Multi-Paradigm Evaluation of (Exact) Solvers in Graphical Model Discrete Optimization

David Allouche[1],**Simon de Givry**[1], **Barry Hurley**[2],
George Katsirelos[1], **Barry O'sullivan**[2], **Thomas Schiex**[1]
[1]MIAT – INRA, Toulouse, France
[2]Insight Centre for Data Analytics, University College Cork, Irlande

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PLAN

- Combinatorial optimization languages
- Translations between formalisms
- Local consistency and dominance rules
- Benchmarks and experimental results
- Portfolio approach at UAI Competition 2014
- Conclusions

Combinatorial optimization languages

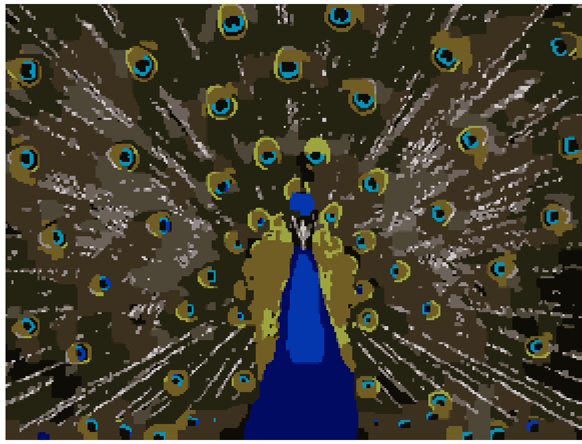
Graphical model (X,D,F)

- *X,* a set of ***n*** variables
- *D,* finite domains of maximum size ***d***
- $F=\{f_{S1},\dots,f_{Se}\}$, *a set of* ***e*** local functions

  with $S_i \subseteq X$ and maximum arity $\boldsymbol{a} = \max_{f_S \in F} |S|$,

  usually defined in extension

Markov Random Field (MRF)

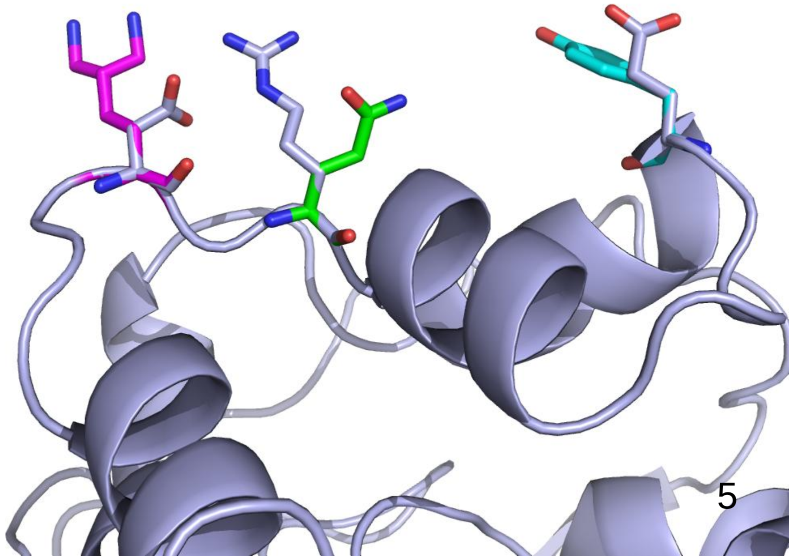


- MAP (maximum a posteriori) query

$$P(X = t) = \frac{\prod_{i=1}^{e} p_{S_i}\left(t[S_i]\right)}{Z} = \frac{\exp\left(-\sum_{i=1}^{e} f_{S_i}\left(t[S_i]\right)\right)}{Z}$$

NP-hard

→ solvers: daoopt, mplp2,..



Cost Function Network (CFN)

- Minimum cost assignment query
  (aka Weighted Constraint Satisfaction Problem)

$$score(X = t) = \sum_{i=1}^{e} f_{S_i}(t[S_i])$$

→solvers: toulbar2, gecode, mistral, opturion,..

Special cases:

- Boolean variables & weighted clauses (Max-SAT)
- 01 variables, linear constraints & objective (01LP)

NP-hard

→ solvers: maxhs, cplex,..

A Simple MRF Example

- Problem (X,D,*P*)
  - X= {x,y}
  - D_x={a,b}, D_y={a,b,c}
  - p(x,y)
    - p(x=a,y=a)=p(a,a)=p(b,b)=1
    - p(a,b)=p(b,a)=0.5
    - p(a,c)=p(b,c)=0

  - MAP solution (x=a, y=a) with normalized probability 1/3

An Equivalent CFN Example

- Problem (X,D,F)
  - X= {x,y}
  - D_x={a,b}, D_y={a,b,c}
  - f(x,y)
    - f(x=a,y=a)=f(a,a)=f(b,b)= 0
    - f(a,b)=f(b,a)= $\lfloor -100 \log(0.5) \rfloor$
    - f(a,c)=f(b,c)= $+\infty$

  - Optimal solution (x=a, y=a) with minimum cost 0

Translations between formalisms

Encodings

Instances\Output	MRF	CFN	Max-SAT	01LP	CP
MRF	-	Energies scaled to non negative integers	Through CFN	Through CFN	Through CFN
CFN	Exponentiating costs	-	**Direct/tuple encodings**	**Direct/tuple encodings**	**new cost variable & table constraint per cost function** (no large costs)
Max-SAT	Through CFN (large arity clauses cannot be represented in extension)	Direct (large arity clauses represented in a compact way)	-	Through CFN (tuple encoding cannot be used)	new cost variable & reified logical expression per weighted clause
CP	Through CFN	Global constraints decomposed into ternary constraints, objective variables *decomposed* into a sum of cost functions (no large domains)	Through CFN	Through CFN (global constraints not decomposed into linear constraints)	-

- CFN→CP: Petit, T., Regin, J., Bessiere, C.: *Meta constraints on violations for over constrained problems*. In Proceedings of IEEE ICTAI'2000, Vancouver, Canada (2000)
- CP→CFN: Allouche, D., Bessiere, C., Boizumault, P., de Givry, S., Gutierrez, P., Loudni, S., Metivier, J., Schiex, T. *Decomposing global cost functions*. In: Proc. of AAAI-12. Toronto, Canada (2012)

Direct Encoding from CFN to 01LP

- Variables and Domains

$\forall i \in X$ with $|D_i|>2$, $\forall r \in D_i$, integer variable $0 \leq d_{ir} \leq 1$ with $\sum_{r \in D_i} d_{ir} = 1$

- Cost Functions

$\forall f_S \in F$, $\forall t \in D_S$ with $|S|>1$, $0 < f_S(t) < +\infty$, variable $0 \leq p_{St} \leq 1$

with $\sum_{i \in S} (1 - d_{it[i]}) + p_{St} \geq 1$ and linear objective $\sum f_S(t)\, p_{St}$

Tuple Encoding from CFN to 01LP

- Variables and Domains

$\forall i \in X$ with $|D_i|>2$, $\forall r \in D_i$, integer variable $0 \leq d_{ir} \leq 1$ with $\sum_{r \in D_i} d_{ir} = 1$

- Cost Functions

$\forall f_S \in F$, $\forall t \in D_S$ with $|S|>1$, $0 \leq f_S(t) < +\infty$, variable $0 \leq p_{St} \leq 1$

with $\forall i \in S, \forall r \in D_i$ $d_{ir} = \sum_{t \in D_S,\ t[i]=r,\ f(t)<+\infty} p_{St}$

and linear objective $\sum f_S(t)\ p_{St}$

Equivalent to the *local polytope* in MRFs

An Equivalent CFN Example

- Problem (X,D,F)
  - X= {x,y}
  - D_x={a,b}, D_y={a,b,c}
  - f(x,y)
    - f(x=a,y=a)=f(a,a)=f(b,b)= 0
    - f(a,b)=f(b,a)= $\lfloor -100 \log(0.5) \rfloor = 30$
    - f(a,c)=f(b,c)= $+\infty$

  - Optimal solution (x=a, y=a) with minimum cost 0

An Equivalent 01LP Example

- Direct encoding
  - Domain variables: x, ya, yb, yc (x=0⇔x=a and x=1⇔x=b)
  - Tuple variables: p_{xayb}, p_{xbya}

  - Min $30*p_{xayb} + 30 * p_{xbya}$

  such that
    - $ya + yb + yc = 1$
    - $x - yb + p_{xayb} >= 0$
    - $-x - ya + p_{xbya} >= -1$
    - $x - yc >= 0$
    - $-x - yc >= -1$

Optimal solution (x=0, ya=1) with minimum cost 0

An Equivalent 01LP Example

- Tuple encoding
  - Domain variables: x, ya, yb, yc ($x=0 \Leftrightarrow x=a$ and $x=1 \Leftrightarrow x=b$)
  - Tuple variables: p_{xaya},p_{xayb} , p_{xbya},p_{xbyb}

  - Min $30*p_{xayb} + 30 * p_{xbya}$

  such that
    - $ya + yb + yc = 1$
    - $1 – x = p_{xaya} + p_{xayb}$
    - $x = p_{xbya} + p_{xbyb}$
    - $ya = p_{xaya} + p_{xbya}$
    - $yb = p_{xayb} + p_{xbyb}$
    - $yc = 0$

Weighted Partial MaxSAT

- X, a set of ***n*** Boolean variables
- $D = \{true, false\}^n$, finite domains of size ***2***
- $F=\{f_{S1},\dots,f_{Se}\}$, *a set of* ***e*** *weighted clauses* such that f_{Si} is associated to a clause $(l_1 \text{ or } l_2 \text{ or } \dots \text{ or } l_{|Si|})$
  f_{Si} : true → 0
  false → $\mathbf{N} \cup \{\infty\}$

$$score(X = t) = \sum_{i=1}^{e} f_{S_i}(t[S_i])$$

- Minimum cost assignment query (satisfying all the hard clauses)

NP-hard

An Equivalent MaxSAT Example

- Direct encoding
  - Domain variables: x, ya, yb, yc *(x=false ⇔x=a and x=true ⇔x=b)*

  Hard clauses:
    - (–ya or –yb), (–ya or –yc), (–yb or –yc)
    - (ya or yb or yc)
    - (x or –yc), (–x or –yc)

  Soft clauses:
    - (x or –yb, 30)
    - (–x or –ya, 30)

Optimal solution (–x, ya, –yb, –yc) with minimum cost 0

An Equivalent MaxSAT Example

- Tuple encoding
  - Domain variables: x, ya, yb, yc
  - Tuple variables: p_{xaya}, p_{xayb} , p_{xbya}, p_{xbyb}

  Hard clauses:
  - (–ya or –yb), (–ya or –yc), (–yb or –yc), (ya or yb or yc)
  - (–p_{xaya} or –x), (–p_{xaya} or ya), (–p_{xayb} or –x), (–p_{xayb} or yb), (–p_{xbya} or x), (–p_{xbya} or ya), (–p_{xbyb} or x), (–p_{xbyb} or yb)
  - (x or p_{xaya} or p_{xayb}), (–x or p_{xbya} or p_{xbyb}), (–ya or p_{xaya} or p_{xbya}), (–yb or p_{xayb} or p_{xbyb}), (–yc)

  Soft clauses:
  - (p_{xayb} , 30)
  - (p_{xbya}, 30)

Constraint Programming (CP)

- $X \cup F \cup \{o\}$, a set of ***n* + e + *1*** variables
- D, finite domains of maximum size ***d***
- $C=\{c_{S1},\ldots,c_{Se}\}$, a set of **e** constraints with $S_i \subseteq X \cup F$
- Goal constraint: $o = Sum_{f \in F} f$

Score(X=x, F=f, o=v) = v

- Minimum cost assignment query (satisfying all the constraints)

NP-hard

An Equivalent CP Example

- Problem (X,D,C)
  - X= {x,y,f,o}
  - D_x={a,b}, D_y={a,b,c} , D_f= [0,M], D_o= [0,M]
  - Constraint c(x,y,f) represented by a list of allowed tuples
    - c(x=a,y=a,f=0)=c(a,a,0), c(b,b,0),
    - c(a,b,30), c(b,a,30),
  - o = f

  - Optimal solution (x=a, y=a, f=0, o=0) with minimum cost 0

Local consistency and dominance rules

Local consistency by Equivalence Preserving Transformation

- Node consistency (NC)
  - $\forall f_x \in F, \forall a \in D_x, f_\varnothing + f_x(a) < k$
  - $\forall f_x \in F, \exists a \in D_x, f_x(a) = 0$
- Arc consistency (AC)
  - $\forall f_{xy} \in F, \forall a \in D_x, \exists b \in D_y, f_{xy}(a,b) = 0$
- Directional arc consistency (DAC)
  - $\forall f_{xy} \in F$ *such that* $x < y$, $\forall a \in D_x, \exists b \in D_y, f_{xy}(a,b) + f_y(b) = 0$
- Full directional arc consistency: FDAC = NC + AC + DAC
- Existential arc consistency (EAC) *Implied by strict arc consistency*
  - $\forall x \in X, \exists a \in D_x, \forall f_{xy} \in F, \exists b \in D_y, f_x(a) + f_{xy}(a, b) + f_y(b) = 0$
- Full existential directional arc consistency: EDAC = FDAC + EAC

Sequence of integer arc EPTs maximizing $f_\varnothing$ *is NP-hard*

(Cooper & Schiex, AI 2004)

Local consistency by Equivalence Preserving Transformation using rationals

- Optimal set of simultaneously applied EPTs (OSAC)
  - Dual of the *local polytope* 01LP relaxation
- Improving sequence of EPTs (VAC)
  - Augmenting DAG (Koval, Schlesinger, 1976)
  - Dynamic VAC (Nguyen et al., ICTAI 2014)

Cohérence locale	Complexity time	space	Classes polynomiales
NC*	$O(nd)$	$O(nd)$	-
BAC$^\emptyset$	$O(n^2d^3)$	$O(e)$	-
AC*	$O(n^2d^2 + ed^3)$	$O(ed)$	-
DAC	$O(ed^2)$	$O(ed)$	Tree
FDAC*	$O(end^3)$	$O(ed)$	Tree
EDAC*	$O(ed^2 \max(nd, k))$	$O(ed)$	
VAC$_\epsilon$	$O(ed^2k/\epsilon)$	$O(ed)$	Tree, submodular funct.
OSAC	$poly(ed + n)$	$poly(ed^2 + nd)$	Tree, submodular funct.

Dominance rules in comb. optimization Value substitutability

- AI & OR
  - E Freuder. *Eliminating interchangeable values in constraint satisfaction problems*. In AAAI 1991
  - A Koster. *Frequency assignment: Models and Algorithms*. Ph.D. thesis, 1999
  - R Niedermeier, P Rossmanith. *New upper bounds for maximum satisfiability*. J. Algorithms 36(1), 2000
  - S Bistarelli, B Faltings, N Neagu. *Interchangeability in Soft CSPs*. In CP 2002
  - A Jouglet, J Carlier. *Dominance rules in combinatorial optimization problems*. EJOR 212(3), 2011
  - G Chu, P Stuckey. *A generic method for identifying and exploiting dominance relations*. In CP 2012
  - C Lecoutre, O Roussel, D Dehani. *WCSP Integration of Soft Neighborhood Substitutability*. In CP 2012
- Computational Protein Design
  - J Desmet, M Maeyer, B Hazes, I Lasters,. *The dead-end elimination theorem and its use in protein side-chain positioning*. Nature 356, 1992
  - R Goldstein. *Efficient rotamer elimination applied to protein side-chains and related spin glasses*. Biophysical Journal 66(5), 1994
  - N Pierce, J Spriet, J Desmet, S Mayo. *Conformational splitting: A more powerful criterion for dead-end elimination*. Journal of Computational Chemistry. 21(11), 2000
  - I Georgiev, R Lilien, B Donald. *Improved pruning algorithms and divide-and-conquer strategies for dead-end elimination, with application to protein design*. Bioinformatics 22(14), 2006
- Computer Vision and Pattern Recognition
  - A Shekhovtsov. Exact and Partial Energy Minimization in Computer Vision, PhD, 2013
  - P Swoboda, B Savchynskyy, J H. Kappes, C Schnörr. Partial Optimality by Pruning for MAP-inference with General Graphical Models. In CVPR 2014

Dead-End Elimination rules

Prune value (*x,b*), dominated by (*x,a*) if:

- **Rule 1**: (Desmet *et al*, Nature 1992)

$$\sum_{f_S \in \Gamma(x)} \max_{t \in l(S \setminus \{x\})} f_S(t \cup \{(x,a)\}) \le \sum_{f_S \in \Gamma(x)} \min_{t \in l(S \setminus \{x\})} f_S(t \cup \{(x,b)\})$$

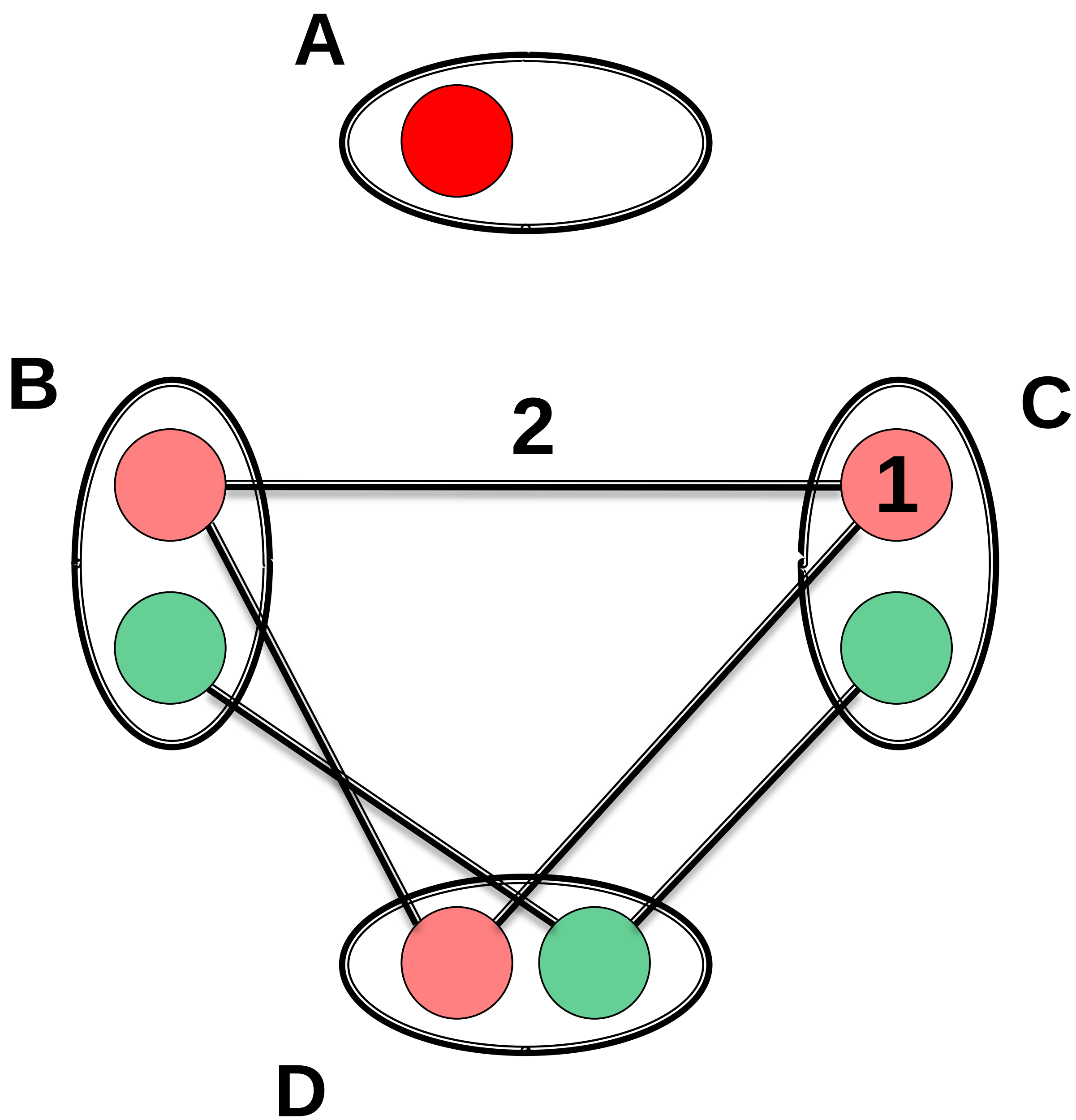
- **Rule 2**: (Goldstein, Bio. J. 1994) (Koster, 1999)

$$\sum_{f_S \in \Gamma(x)} \max_{t \in l(S \setminus \{x\})} f_S(t \cup \{(x,a)\}) - f_S(t \cup \{(x,b)\}) \le 0$$

Rule 2 is always stronger than rule 1 and it has been improved in (Givry et al, CP 2013 ; Allouche et al., AI 2014)

Rule 1&2 enforced partially in $O(ned^2)$

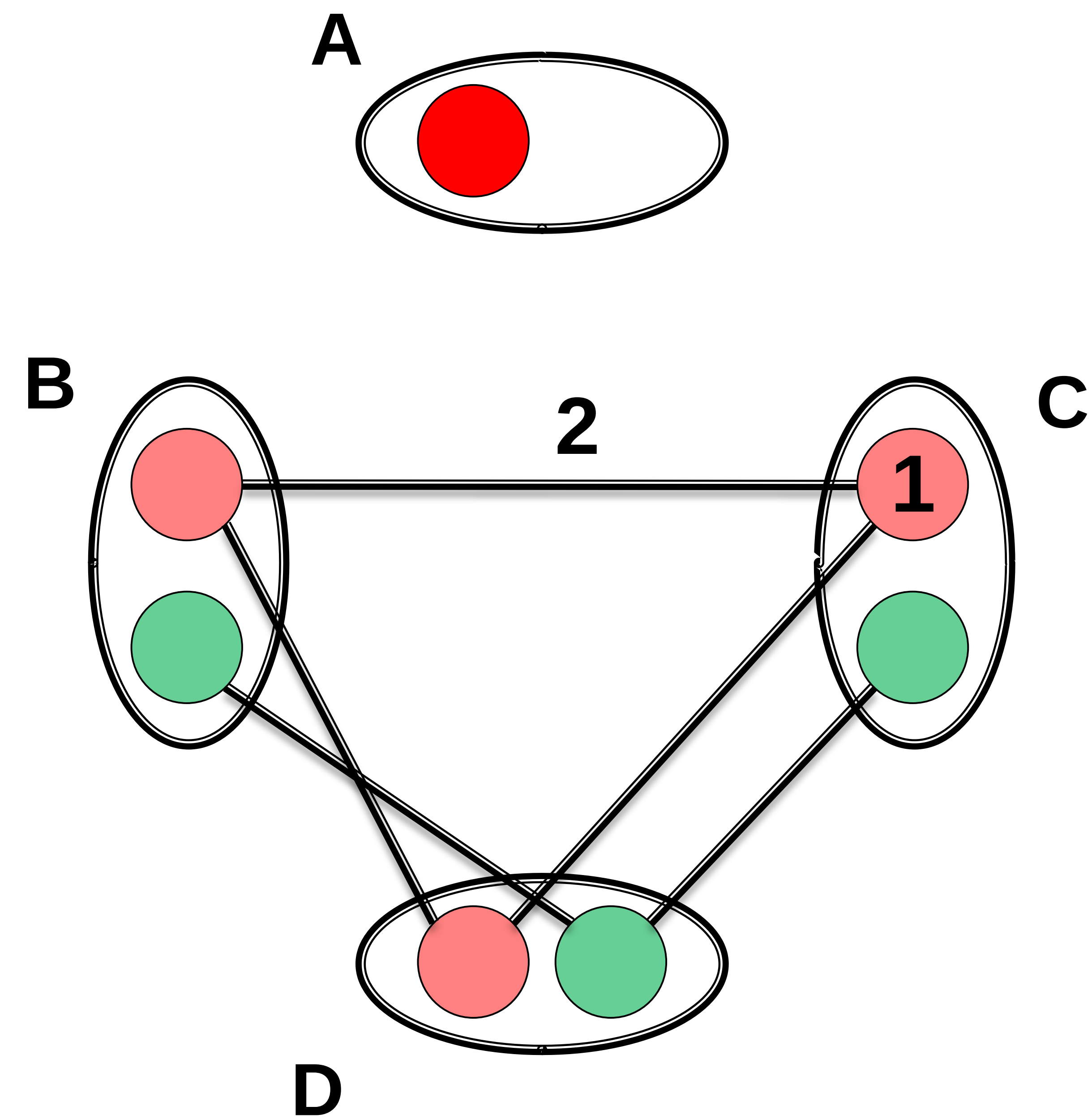
Pruning by dominance



$f_\varnothing = 1$

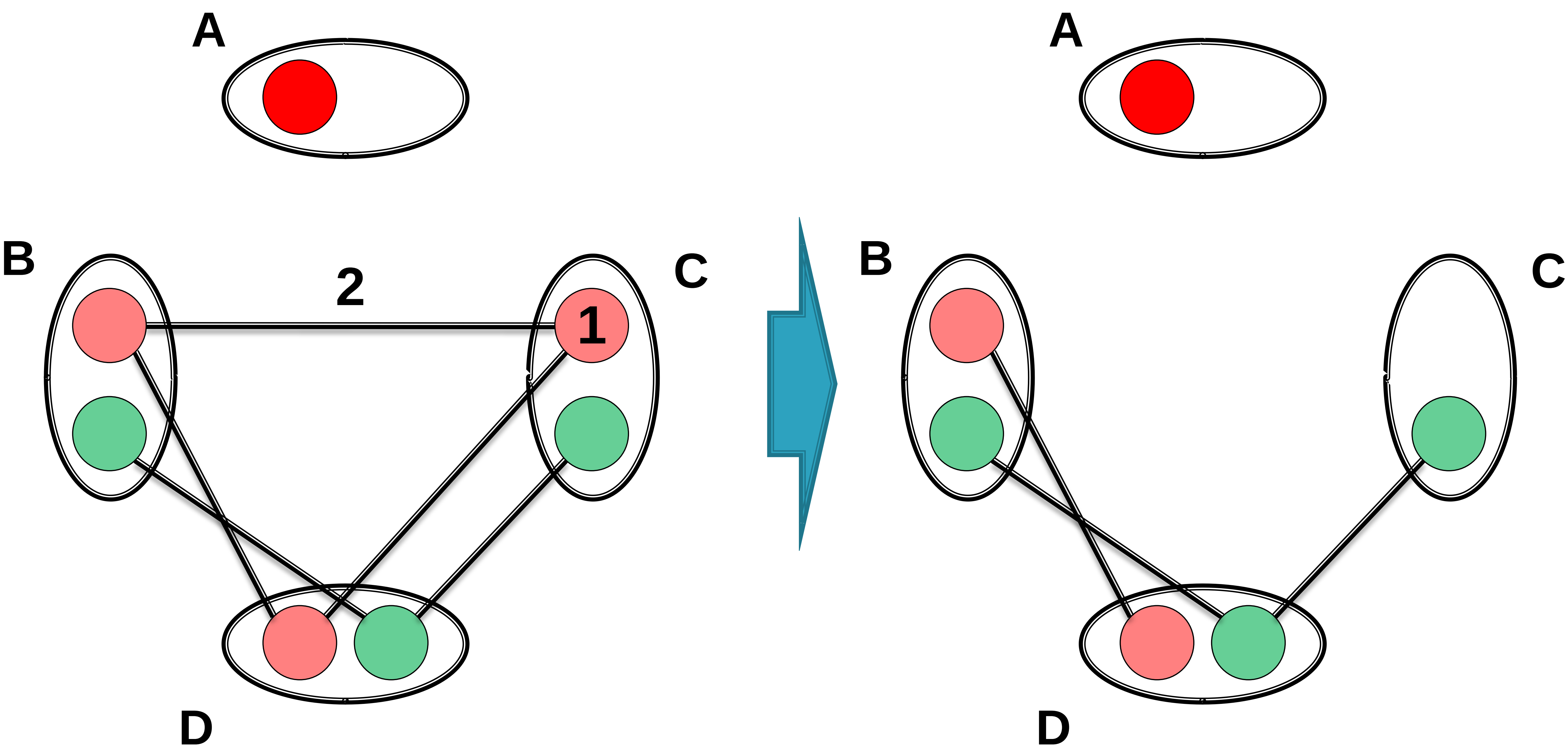
Problem is EDAC

$k = 5$



$f_\varnothing = 1$

Pruning by dominance



$f_{\varnothing}$ = 1

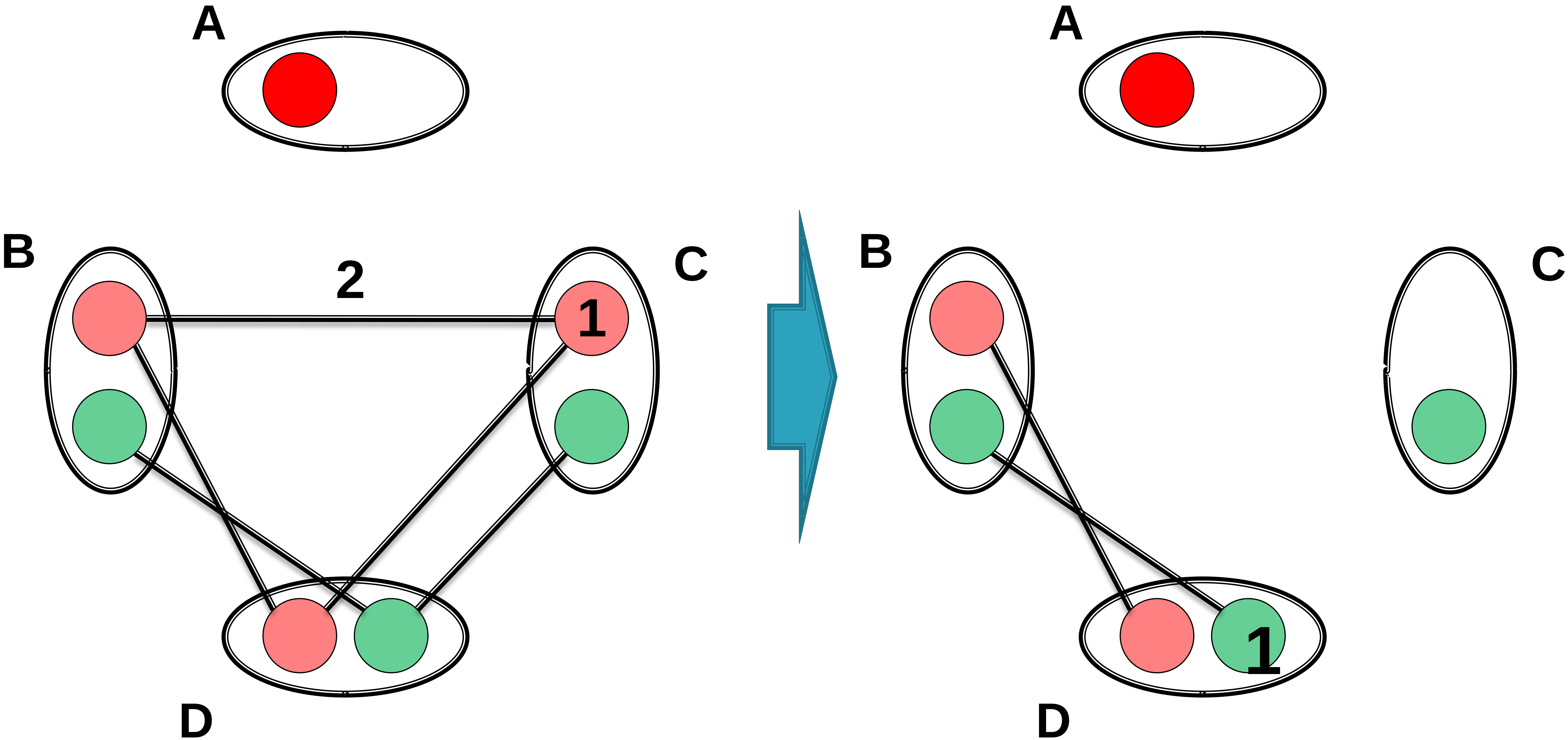
Problem is EDAC

k = 5

Rule 1 and Rule 2(+) find
(*C*,*red*) is dominated by (*C*,*green*)

$f_{\varnothing}$ = 1

Pruning by dominance

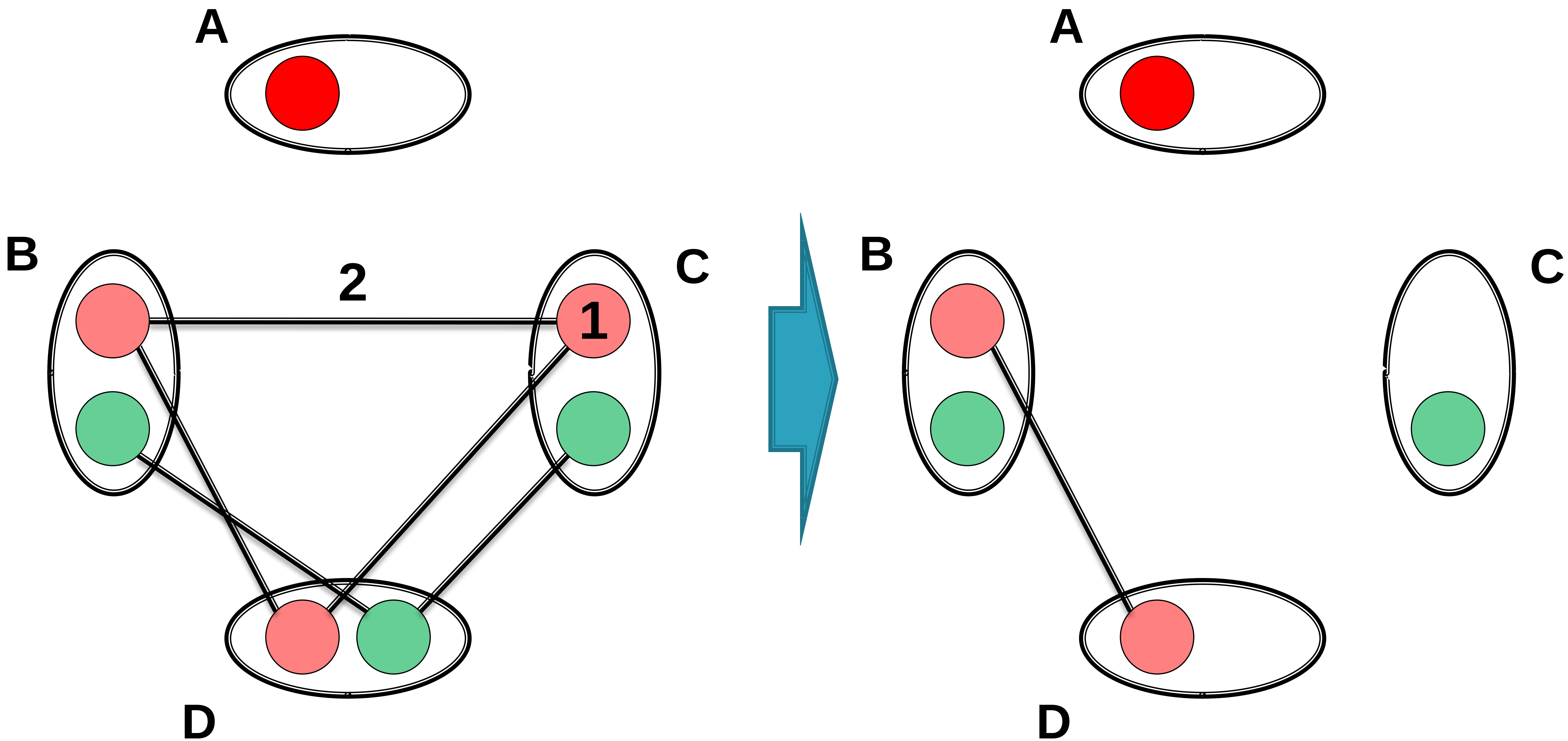


$f_\varnothing = 1$ $k = 5$ $f_\varnothing = 1$

Problem is EDAC

Rule 1 and Rule 2(+) find (*C*,*red*) is dominated by (*C*,*green*)

Pruning by dominance

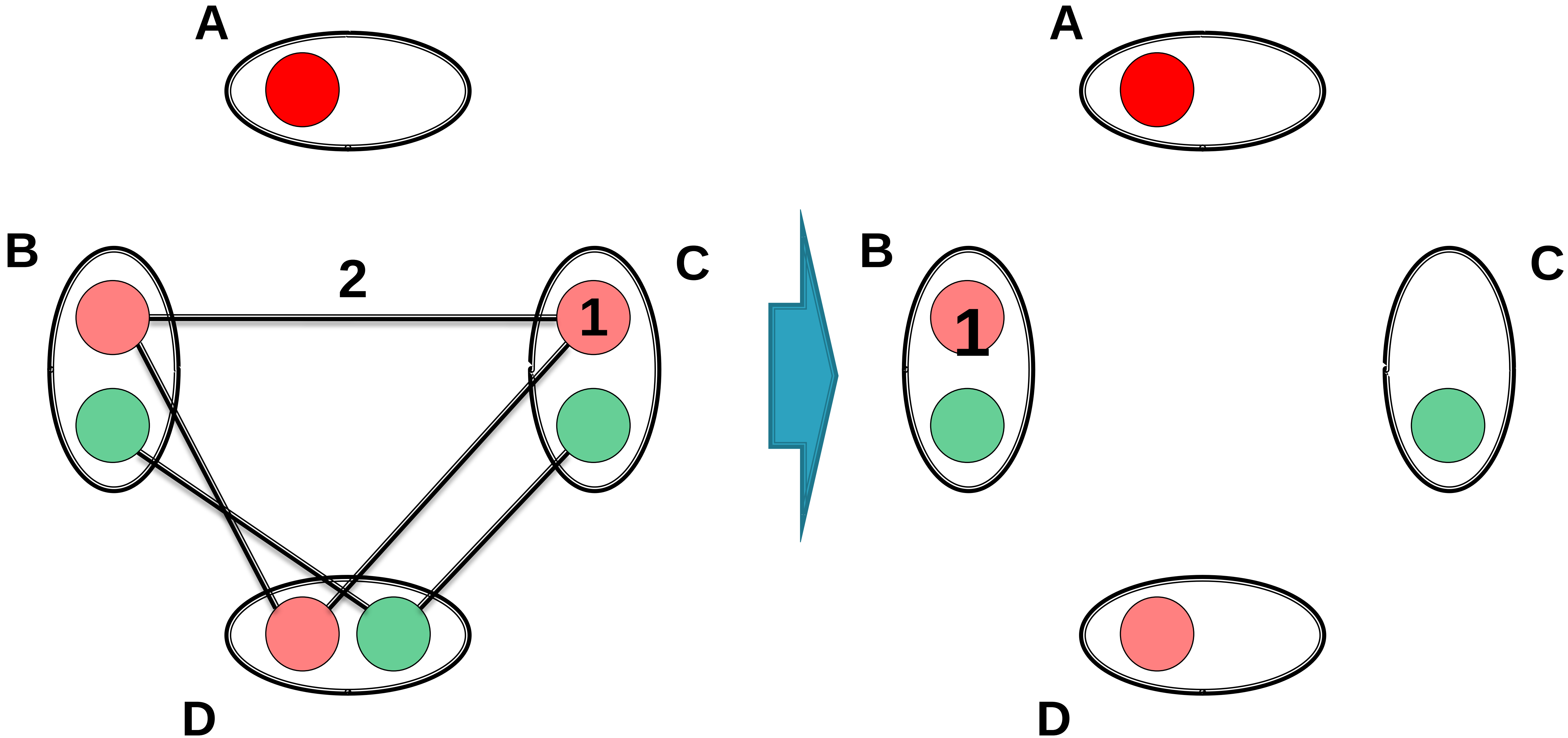


$f_{\varnothing}$ = 1 **k = 5** **$f_{\varnothing}$ = 1**

Problem is EDAC

Rule 1 and Rule 2(+) find
(*C*,*red*) is dominated by (*C*,*green*)

Pruning by dominance



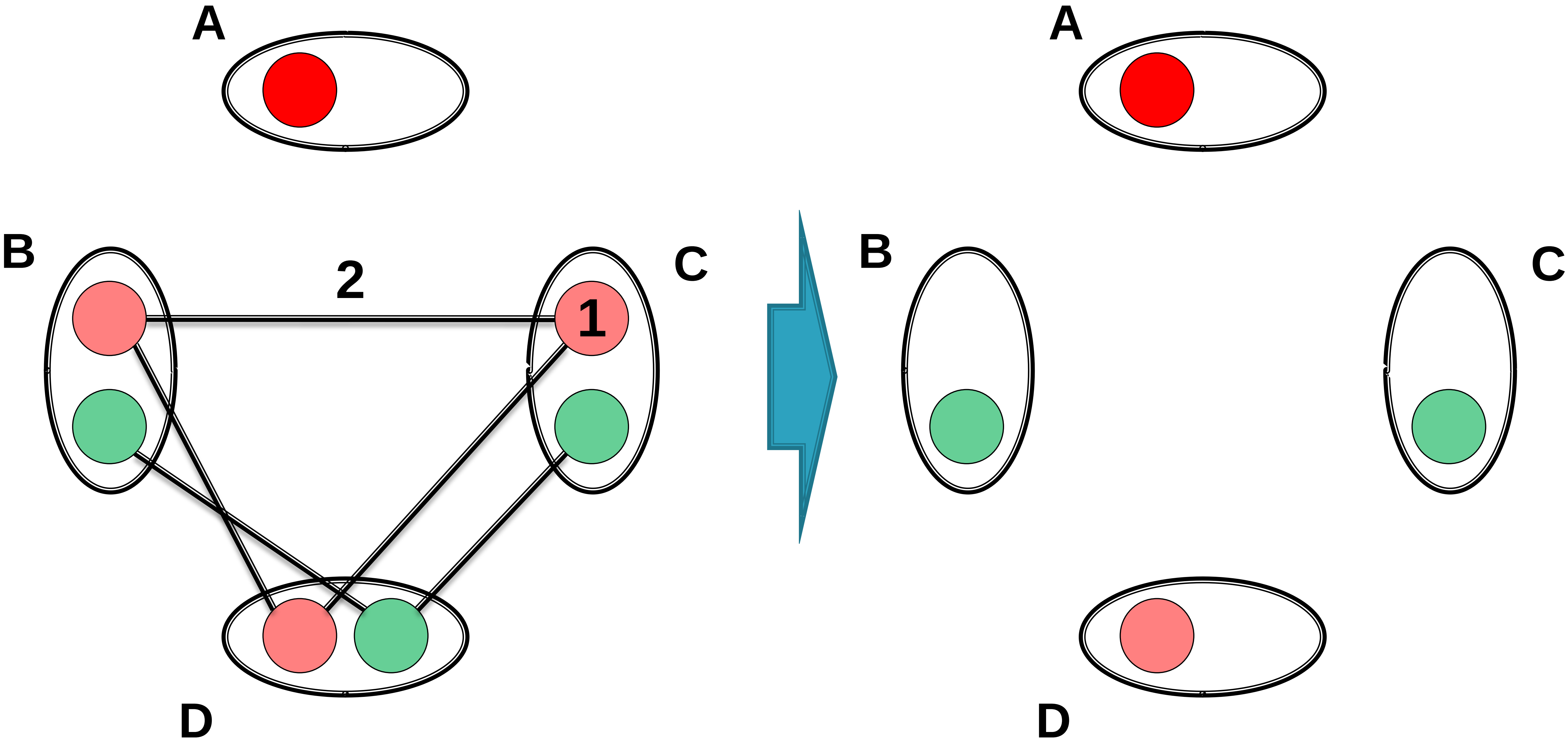
$f_\varnothing = 1$ $k = 5$ $f_\varnothing = 1$

Problem is EDAC

Rule 1 and Rule 2(+) find (*C*,*red*) is dominated by (*C*,*green*)

DEE cannot improve VAC lb (only speed-up)

Pruning by dominance



$f_\varnothing$ = 1 **k = 5** **$f_\varnothing$ = 1**

Problem is EDAC

Rule 1 and Rule 2(+) find
(*C*,*red*) is dominated by (*C*,*green*)

Benchmarks and experimental results

Benchmarks & Solvers

- MRF
  - UAI Evaluation 2008-2010 (genetic linkage analysis)
    - Winner 2010 (20min): **toulbar2**
  - Probabilistic Inference Challenge 2011
    - winner: **daoopt***
  - Computer Vision and Pattern Recognition OpenGM2 2013
    - winner: **TRWS, MCA, mplp2,..**

Problem	#inst.	n	d	e	a
MRF (`uai`)	319				
Linkage	22	1289	7	2184	5
DBN	108	1094	2	22793	2
Grid	21	6400	2	19200	2
ImageAlignment	10	400	93	3563	2
ObjectDetection	37	60	21	1830	2
ProteinFolding	21	1972	503	8816	2
Segmentation	100	237	21	886	2
MRF/CVPR (`hdf5`)	1453				
ChineseChars	100	17856	2	553726	2
ColorSeg	3	414720	4	2069714	2
ColorSeg-4	9	86400	12	258600	2
ColorSeg-8	9	86400	12	430202	2
GeomSurf-3	300	1133	3	5039	3
GeomSurf-7	300	1133	7	5039	3
InPainting-4	2	14400	4	42960	2
InPainting-8	2	14400	4	71282	2
Matching	4	20	20	210	2
MatchingStereo	2	166222	20	497849	2
ObjectSeg	5	68160	8	203947	2
PhotoMontage	2	514080	7	1540689	2
SceneDecomp	715	208	8	769	2

Benchmarks & Solvers

- WCSP
  - CFNLib
  - Max-CSP
    - Winner Max-CSP Competition 2008 : toulbar2

Problem	#inst.	n	d	e	a
CFN (wcsp)	281				
Auction *	170	246	2	11528	2
CELAR *	16	458	44	2335	2
Pedigree *	10	10017	28	18875	3
ProteinDesign	10	18	198	171	2
SPOT5	20	1057	4	21786	3
Warehouse *	55	1100	300	101100	2
Max-CSP (xcsp)	503				
BlackHole	37	205	50	1651	2
Coloring	22	450	6	6164	2
Composed	80	83	10	785	2
EHI	200	315	7	4715	2
Geometric	100	50	20	605	2
Langford	4	33	29	517	2
QCP	60	264	9	2662	2

*: ad-hoc encoding in minizinc

Benchmarks & Solvers

- Max-SAT
  - Max-SAT Evaluation 2013
    - Winner crafted instances : maxhs

Problem	#inst.	n	d	e	a
WPMS (`wcnf`)	427				
MIPLib	12	24776	2	107956	93
MaxClique	62	3321	2	378247	2
Haplotyping	100	216117	2	1188223	580
PackupWeighted	99	25554	2	70677	177
PlanningWithPref	29	69409	2	771883	372
TimeTabling	25	903884	2	2912882	36
Upgradeability	100	18169	2	105097	77

Benchmarks & Solvers

- CP
  - CSP Competition 2009 (no instances)
    - Winner (constraints in extension): **mistral**
  - MiniZinc Challenge 2012&2013
    - Winner free search 2012 : **gecode**
    - Winner free search 2013 : **opturion/cpx**

Problem	#inst.	n	d	e	a
CP (`minizinc`)	35				
AMaze	6	1573	17	3173	4
FastFood	6(1)	2	5	3	2
Golomb	6(3)	44	163	717	3
OnCallRostering	5(3)	2205	89	4513	4
ParityLearning	7	759	20	1440	4
VehicleRoutingProb.	5	11531	100	22999	4

Experimental settings

- 2-digit precision for MRF instances
  (CP solver domains on 32-bit)
- Default parameters (except daoopt, toulbar2 v0.9.6)
- 1 AMD Operon 6176 à 2.3 GHz et 8GB
- Time limit: 20 minutes (except CVPR: 1 hour)

- 01LP solver cplex version 12.4 (EP(A)GAP,EPINT set to zero)

- Benchmarks & detailed results:
  http://genoweb.toulouse.inra.fr/~degivry/evalgm

Problem	Nb.	daoopt	mplp2	toulbar2	cplex	$cplex_t$	maxhs	$maxhs_t$	opturion	gecode	mistral
MRF (uai)	319	144	123	**219**	152	205	104	72	1	0	1
Linkage	22	16	1	13	14	**22**	20	20	0	0	1
DBN	108	60	0	**77**	64	65	30	0	0	0	0
Grid	21	2	2	0	**15**	0	4	2	0	0	0
ImageAlignment	10	9	**10**	**10**	0	9	0	0	0	0	0
ObjectDetection	37	0	0	0	0	0	0	0	0	0	0
ProteinFolding	21	0	10	**19**	9	9	0	0	0	0	0
Segmentation	100	57	**100**	**100**	50	**100**	50	50	1	0	0
WPMS (wcnf)	427	11	0	195	269	N/A	**277**	N/A	27	19	15
MIPLib	12	2	0	**3**	**3**	N/A	**3**	N/A	2	**3**	2
MaxClique	62	9	0	33	**38**	N/A	36	N/A	10	15	13
Haplotyping	100	N/A	N/A	1	18	N/A	**25**	N/A	MZN	MZN	MZN
PackupWeighted	99	N/A	N/A	52	**99**	N/A	85	N/A	1	0	0
PlanningWPref	29	N/A	N/A	6	11	N/A	**27**	N/A	14	1	0
TimeTabling	25	N/A	N/A	0	0	N/A	**1**	N/A	MZN	MZN	MZN
Upgradeability	100	N/A	N/A	**100**	**100**	N/A	**100**	N/A	N/A	N/A	N/A
CFN (wcsp)	281	188	44	**247**	241	235	227	195	70	130	111
Auction	170	158	0	167	**170**	**170**	**170**	166	61	104	108
CELAR	16	3	0	**12**	0	3	0	0	1	0	1
Pedigree	10	4	0	**10**	5	9	5	5	4	0	0
ProteinDesign	10	4	7	**9**	0	6	0	0	0	0	0
SPOT5	20	6	0	4	**16**	10	5	5	1	0	2
Warehouse	55	13	37	45	**50**	37	47	19	3	26	0
Max-CSP (xcsp)	503	173	0	216	199	50	**238**	150	118	6	96
BlackHole	37	10	0	10	**30**	10	10	10	10	0	10
Coloring	22	16	0	**17**	**17**	15	12	12	8	4	6
Composed	80	26	0	**80**	**80**	15	**80**	15	**80**	0	**80**
EHI	200	0	0	0	0	0	0	0	0	0	0
Geometric	100	91	0	**93**	49	0	88	77	9	0	0
Langford	4	**2**	0	**2**	**2**	1	**2**	**2**	1	**2**	0
QCP	60	28	0	14	21	9	**46**	34	10	0	0
CP (minizinc)	35	10	1	13	2	5	16	8	18	**26**	18
AMaze	6	0	0	2	0	2	**6**	3	5	4	2
FastFood	6	1	1	1	1	1	1	1	**6**	**6**	5
Golomb	6	0	0	3	0	0	3	1	4	**6**	5
OnCallRoster.	5	2	0	2	1	2	**3**	**3**	2	2	2
ParityLearning	7	**7**	0	5	0	0	3	0	1	**7**	4
VRP	5	0	0	0	0	0	0	0	0	**1**	0
Nb. of 1st Pos.	33	2	2	**13**	12	3	11	2	2	6	1

Problem	Nb.	daoopt	mplp2	toulbar2	cplex	cplex$_t$	maxhs	maxhs$_t$	trws	bundle	mca	mcbc
CVPR (hdf5)	1453	1272	**1339**	1301	372	1329	311	1008	728	1143	26	56
ChineseChars	100	0	0	0	0	0	0	0	0	N/A	N/A	**56**
ColorSeg	3	1	**3**	0	0	1	0	0	1	0	**3**	N/A
ColorSeg-4	9	0	**8**	0	0	3	0	0	7	7	**8**	N/A
ColorSeg-8	9	0	4	0	0	2	0	0	2	1	**8**	N/A
GeomSurf-3	300	**300**	**300**	**300**	**300**	**300**	236	291	N/A	277	N/A	N/A
GeomSurf-7	300	252	**300**	281	72	**300**	74	2	N/A	180	N/A	N/A
InPainting-4	2	0	**1**	**1**	0	**1**	0	0	**1**	**1**	**1**	N/A
InPainting-8	2	0	**1**	0	0	0	0	0	0	0	**1**	N/A
Matching	4	**4**	**4**	**4**	0	3	0	0	0	0	N/A	N/A
MatchingStereo	2	0	0	0	0	0	0	0	0	**1**	N/A	N/A
ObjectSeg	5	0	3	0	0	4	0	0	**5**	3	**5**	N/A
PhotoMontage	2	0	0	0	0	0	0	0	0	0	N/A	N/A
SceneDecomp	715	**715**	**715**	**715**	0	**715**	1	**715**	712	673	N/A	N/A
Nb. of 1st Pos.	13	3	**8**	4	1	4	0	1	2	2	6	1

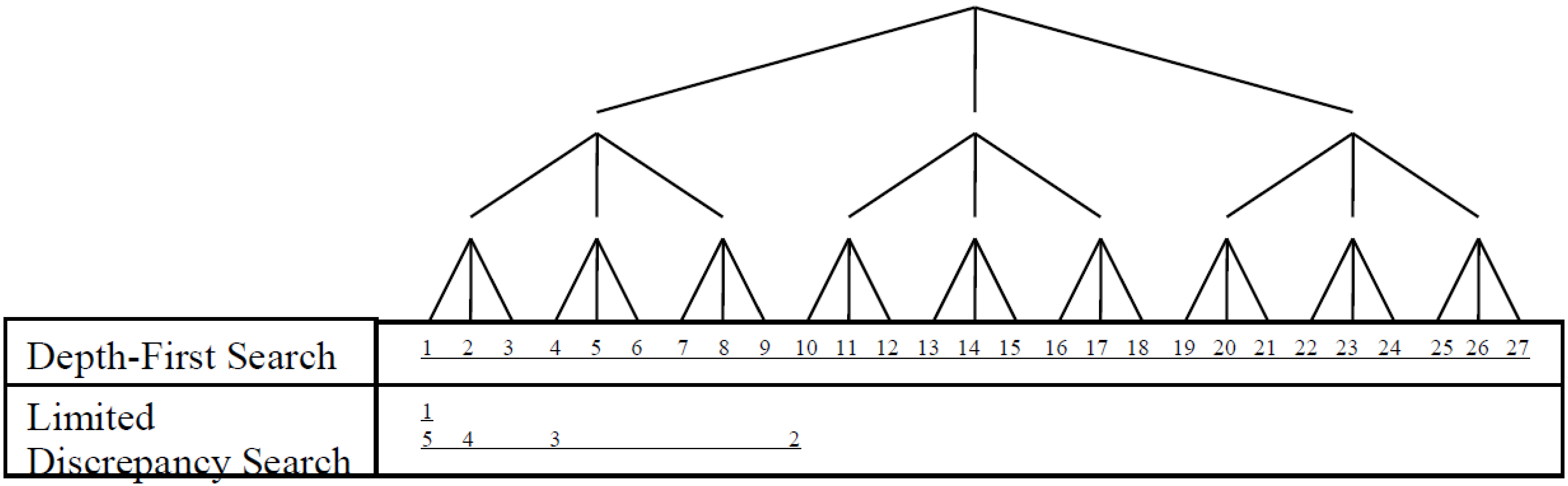
DEE reduces ChineseChars from 17,856 variables to at most 665 unassigned variables toulbar2 version 0.9.7

Problem	Nb.	mplp2	cplex$_t$	toulbar2	tb2-vac-lds1 (options –A –V –l=1)	tb2-vac-incop-lds1 (options –A –V –i –l=1)	tb2-ternary (option –t)
CVPR (`hdf5`)	1453	**1339**	1329	1301	1337	1332	1323
ChineseChars	100	0	0	0	0	0	**9 (256)**
ColorSeg	3	**3 (46.3)**	1 (71.9)	0	2 (1447)	1 (118)	0
ColorSeg-4	9	**8 (1202)**	3 (201)	0	6 (1463)	4 (1400)	1 (59)
ColorSeg-8	9	4 (1577)	2 (271)	0	**5 (1314)**	5 (1907)	0
GeomSurf-3	300	300 (0.43)	300 (0.33)	**300 (0.03)**	300 (0.06)	300 (0.06)	**300 (0.03)**
GeomSurf-7	300	300 (4.06)	300 (3.71)	281 (77.2)	**300 (2.67)**	300 (6.01)	293 (22.9)
InPainting-4	2	1 (157)	1 (189)	**1 (1.46)**	1 (12.5)	1 (34.9)	1 (1.90)
InPainting-8	2	**1 (915)**	0	0	0	1 (982)	0
Matching	4	4 (9.82)	3 (747)	**4 (7.75)**	4 (9.32)	4 (10.1)	4 (41.8)
MatchingStereo	2	0	0	0	0	0	0
ObjectSeg	5	3 (1791)	4 (366)	0	**4 (2762)**	1 (3454)	0
PhotoMontage	2	0	0	0	0	0	0
SceneDecomp	715	715 (0.13)	715 (0.82)	**715 (0.03)**	715 (0.09)	715 (0.09)	**715 (0.03)**
Nb. best sol.	1453	1337	*1338*	1306	1339	1401	1327

Brain0_9mm solved in 2700 seconds with tb2-vac,
DEE reducing from 785,540 to 62,687 unassigned vars

GeomSurf-7 in 300 (1.08) with tb2-dynamic-vac

Limited Discrepancy Search



Limited Discrepancy Search. W. Harvey, M. Ginsberg. Proc. of IJCAI 1995

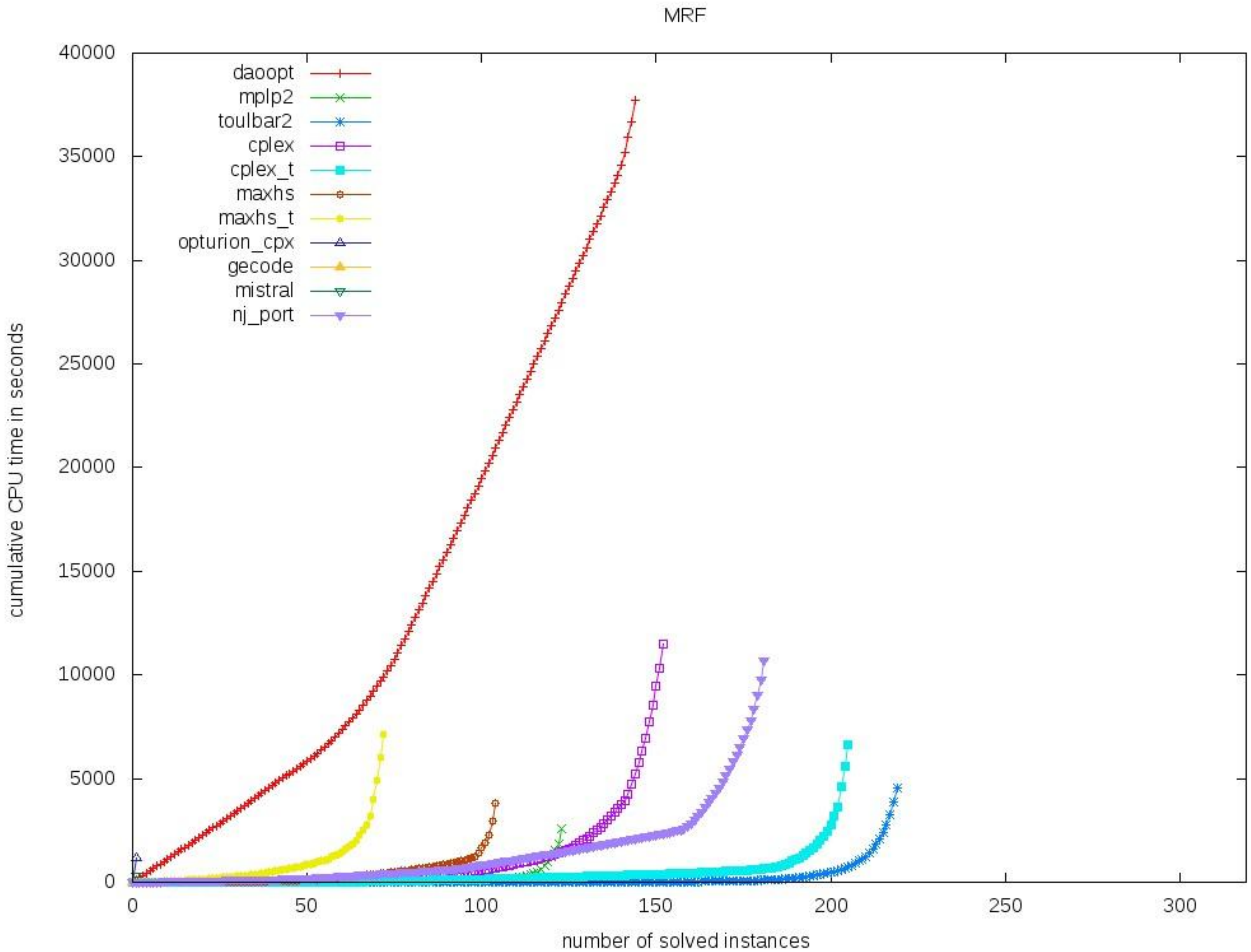
INCOP local search *IDWalk*

- *IDWalk* performs *S* moves and returns the best solution found during the walk.
- A move examines at most *Max* candidate neighbors at random (flips among variables in conflicts):
  - If the cost of a neighbor is less than or equal to the cost of the current solution, then it is selected (intensification)
  - If no neighbors are selected, then chose one at random (diversification)

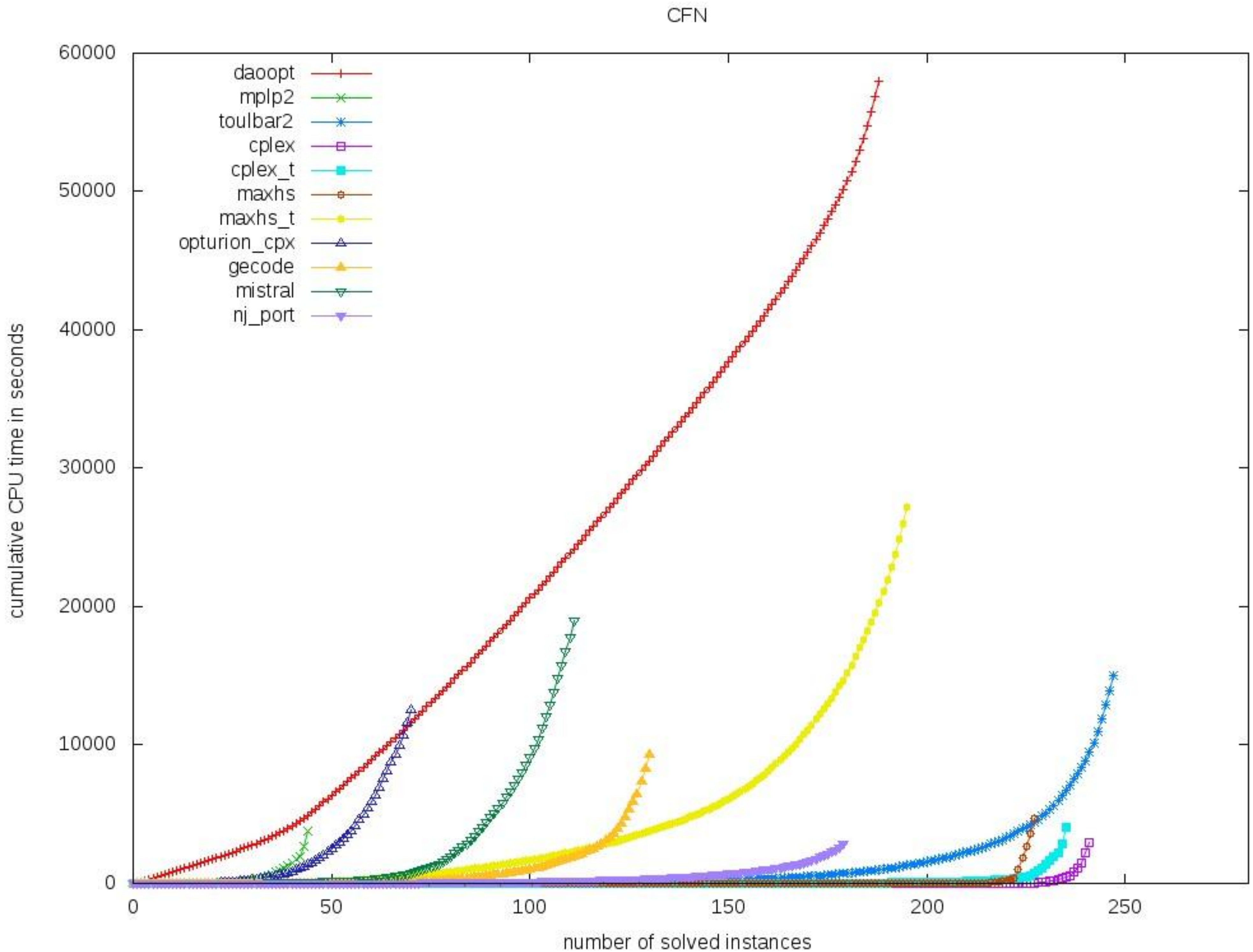
ID Walk: a Candidate List Strategy with a Simple Diversification Device.
B. Neveu, G. Trombettoni, F. Glover. LNCS 3258, Springer, p. 423--437, CP 2004

S= 100,000 ; *Max*=200 ; 3 repeats

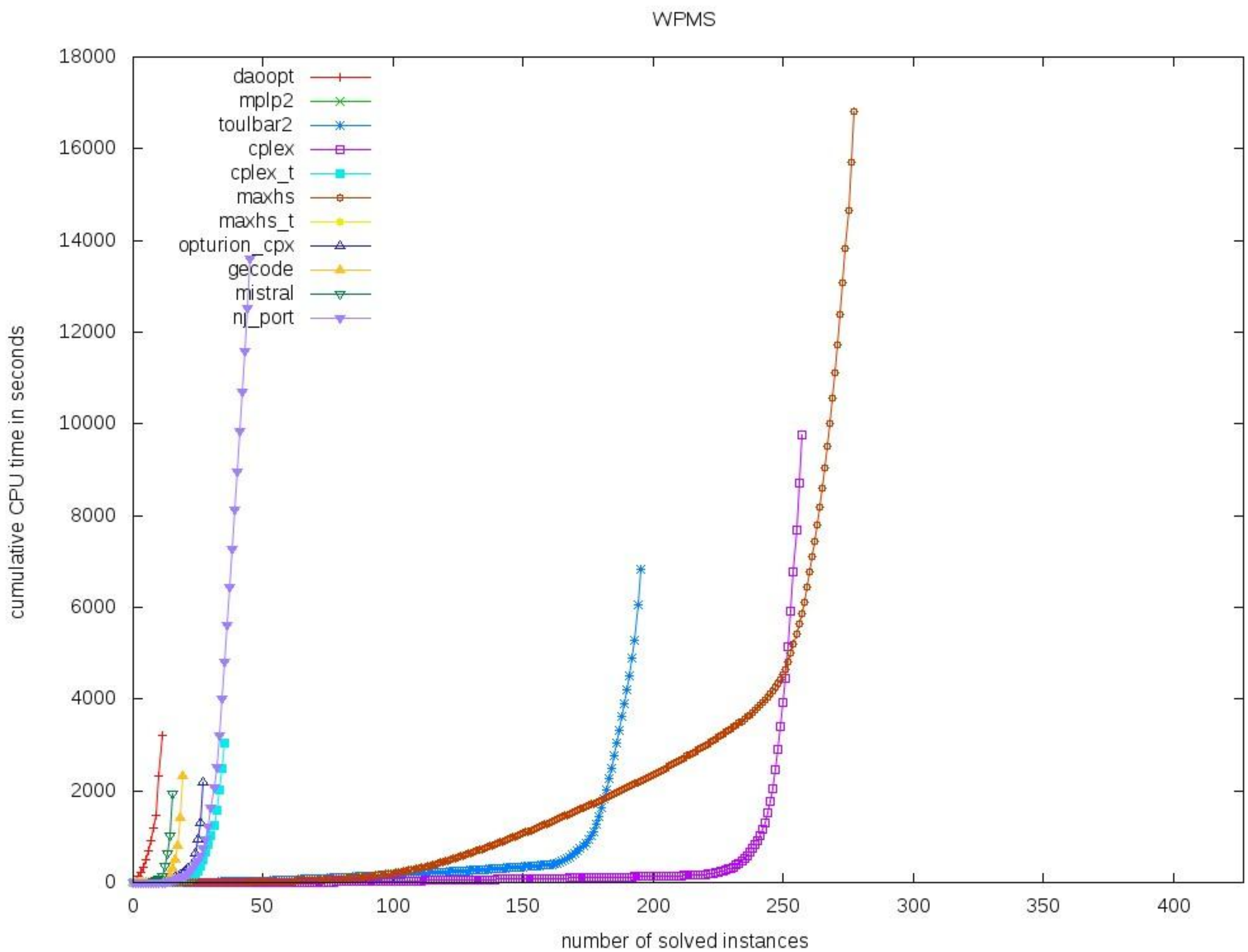
MRF


MRF
cumulative CPU time in seconds
number of solved instances
0
5000
10000
15000
20000
25000
30000
35000
40000
0
50
100
150
200
250
300
daoopt
mplp2
toulbar2
cplex
cplex_t
maxhs
maxhs_t
opturion_cpx
gecode
mistral
nj_port

CFN

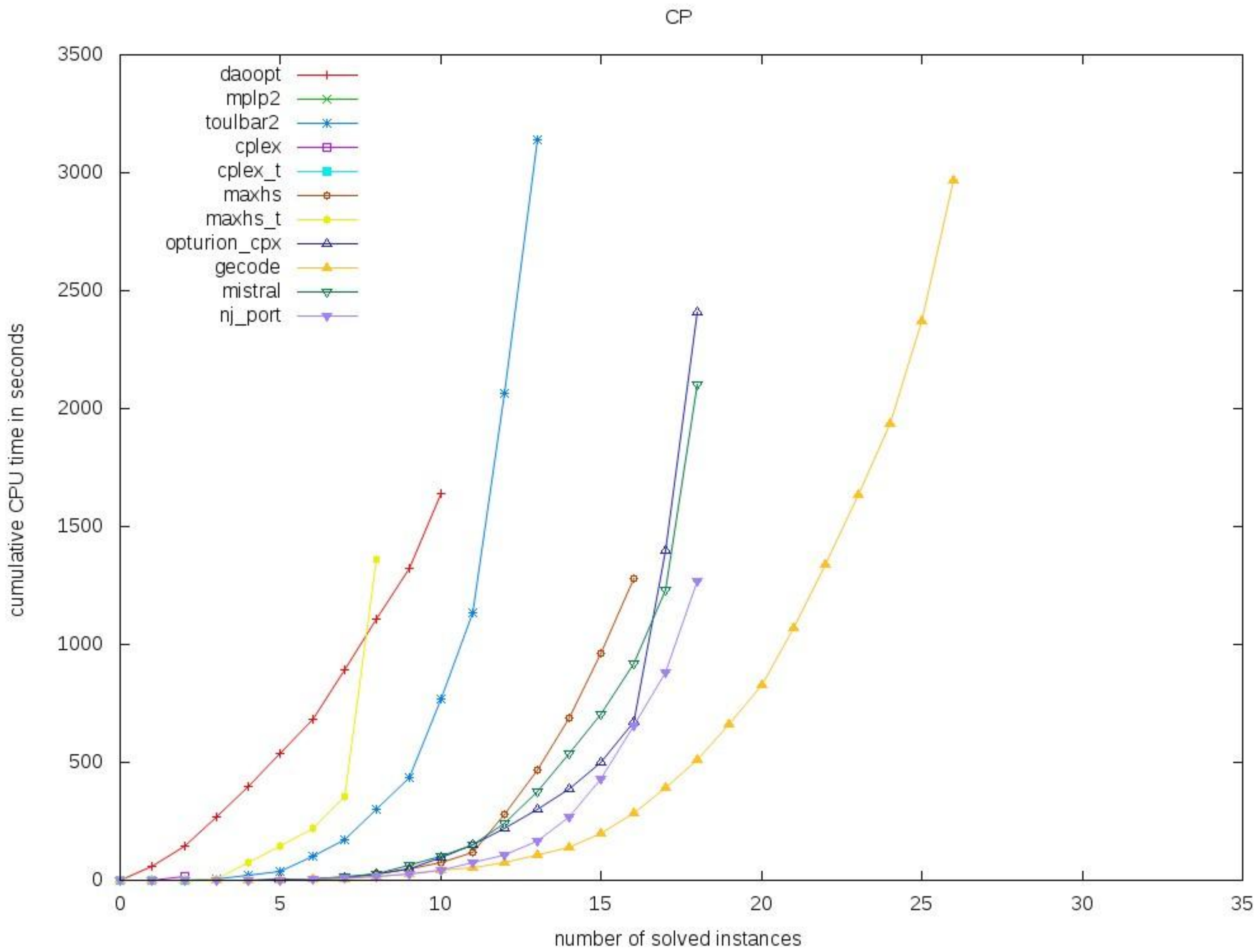

CFN
daoopt
mplp2
toulbar2
cplex
cplex_t
maxhs
maxhs_t
opturion_cpx
gecode
mistral
nj_port
cumulative CPU time in seconds
number of solved instances
0
10000
20000
30000
40000
50000
60000
0
50
100
150
200
250

Max-SAT



WPMS
daoopt
mplp2
toulbar2
cplex
cplex_t
maxhs
maxhs_t
opturion_cpx
gecode
mistral
nj_port
cumulative CPU time in seconds
number of solved instances
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
0
50
100
150
200
250
300
350
400

CP



CP
daoopt
mplp2
toulbar2
cplex
cplex_t
maxhs
maxhs_t
opturion_cpx
gecode
mistral
nj_port
cumulative CPU time in seconds
number of solved instances
0
500
1000
1500
2000
2500
3000
3500
0
5
10
15
20
25
30
35

Conclusions

- 3018 instances in 5 formats (uai,wcsp,wcnf,lp,mzn)
  http://genoweb.toulouse.inra.fr/~degivry/evalgm
  - Largest instances solved in 20min (mplp2): CVPR/ColorSeg (n=414720, d=4)
  - Smallest instances unsolved in 20min : MRF/ObjectDetection (n=60,d=21)
- toulbar2 solver https://mulcyber.toulouse.inra.fr/projects/toulbar2/
- Multi-solver approach using numberjack:
  http://numberjack.ucc.ie/
  CP platform in python with C/C++ solvers
  (mistral1/2,minisat,toulbar2,clasp,glucose,..,scip,cplex,gurobi)

Portfolio approach dedicated to UAI Competition 2014:
https://github.com/9thbit/uai-proteus

References

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- David Allouche, Jessica Davies, Simon de Givry, George Katsirelos, Thomas Schiex, Seydou Traoré, Isabelle André, Sophie Barbe, Steve Prestwich, and Barry O'Sullivan. Computational protein design as an optimization problem. *Artificial Intelligence*, 212:59–79, 2014

- M. Cooper, S. de Givry, M. Sanchez, T. Schiex, M. Zytnicki, and T. Werner. Soft arc consistency revisited. *Artificial Intelligence*, (7-8):449–478, 2010