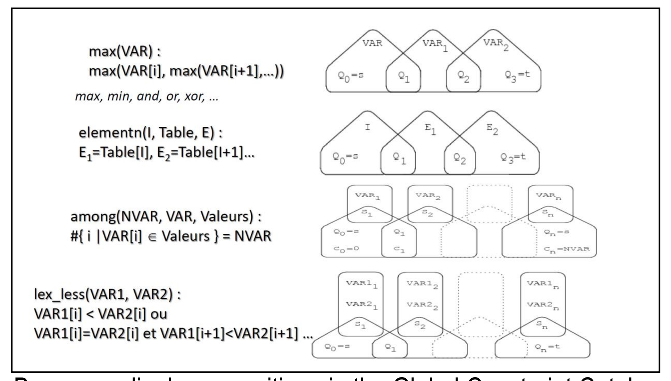
Filtering Decomposable Global Cost Functions

Strong local consistencies for filtering weighted constraint networks
ANR -10-BLA-0214 French Research Project (FiCoLoFo)



Berge-acyclic decompositions in the Global Constraint Catalog

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Summary

Combinatorial optimization problems are naturally expressed as cost function networks. Recently, global cost functions have been introduced with dedicated flow-based filtering algorithms (Lee and Leung, JAIR 2012), leading to a new Cost Function Programming paradigm. In this paper, we explore the possibility of decomposing global cost functions by adding intermediate variables in such a way that enforcing soft local consistency on the decomposition achieves the same level of consistency on the original global cost function. We give conditions under which directional and virtual arc consistency offer such guarantees. We conclude by experiments on decomposable cost functions showing that decompositions may be very useful to easily integrate efficient – *incremental* – global cost functions in solvers.

Preliminaries: Cost Function Networks and Filtering Global Cost Functions using Soft Local Consistency

Combinatorial optimization for cost function networks
(Shapiro, Haralick, IEEE PAMI 81)

- *n* variables $X = \{X_1, \dots, X_n\}$
  - finite domains $X_i \in D_i, |D_i| \le d$
- *e* local/global functions $F = \{f_1, \dots, f_e\}$
  - scope, function with costs
    arity **r**
- Costs ∈ {0,...,+∞} finite or infinite integer (maximum cost **k**)

A	B	D	f(A,B,D)
1	2	3	12
1	3	2	7
2	1	3	9
…	…	…	f(…)

$f(A,B,D) = 3A^2 \times B \times D$

A	B	D	f(A,B,D)
1	2	3	0
1	3	2	+∞
2	1	3	0
…	…	…	f(…)

AllDifferent(A,B,D) +∞ 0

Minimize $\sum_F f_i(X)$ **NP-hard**

Existing global cost functions

- Soft (*relaxed*) constraints (Petit et al, CP 2001), (Hoeve et al, J. Heuristics 2006)
  - softAllDifferent, softGCC, softAmong, softRegular,…
  - Semantic of violation (number of variables to be changed, number of violated elementary constraints, …)
- Constraints integrating unary costs (Régin, Constraints 2002), (Trick, AOR 2003), (Demassey et al, Constraint 2006)
  - GCC_with_costs, knapsack, costRegular, …
- Functions integrating complex costs (Katsirelos et al, AOR 2011)
  - weightedRegular (costs associated to automaton transitions)

Filtering of cost functions

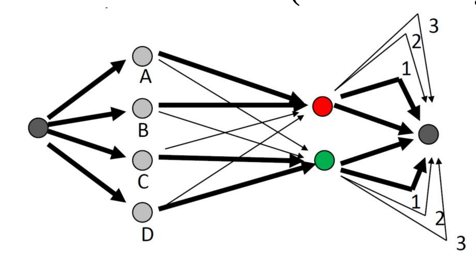
(Petit et al, ICTAI 2000)

- Using a cost variable associated to each global constraint
- Using equivalence preserving problem transformations exploiting special cost functions: $f_\varnothing$ and $f(X_i)$, $\forall X_i \in X$
  - Chaotic transformations (AC, DAC, FDAC, EDAC)
  - Planned transformations (*OSAC*, VAC)

See next and below

Filtering softAllDiff

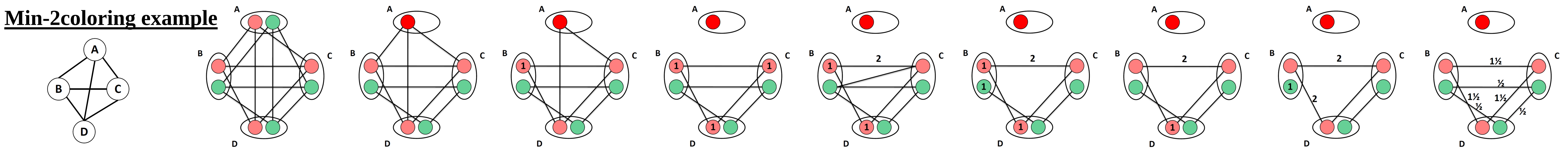
(Lee and Leung, JAIR 2012)



(each edge has a unit capacity)

Monolithic approach based on min-cost max-flow algorithm

Min-2coloring example



X={A,B,C,D}
F={f(A,B), f(A,C), f(A,D), f(B,C), f(B,D), f(C,D)}

micro-structure
(each edge has a unit cost)

assign *red* to **A**

Arc consistent equivalent problem (**AC**)
(Schiex, CP 2000)

$f_\varnothing = 1$ $f_\varnothing = 1$
Directional arc consistent problem (**DAC**)
(Cooper, FSS 2003)
A < B < C < D
O(e d²)

$f_\varnothing = 1\frac{1}{2}$
Virtual arc consistent problem (**VAC**)
(Cooper et al, AIJ 2010)
O(e k d²/ε)

Contribution: Softening Decomposable Global Constraints and Enforcing Soft Local Consistency on Berge-acyclic Decompositions

❶ Definition : cost function decomposition

Polynomial transformation parameterized by **p**

f(T) ⟶ (T∪E, F) *cost function network*

such that

- ∀f'(S)∈F, |S| ≤ p *bounded arity by p*
- $\forall t \in D^T$, $f(t) = \min_{t' \in D^{T \cup E},\, t'[T]=t} \sum_{f'(S) \in F} f'(t'[S])$

preserves cost distribution

❷ Theorem: softening global constraints

- Let c(T) ⟶ (T∪E, C) *constraint network*
- Let ∀c'(S)∈C, **g•**c' *new cost function*

such that

∀ t' ∈ S, **g•**c'(t') ≤ c'(t') *softening of constraint c' by g*

Then

(T∪E, **g•**C) is a relaxation of c(T)

❸ Theorem: DAC solves Berge-acyclic decompositions

f(T) ⟶ (T∪E, F) *Berge-acyclic cost function network*

∃ order $(X_1,\dots,X_m)$ on T∪E such that $X_1 \in T$,

$f(X_1)$ obtained by filtering DAC(T, f(T)∪{f(X_i) | X_i ∈ T})

=

$f(X_1)$ obtained by filtering DAC(T∪E, F∪{f(X_i) | X_i ∈ T})

❹ Theorem: VAC solves Berge-acyclic pb

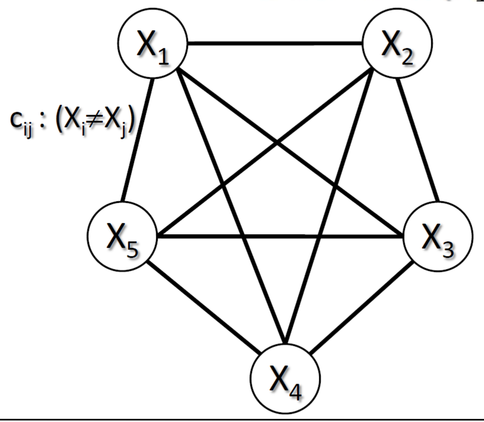
f(T) ⟶ (T∪E, F) *Berge-acyclic network*

$f_\varnothing$ by VAC(T, f(T)∪{f(X_i) | X_i ∈ T})

=

$f_\varnothing$ by VAC(T∪E, F∪{f(X_i) | X_i ∈ T})

softAllDiff(X_1, X_2, X_3, X_4, X_5)



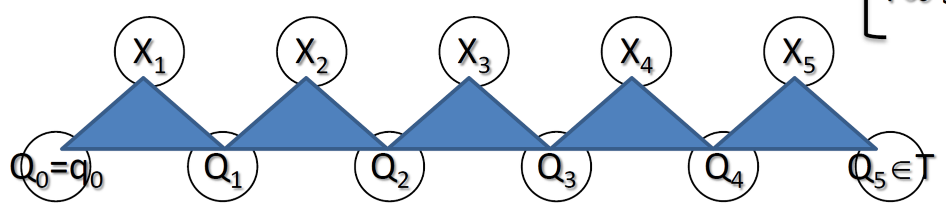
$c_{ij} : (X_i \neq X_j)$

$$\mathbf{g}\bullet c_{ij}(X_i, X_j) = \begin{cases} 0 & \text{si } X_i \neq X_j \\ 1 & \text{si } X_i = X_j \end{cases}$$

NON Berge-acyclic decomposition

softRegular(X_1, X_2, X_3, X_4, X_5, (Q, Σ, δ, q_0, T))

$$\mathbf{g}\bullet\delta(Q_{i-1}, X_i, Q_i) = \begin{cases} 0 & \text{si } (q_{i-1}, x_i, q_i) \in \delta \\ 1 & \text{si } (q_{i-1}, v, q_i) \in \delta, v \neq x_i \\ +\infty & \text{sinon} \end{cases}$$



$D_{Xi} \in Q$

$D_{Qi} \in \Sigma$

Berge-acyclic decomposition

(X,F) is Berge-acyclic iif its incidence graph (X ∪ F, E_F) is acyclic ({X_i,f(T)}∈E_F if X_i∈T)

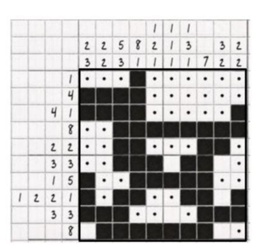
Experimental Results: Random Problem, Soft Nonogram, Market Split Problem, Crop Allocation Problem, Capacitated Warehouse Location Problem

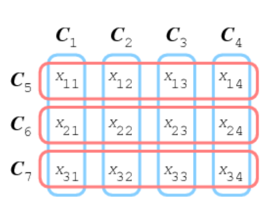
Simple problem with **1** random **softRegular** and random unary costs

n	|Σ|	|Q|	Monolithic filter	Monolithic solve	Decomposed filter	Decomposed solve
25	5	10	0.12	0.51	0.00	0.00
		80	2.03	9.10	0.08	0.08
25	10	10	0.64	2.56	0.01	0.01
		80	10.64	43.52	0.54	0.56
25	20	10	3.60	13.06	0.03	0.03
		80	45.94	177.5	1.51	1.55
50	5	10	0.45	3.54	0.00	0.00
		80	11.85	101.2	0.17	0.17
50	10	10	3.22	20.97	0.02	0.02
		80	51.07	380.5	1.27	1.31
50	20	10	15.91	100.7	0.06	0.07
		80	186.2	1,339	3.38	3.47

*All problems solved using **toulbar2** solver*
http://mulcyber.toulouse.inra.fr/projects/toulbar2

Nonogram
(CSPlib #12)
structure:





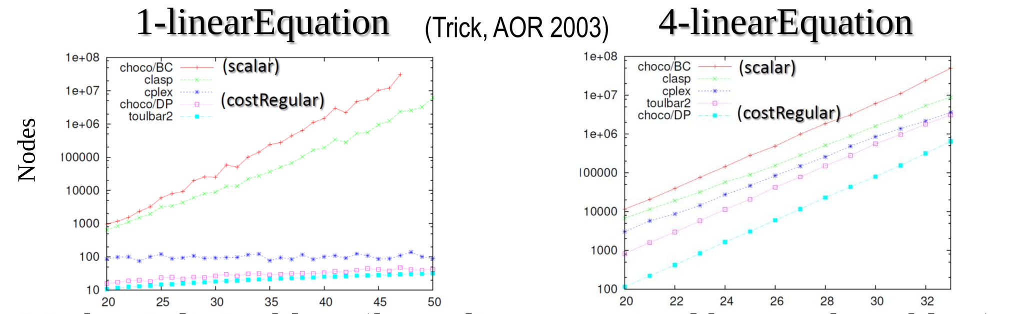
Randomized softened nonograms
2n softRegular

Size	Monolithic Solved	Monolithic Time	Decomposed Solved	Decomposed Time
6 × 6	100%	1.98	100%	0.00
8 × 8	96%	358	100%	0.52
10 × 10	44%	2,941	100%	30.2
12 × 12	2%	3,556	82%	1,228
14 × 14	0%	3,600	14%	3,316

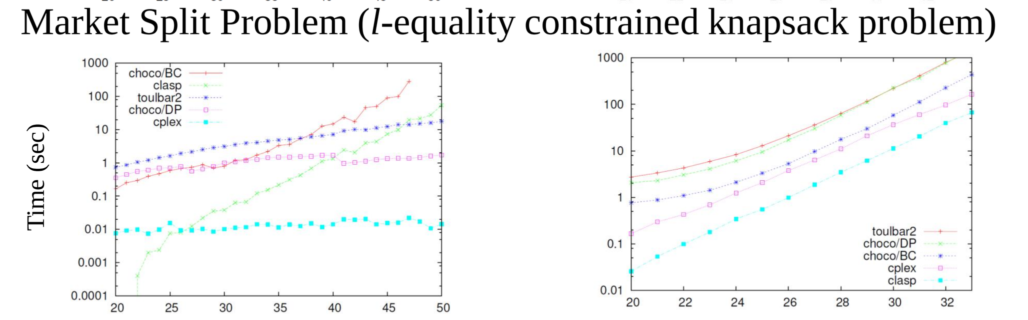
White-noise nonograms
2n regular with unary costs

Size	choco* (scalar) Solved	choco* (scalar) Time	toulbar2 (decomposed) Solved	toulbar2 (decomposed) Time
20 × 20	100%	1.88	100%	0.93
25 × 25	100%	14.78	100%	3.84
30 × 30	96%	143.6	96%	99.01
35 × 35	80%	459.9	94%	218.2
40 × 40	46%	1,148	66%	760.8
45 × 45	14%	1,627	32%	1.321

1-linearEquation (Trick, AOR 2003) 4-linearEquation



Market Split Problem (*l*-equality constrained knapsack problem)



*toulbar2 using exponential-size decomposed **sum** global constraint with domain size up to d=1000!*

Crop Allocation Farming Problem
using **regular**, **same** & **softGcc**

	n	e	Optimum	Monolithic Time(s)	Monolithic Nodes	Decomposed Time(s)	Decomposed Nodes	0/1 ILP (SCIP) Time(s)	0/1 ILP (SCIP) Nodes
B1234-LU15*	135	360	704	10.33	118	0.13	30	**0.09**	1
B1234-LU30*	270	712	1560	147.82	384	0.69	212	**0.33**	1
B1234-LU60*	540	1384	3852	2952.05	2700	**7.59**	2508	63.48	575
B1234-LU120*	1080	2728	7520	-	-	651.18	58192	**153.64**	932

Capacitated Warehouse Location Problem
using exponential-size decomposed **sum**

Problem Size	Mistral	Toulbar2* 0/1 vars	0/1 ILP (SCIP)
5 x 10	0.04 second	0.06 second	**0.01** second
	13,273 nodes	310 nodes	1 node
19 x 20	-	86.23 seconds	**0.03** second
	-	752,075	1 node

*Experiments using **NumberJack** (common model in python accessing multiple solvers)*

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