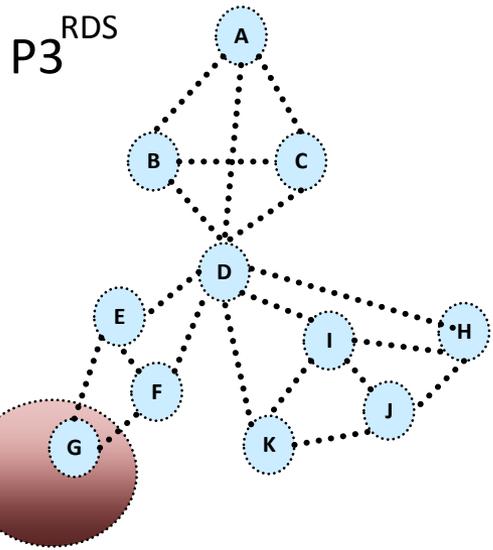
$d = 44$

$e = 1539$

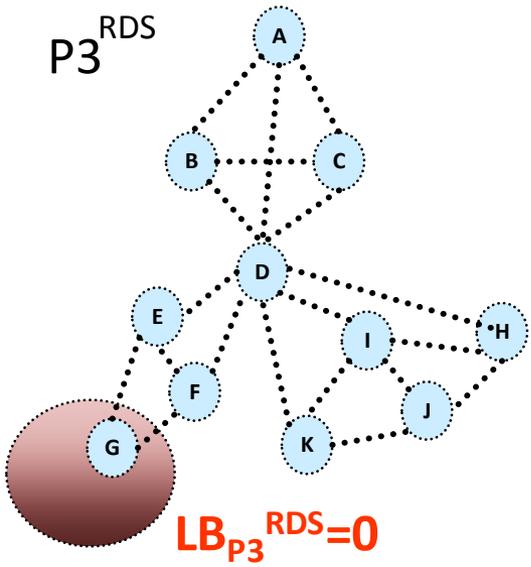
$w = 85$

$s = 4$

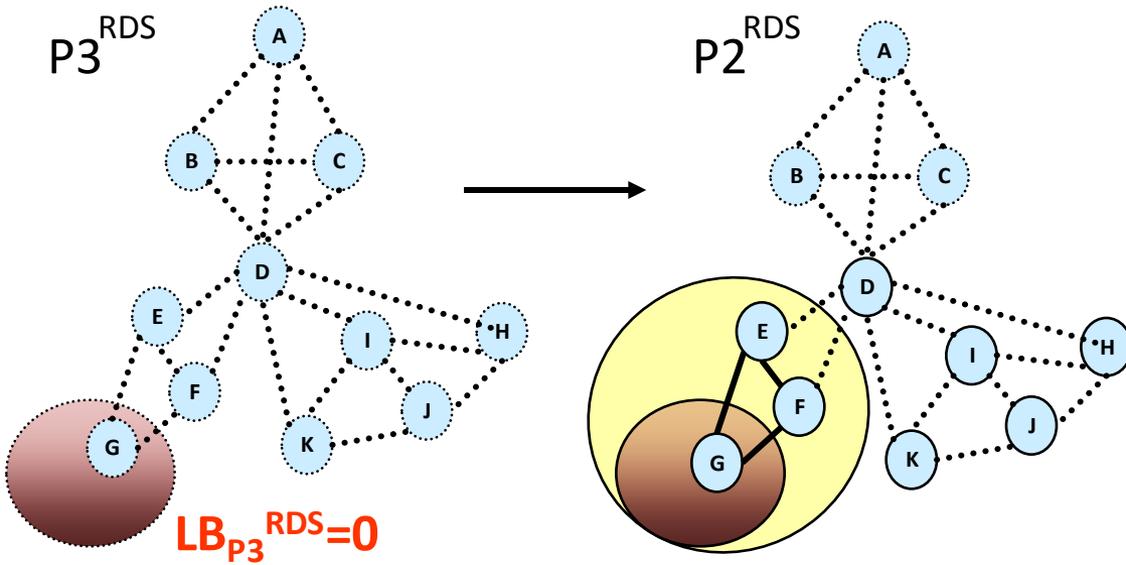
Russian Doll Search with Tree Decomposition



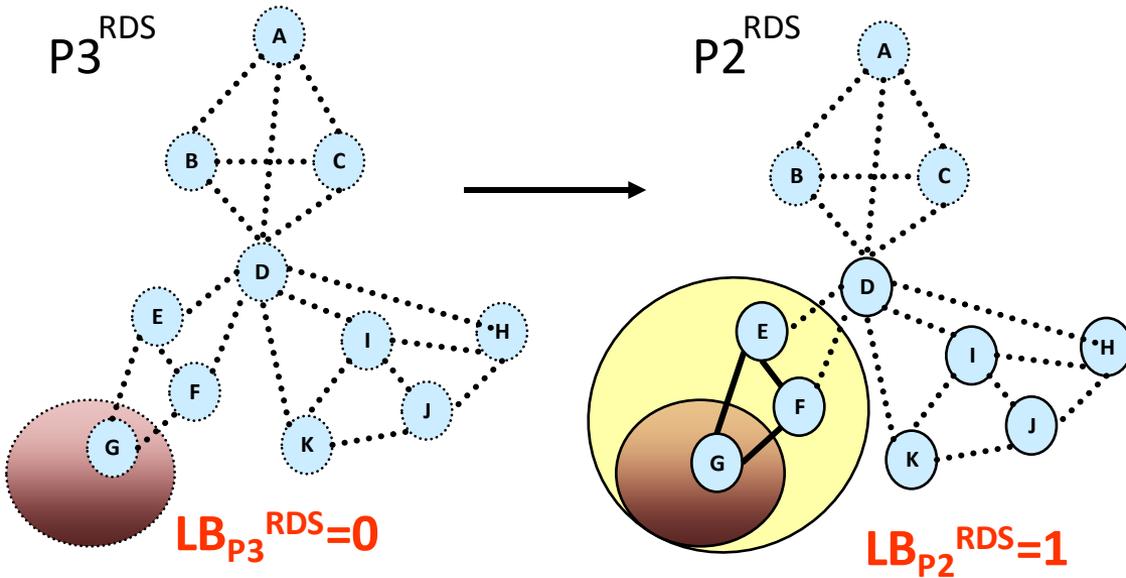
Russian Doll Search with Tree Decomposition



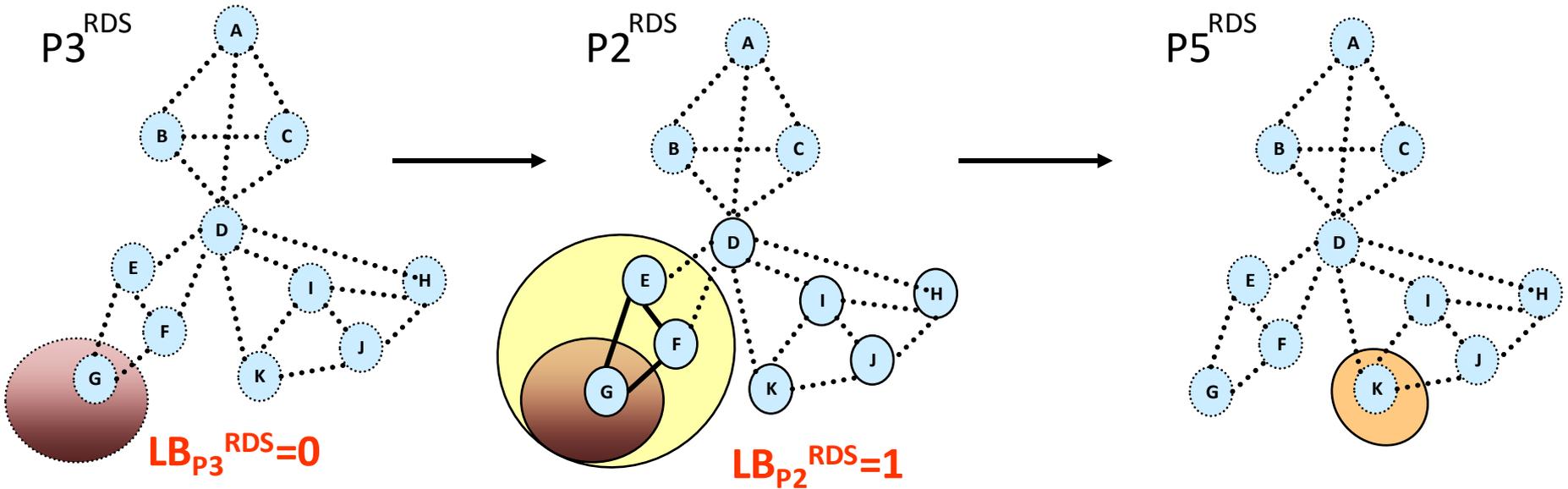
Russian Doll Search with Tree Decomposition



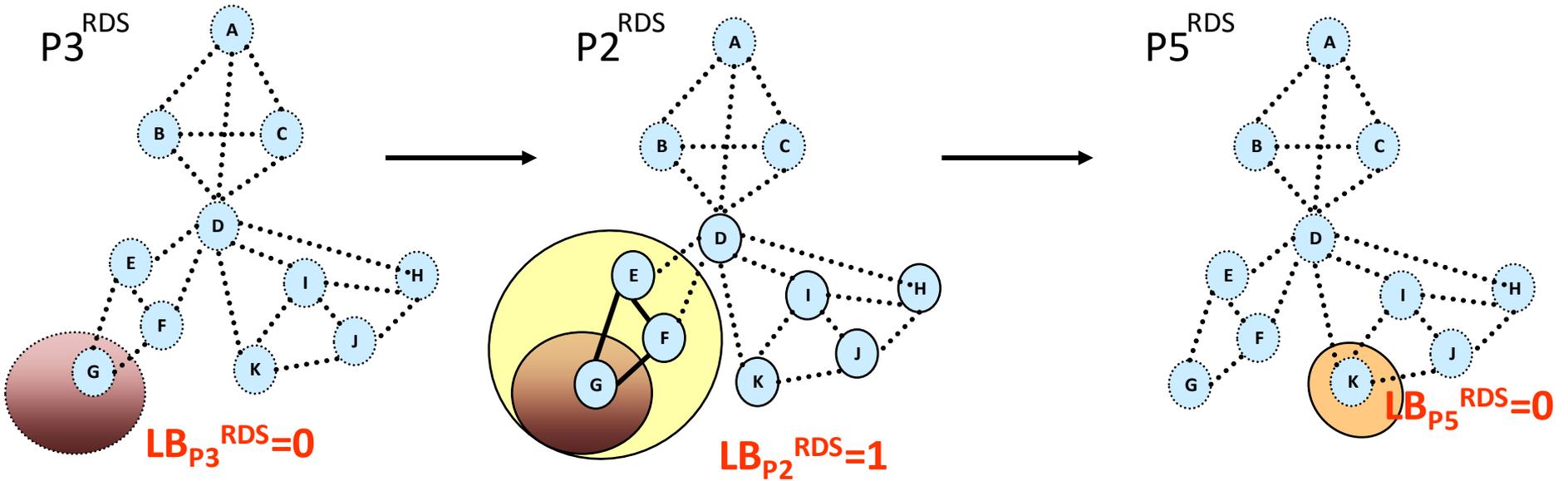
Russian Doll Search with Tree Decomposition



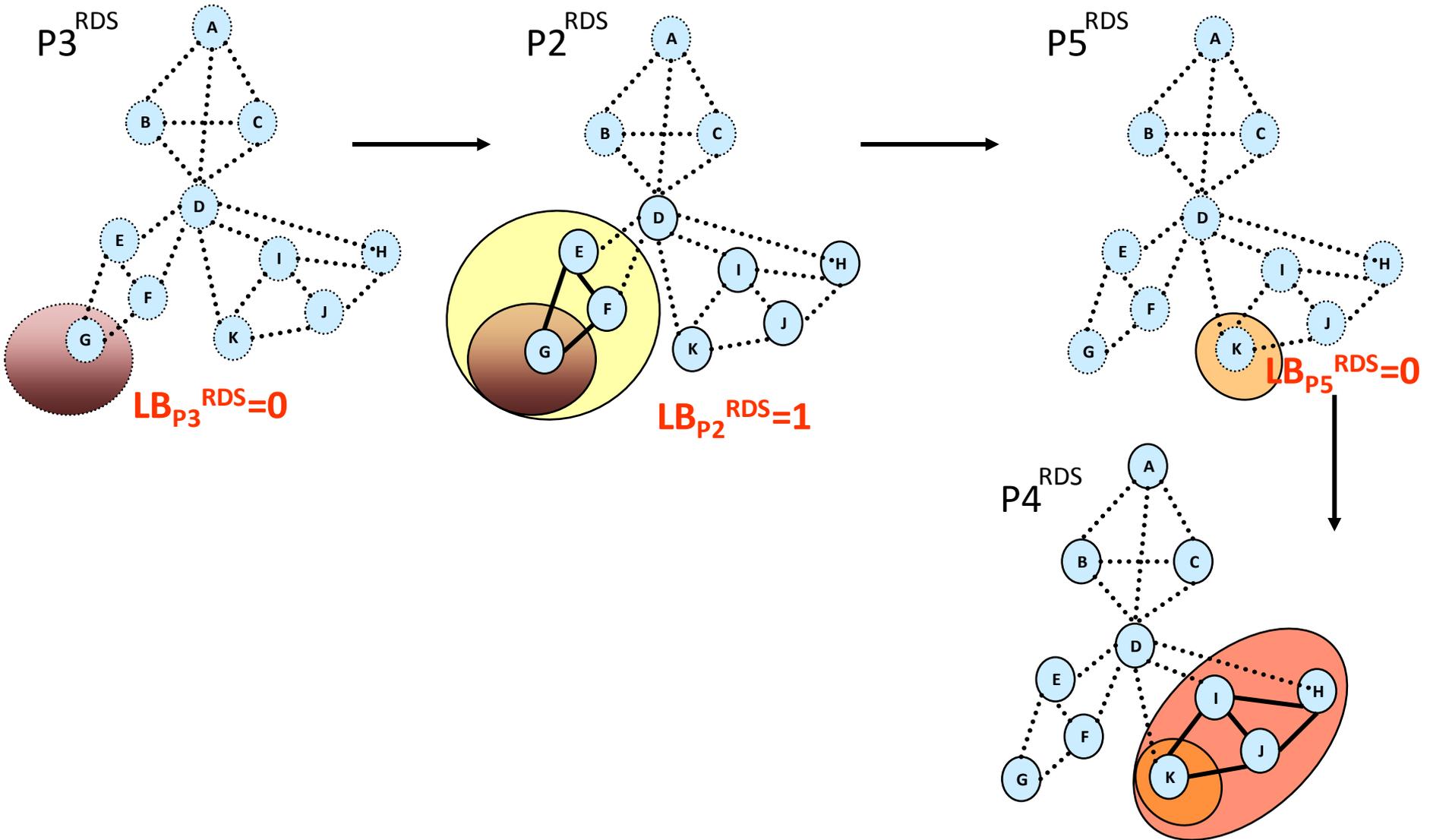
Russian Doll Search with Tree Decomposition



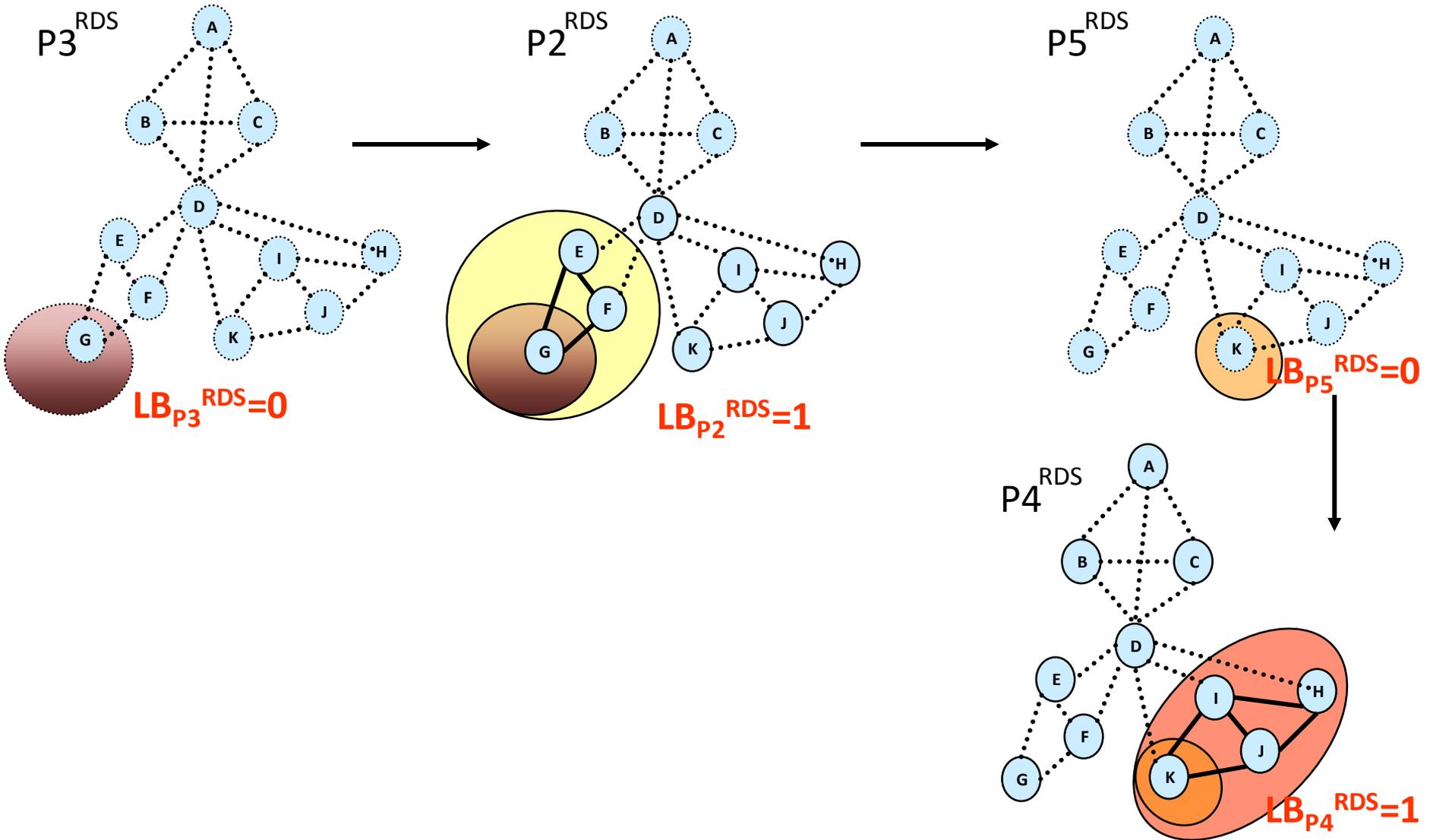
Russian Doll Search with Tree Decomposition



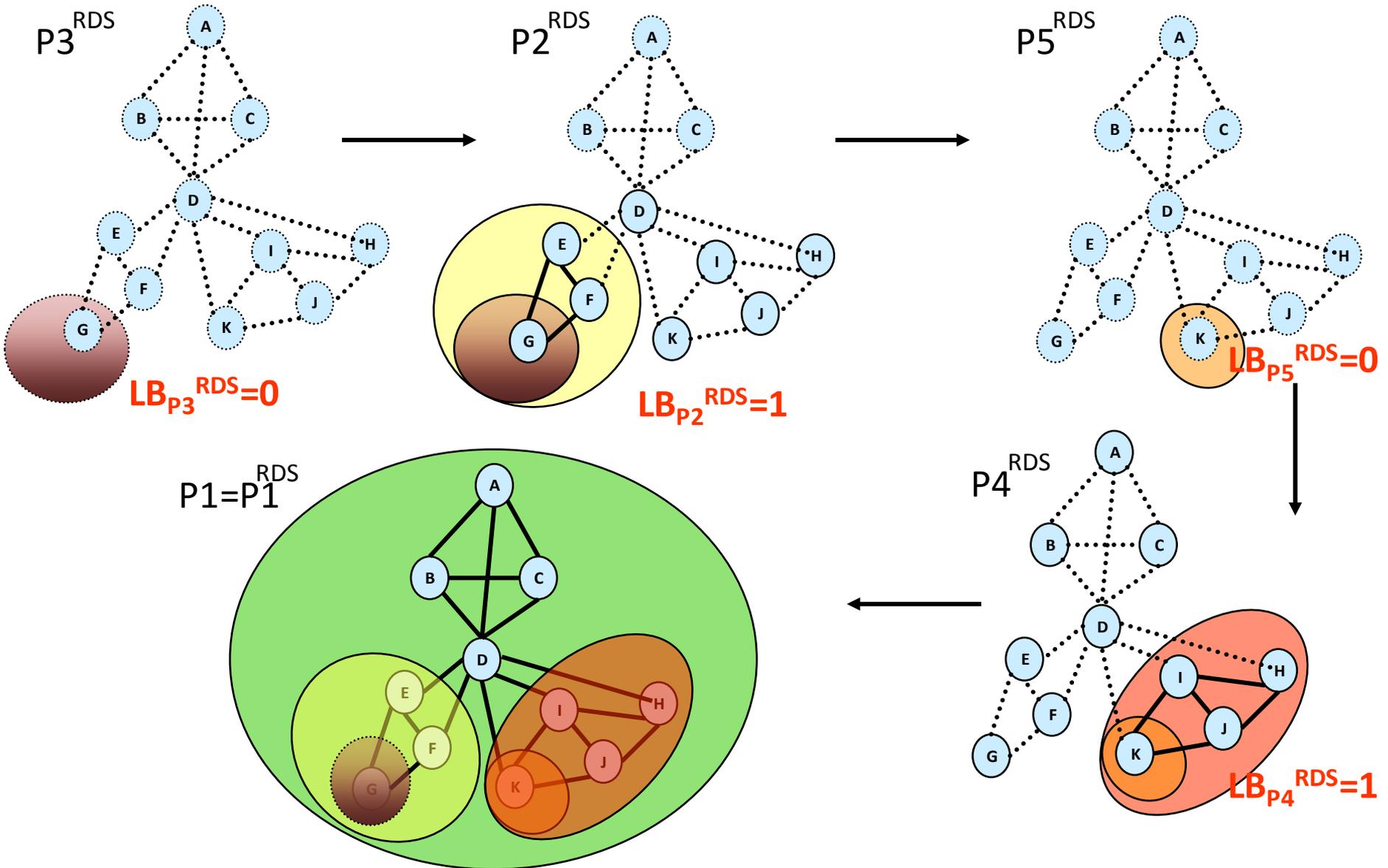
Russian Doll Search with Tree Decomposition



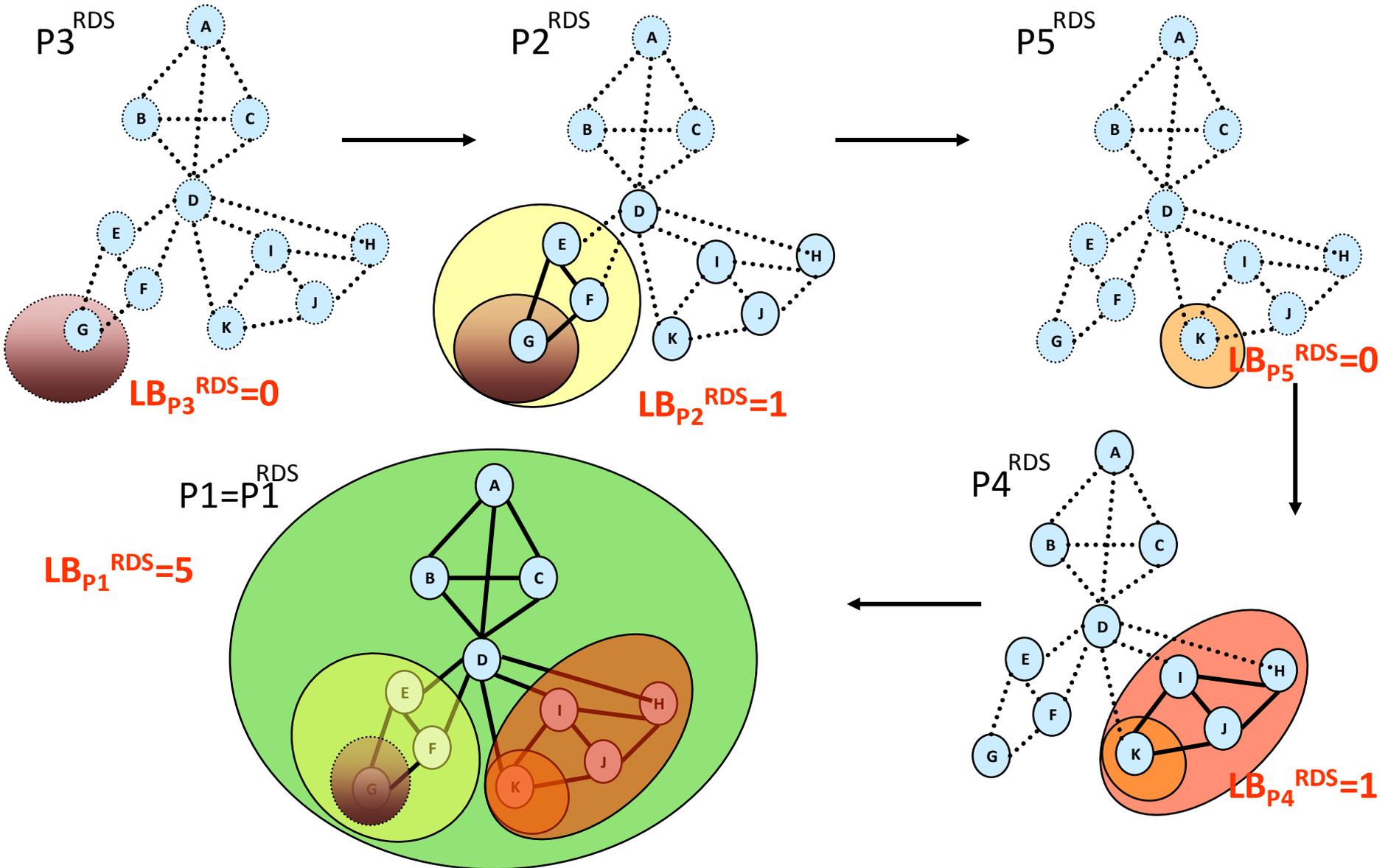
Russian Doll Search with Tree Decomposition



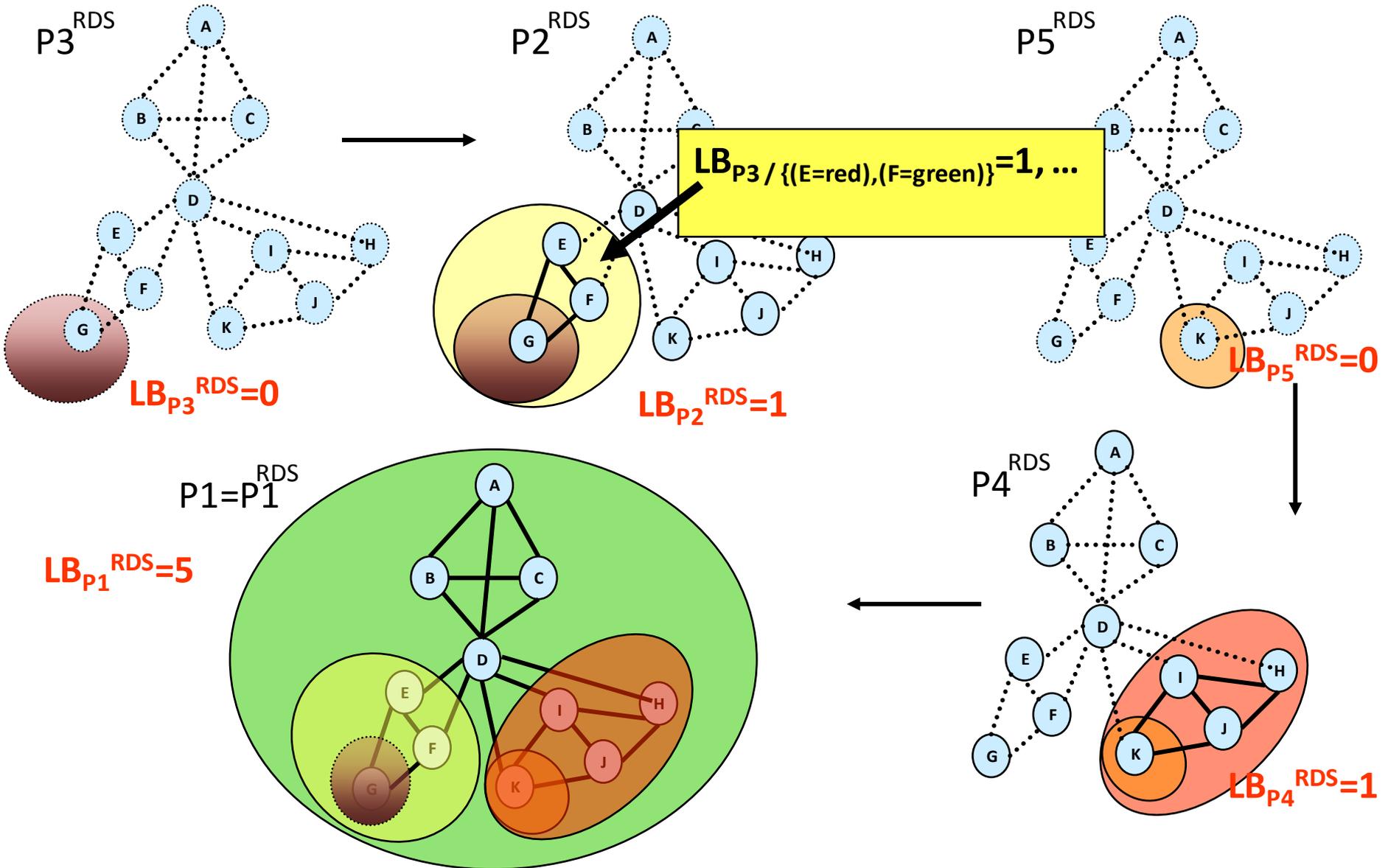
Russian Doll Search with Tree Decomposition



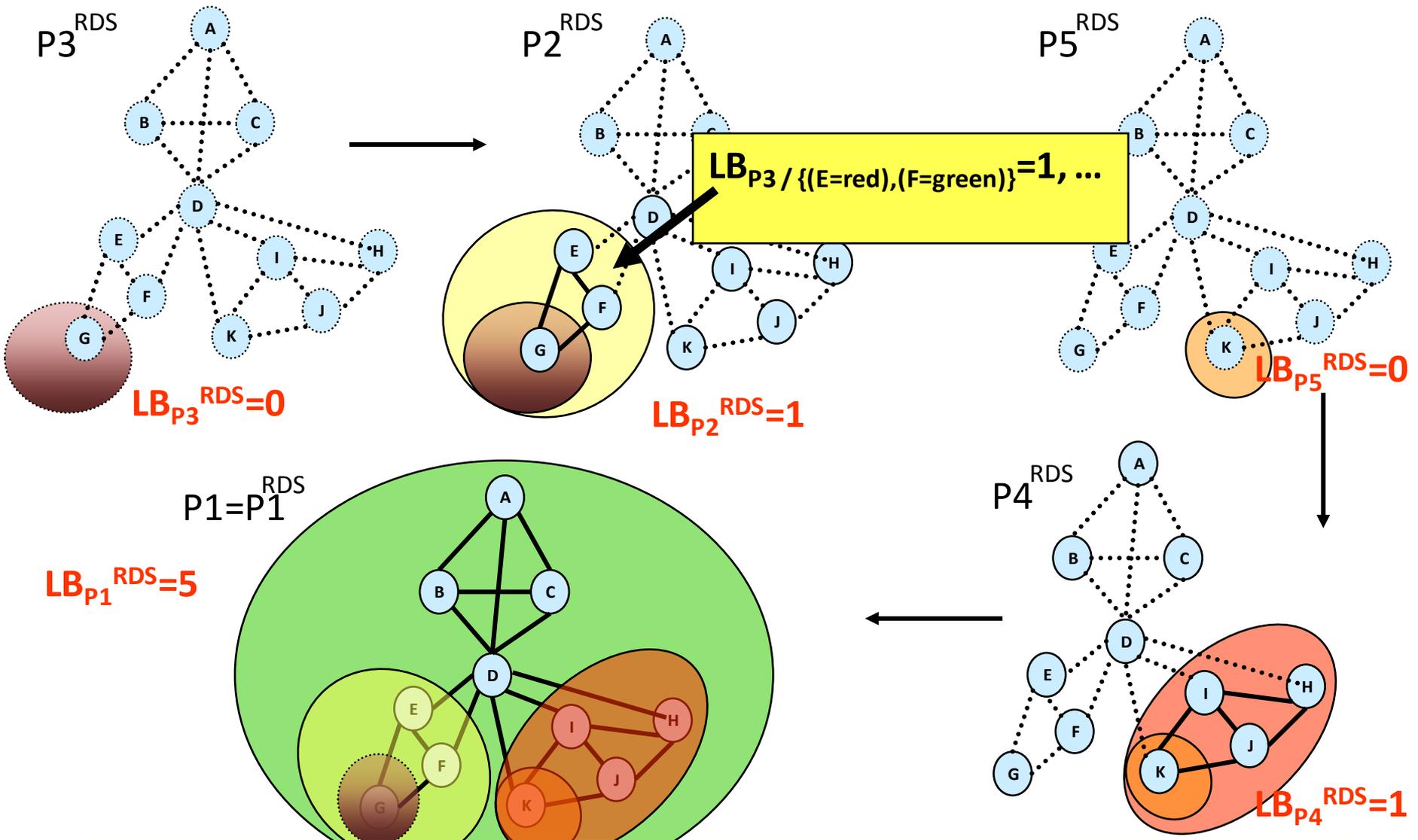
Russian Doll Search with Tree Decomposition



Russian Doll Search with Tree Decomposition

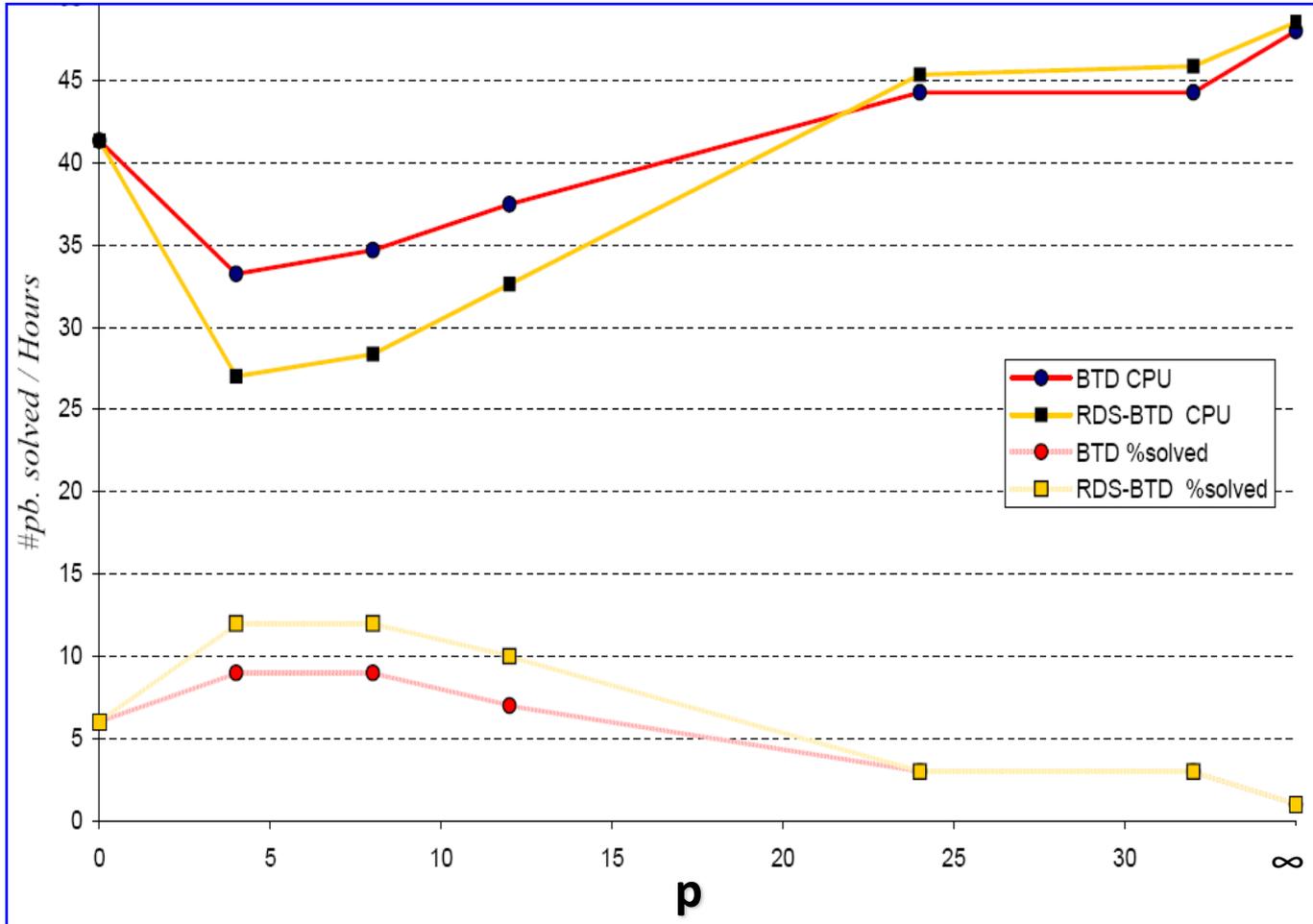


Russian Doll Search with Tree Decomposition



Same complexities as BTM: Time: $O(\exp(w+1))$ Space: $O(\exp(s))$
 $w = \text{tree-width} = \text{largest cluster size} - 1$, $s = \text{largest separator size}$

TAGSNP

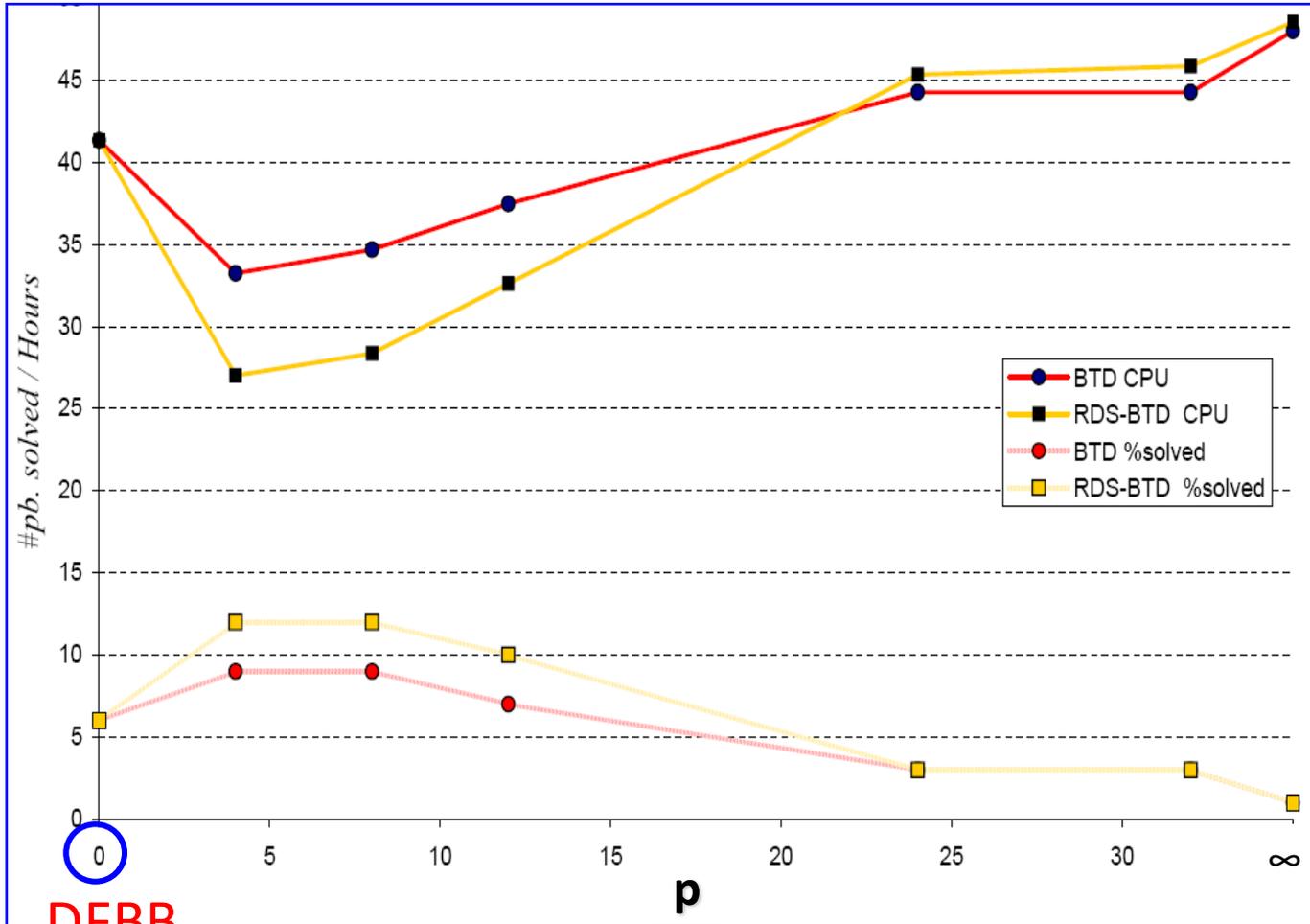


$171 \leq n/2 \leq 777$
 $30 \leq d \leq 266$
 $6\% \leq m \leq 37\%$
 $14\% \leq w \leq 23\%$

toulbar2 v.0.8
 avec minorant
 initial par FESTA
 et décomposition
 MCS

Time limite : 2 heures

TAGSNP



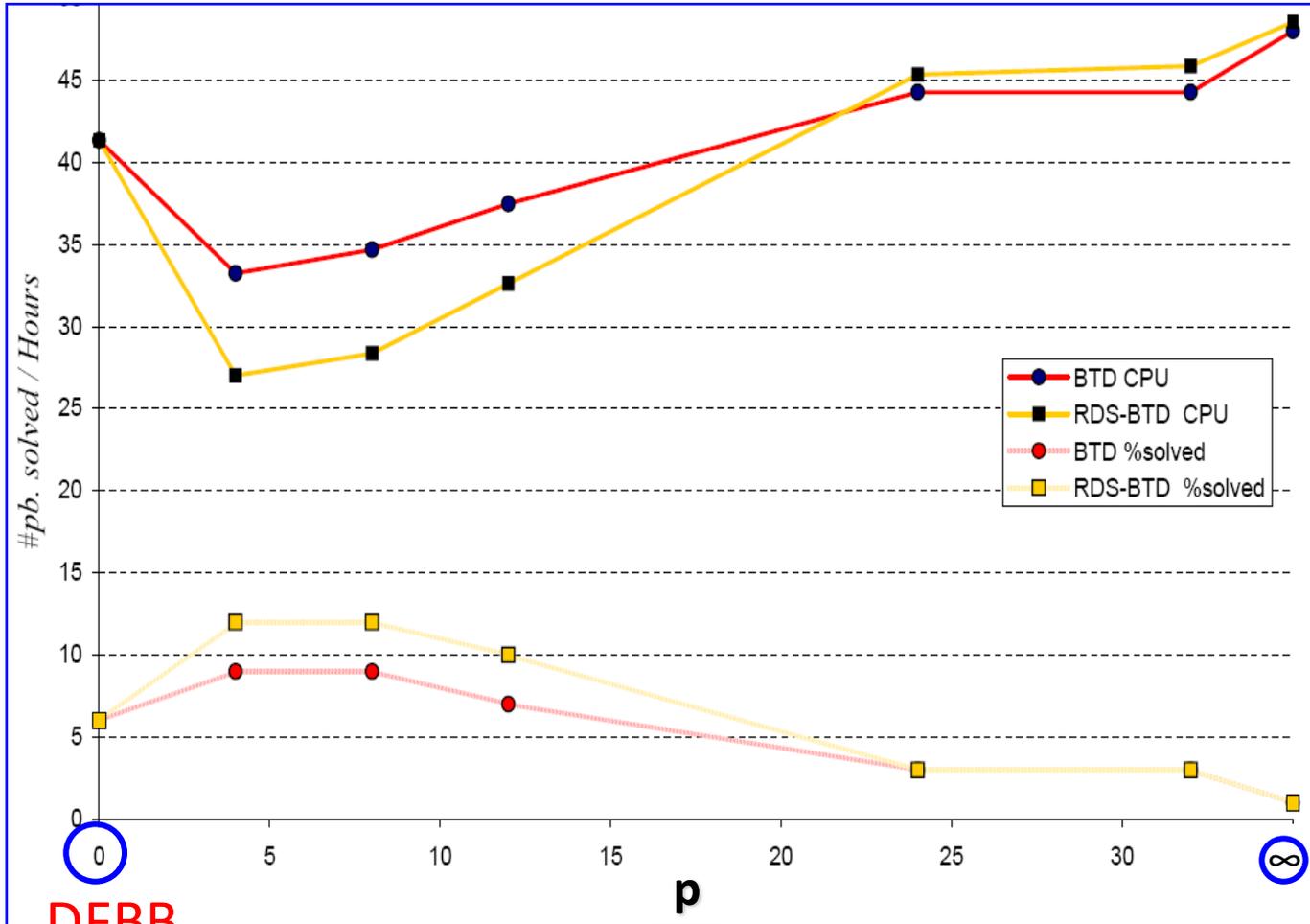
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Time limite : 2 heures

DFBB

TAGSNP



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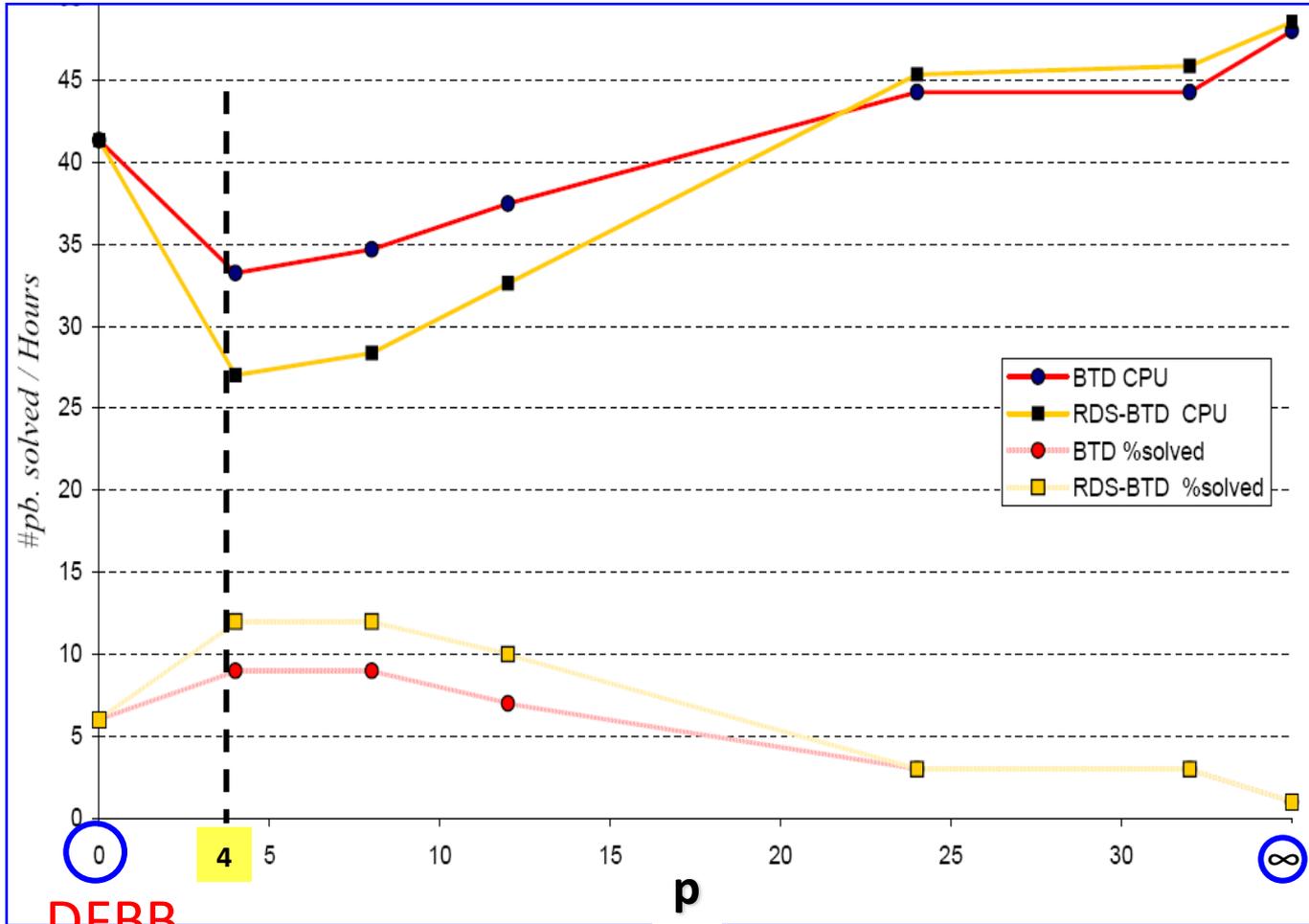
toulbar2 v.0.8
 avec minorant
 initial par FESTA
 et décomposition
 MCS

Time limite : 2 heures

DFBB

BTD
RDS-BTD

TAGSNP



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toulbar2 v.0.8
 avec minorant
 initial par FESTA
 et décomposition
 MCS

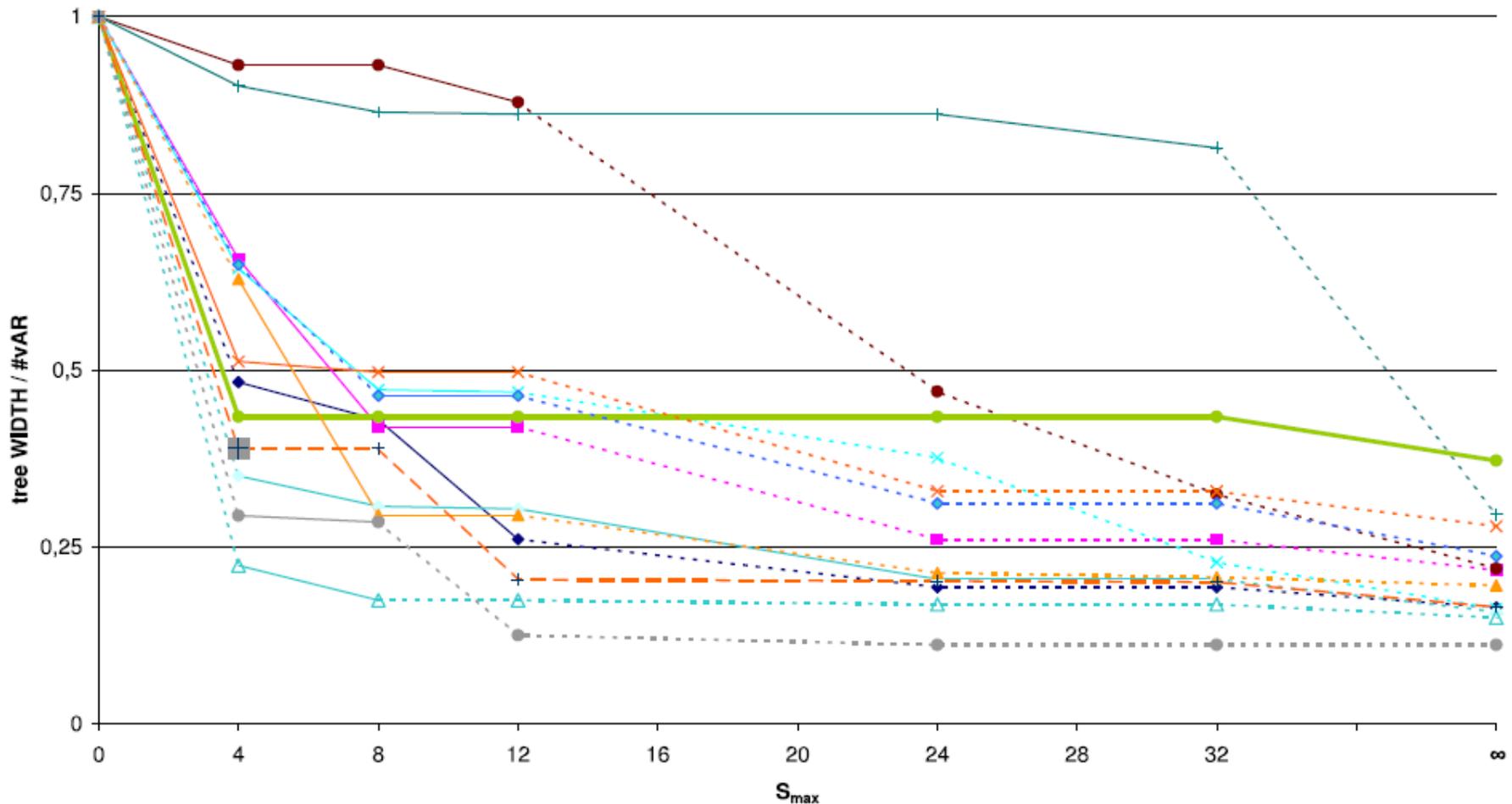
Time limite : 2 heures

DFBB

BTD
RDS-BTD

TAGSNP (Allouche, de Givry, Sanchez, Schiex, WCB 09)

Réglage de la limite sur la taille des séparateurs



Evolution de la largeur d'arbre normalisée par le nombre de variables du problème pour différentes limites sur la taille des séparateurs