# Parallel Hybrid Best-First Search

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**Abstract**

Supplementary materials for the article *Parallel Hybrid Best-First Search* published at the 28th International Conference on Principles and Practice of Constraint Programming (CP-2022). Figures 1, 2 and Tables 1, 2 (without the experimental results on 1,800 cores) are also in the article.

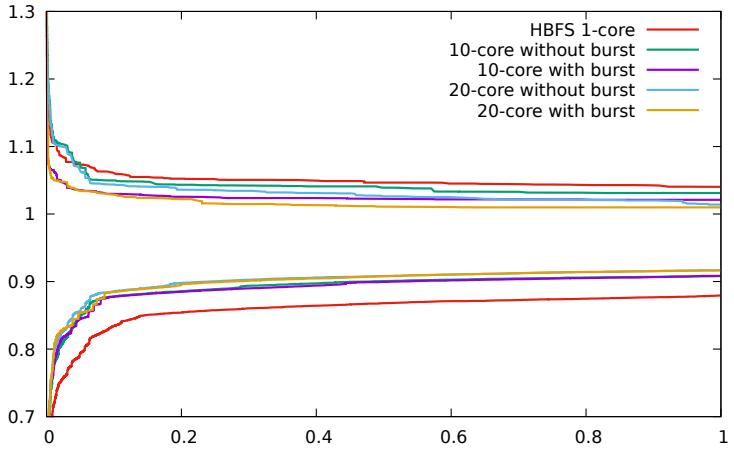


Figure 1: Comparison on a medium-scale computer between sequential versus parallel HBFS with or without burst mode. The x-axis represents normalized time (with 1 corresponding to 3,600 seconds). The y-axis corresponds to normalized lower and upper bounds on 134 instances (with 1 corresponding to the optimum or best known cost).

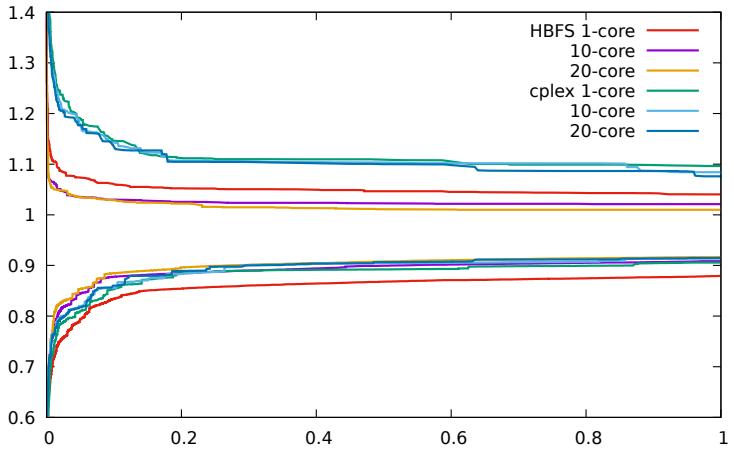


Figure 2: Comparison on a medium-scale computer between toulbar2 using parallel HBFS (with burst mode) and cplex using multiple threads. The x-axis represents normalized time (with 1 corresponding to $3,600$ seconds). The y-axis corresponds to normalized lower and upper bounds on 134 instances (with 1 corresponding to the optimum or best known cost).

| Method | CPD (35) | | Warehouses (15) | | Linkage (22) | | MaxClique (62) | |
|---|---|---|---|---|---|---|---|---|
| | | *Speed-up* | | *Speed-up* | | *Speed-up* | | *Speed-up* |
| HBFS-1 | 30 (43.44s) | | 15 (128.96s) | | 20 (23.24s) | | 37 (364.25s) | |
| HBFS-10 | 30 (8s) | *5.43* | 15 (80.174s) | *1.61* | 21 (3.5s) | *6.64* | 38 (40.24s) | *9.05* |
| HBFS-20 | 30 (4.43s) | *9.81* | 15 (85.39s) | *1.51* | 21 (2s) | *11.62* | 40 (19.9s) | *18.3* |
| cplex-1 | 24 (331.2s) | | 15 (123.83s) | | 22 (8.04s) | | 42 (282.16s) | |
| cplex-10 | 24 (226.51s) | *1.46* | **15 (68.82s)** | *1.8* | 22 (2.56s) | *3.14* | 45 (**55.48s**) | *5.08* |
| cplex-20 | 24 (198.49s) | *1.67* | 15 (72.06s) | *1.72* | **22 (2.29s)** | *3.51* | **46** (71.47s) | *3.95* |
| HBFS-1 (cluster) | 30 (66.46s) | | 15 (392.30s) | | 21 (427.21s) | | 37 (504s) | |
| HBFS-180 (cluster) | **30 (3.7s)** | ***17.96*** | 15 (126s) | ***3.11*** | 22 (4.15s) | ***102.94*** | 45 (6.44s) | ***78.26*** |

Table 1: Number of solved instances within 1 hour (except for sequential HBFS-1 run on the cluster with a larger timeout of 10 hours) and average time in seconds in parentheses. To compute the mean we only consider for a given method (toulbar2 HBFS or cplex) the instances solved with any number of cores on the same computer (server with 3 GHz cores or cluster with 2.3 GHz cores).

| instance | n | d | opt. | *cub* | $\|l\|$ | av. time | max. time | #fail (depth) | EPS-180 | HBFS-180 | HBFS-1800 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| linkage/pedigree19 | 259 | 5 | 4625 | 5684 | 5114 | 20.57 | - | 1 (4) | - | **69.1** | 201 |
| linkage/pedigree40 | 274 | 6 | 7300 | 8838 | 5641 | 101.99 | - | 49 (21) | - | **1680** | 2753 |
| linkage/pedigree51 | 295 | 5 | 6406 | 6802 | 5798 | 0.61 | 497.38 | 0 | 499 | **5.7** | 8.4 |
| cpd/1BRS | 38 | 178 | 4007610 | 4007679 | 956 | 2.94 | 38.90 | 0 | 44 | 37.5 | **15.2** |
| cpd/1CDL | 38 | 170 | 3590514 | 3590825 | 1001 | 6.66 | 79.04 | 0 | 79 | 18.3 | **14.9** |
| cpd/1GVP | 52 | 170 | 5196719 | 5196841 | 979 | 14.59 | 170.66 | 0 | 171 | **17.0** | 24.1 |
| maxclique/brock400_1 | 400 | 2 | 373 | 379 | 6010 | 63.95 | - | 12 ( 149 ) | - | 1812 | **947** |
| maxclique/brock400_2 | 400 | 2 | 371 | 379 | 5975 | 65.27 | - | 18 ( 149 ) | - | 880 | **686** |
| maxclique/san400_0.5_1 | 400 | 2 | 387 | 392 | 6073 | 5.07 | 414.96 | 0 | 3652 | 1220 | **630** |

Table 2: EPS, HBFS-180 and HFBS-1800 results on hard instances. A '-' indicates that some (see #failed) subproblems could not be solved in less than $3,600$ seconds.

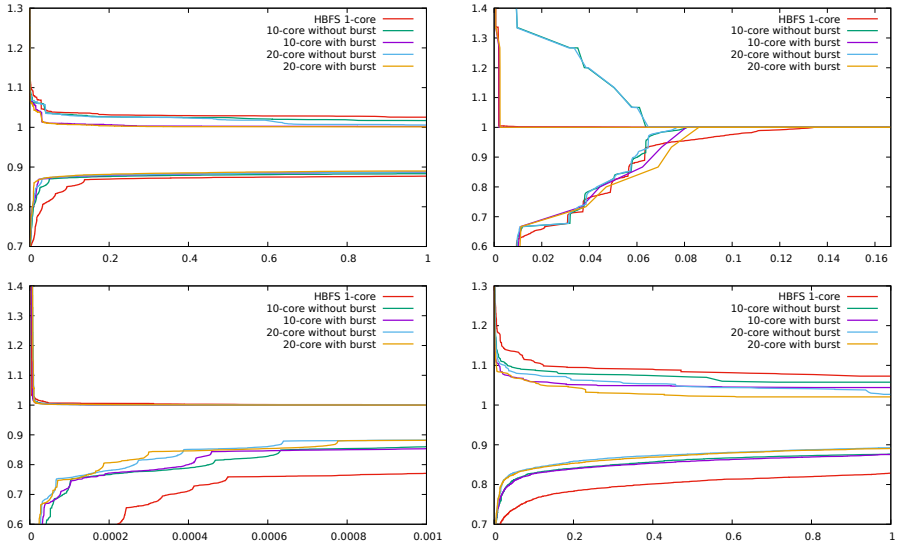


Figure 3: Comparison on a medium-scale computer between sequential versus parallel HBFS with or without burst mode. The x-axis represents normalized time (with 1 corresponding to $3,600$ seconds). The y-axis corresponds to normalized lower and upper bounds on (Upper-Left) 35 CPD, (Upper-Right) 15 Warehouses, (Lower-Left) 22 Linkage, (Lower-Right) 62 MaxClique instances (with 1 corresponding to the optimum or best known cost).

| Instance | n | d | HBFS-1 | HBFS-10 | HBFS-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| 1BK2 | 22 | 145 | 0.038(48) | 0.042(67) | 0.057(67) | 0.904 | 0.666 |
| 1BRS | 38 | 178 | 337.439(154898) | 66.793(210774) | 32.975(181989) | 5.052 | 10.23 |
| 1C9O | 35 | 177 | 0.236(566) | 0.214(601) | 0.236(878) | 1.102 | 1.0 |
| 1CDL | 38 | 170 | 232.863(70451) | 59.952(121263) | 38.367(127756) | 3.884 | 6.069 |
| 1CM1 | 42 | 167 | 1.249(304) | 0.893(391) | 0.931(387) | 1.398 | 1.341 |
| 1CSE | 40 | 114 | 0.011(16) | 0.020(16) | 0.030(16) | 0.55 | 0.366 |
| 1CSK | 18 | 41 | 0.011(39) | 0.016(42) | 0.016(42) | 0.687 | 0.687 |
| 1CSP | 14 | 142 | 0.180(798) | 0.124(1080) | 0.134(1202) | 1.451 | 1.343 |
| 1CTF | 36 | 43 | 0.160(974) | 0.086(1532) | 0.100(2171) | 1.860 | 1.6 |
| 1DKT | 45 | 174 | 0.276(355) | 0.295(466) | 0.317(575) | 0.935 | 0.870 |
| 1ENH | 36 | 156 | - | - | - | - | - |
| 1FNA | 29 | 45 | 0.079(449) | 0.048(701) | 0.064(1063) | 1.645 | 1.234 |
| 1FYN | 21 | 183 | 0.353(106) | 0.393(223) | 0.408(200) | 0.898 | 0.865 |
| 1GVP | 52 | 170 | 497.121(193685) | 77.808(211554) | 34.656(161096) | 6.389 | 14.34 |
| 1HNG | 57 | 168 | 0.427(1646) | 0.319(1433) | 0.344(2192) | 1.338 | 1.241 |
| 1L63 | 73 | 175 | 0.114(199) | 0.129(216) | 0.147(216) | 0.883 | 0.775 |
| 1LZ1 | 53 | 43 | 0.720(5128) | 0.171(5315) | 0.146(5551) | 4.210 | 4.931 |
| 1MJC | 5 | 136 | 0.004(2) | 0.021(2) | 0.025(2) | 0.190 | 0.16 |
| 1NXB | 25 | 35 | 0.007(17) | 0.014(17) | 0.022(17) | 0.5 | 0.318 |
| 1PGB | 31 | 179 | - | - | - | - | - |
| 1PIN | 26 | 185 | 0.832(992) | 0.502(1187) | 0.512(1772) | 1.657 | 1.625 |
| 1POH | 31 | 168 | 0.023(39) | 0.028(35) | 0.043(35) | 0.821 | 0.534 |
| 1RIS | 55 | 165 | 63.632(25676) | 13.198(39604) | 10.677(56152) | 4.821 | 5.959 |
| 1SHF | 20 | 47 | 0.013(16) | 0.020(16) | 0.037(16) | 0.65 | 0.351 |
| 1SHG | 18 | 50 | 0.043(536) | 0.024(883) | 0.031(1169) | 1.791 | 1.387 |
| 1STN | 120 | 180 | - | - | - | - | - |
| 1TEN | 21 | 43 | 0.011(109) | 0.017(101) | 0.025(101) | 0.647 | 0.44 |
| 1UBI | 35 | 141 | 0.869(3687) | 0.248(7492) | 0.203(6723) | 3.504 | 4.280 |
| 2CI2 | 48 | 180 | - | - | - | - | - |
| 2DRI | 34 | 179 | 117.015(131753) | 13.057(126009) | 8.369(152847) | 8.961 | 13.98 |
| 2PCY | 32 | 44 | 0.019(39) | 0.026(34) | 0.041(34) | 0.730 | 0.463 |
| 2RN2 | 52 | 43 | 0.111(272) | 0.109(367) | 0.109(412) | 1.018 | 1.018 |
| 2TRX | 56 | 179 | 0.151(328) | 0.157(310) | 0.175(295) | 0.961 | 0.862 |
| 3CHY | 66 | 56 | 49.168(82898) | 5.157(74429) | 3.891(111411) | 9.534 | 12.63 |
| 3HHR | 115 | 175 | - | - | - | - | - |

Table 3: CPD benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

| Instance | n | d | HBFS-1 | HBFS-10 | HBFS-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| capmo1 | 200 | 100 | 6.210(7450) | 2.176(8182) | 2.253(11381) | 2.853 | 2.756 |
| capmo2 | 200 | 100 | 1.807(3545) | 1.307(4571) | 1.318(5144) | 1.382 | 1.371 |
| capmo3 | 200 | 100 | 5.115(5548) | 2.570(6414) | 2.737(7556) | 1.990 | 1.868 |
| capmo4 | 200 | 100 | 1.548(2219) | 1.144(2607) | 1.177(2842) | 1.353 | 1.315 |
| capmo5 | 200 | 100 | 1.178(2244) | 0.907(2347) | 0.939(2576) | 1.298 | 1.254 |
| capmp1 | 400 | 200 | 76.565(22294) | 36.440(24755) | 39.058(29549) | 2.101 | 1.960 |
| capmp2 | 400 | 200 | 31.269(14218) | 20.458(15449) | 20.909(18265) | 1.528 | 1.495 |
| capmp3 | 400 | 200 | 33.761(14935) | 22.065(16270) | 22.236(17997) | 1.530 | 1.518 |
| capmp4 | 400 | 200 | 60.784(18909) | 39.804(20573) | 40.555(22812) | 1.527 | 1.498 |
| capmp5 | 400 | 200 | 22.758(13788) | 15.436(15183) | 15.614(17100) | 1.474 | 1.457 |
| capmq1 | 600 | 300 | 234.527(41558) | 159.795(46376) | 169.543(50447) | 1.467 | 1.383 |
| capmq2 | 600 | 300 | 376.344(47832) | 253.176(62399) | 268.144(64573) | 1.486 | 1.403 |
| capmq3 | 600 | 300 | 198.975(39463) | 132.376(41554) | 138.671(46572) | 1.503 | 1.434 |
| capmq4 | 600 | 300 | 399.384(45392) | 224.407(52846) | 248.021(58262) | 1.779 | 1.610 |
| capmq5 | 600 | 300 | 484.131(45824) | 290.551(55870) | 309.584(68062) | 1.666 | 1.563 |

Table 4: Warehouses benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

| Instance | n | d | HBFS-1 | HBFS-10 | HBFS-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| pedigree13 | 274 | 3 | 9.728(155015) | 1.510(202376) | 1.081(313231) | 6.442 | 8.999 |
| pedigree18 | 288 | 5 | 0.875(10672) | 0.116(14467) | 0.086(23056) | 7.543 | 10.17 |
| pedigree19 | 259 | 5 | - | 1595.811(226667769) | 715.888(216864808) | - | - |
| pedigree1 | 80 | 4 | 0.011(184) | 0.018(572) | 0.020(795) | 0.611 | 0.55 |
| pedigree20 | 115 | 5 | 0.194(5273) | 0.051(7804) | 0.043(9426) | 3.803 | 4.511 |
| pedigree23 | 83 | 5 | 0.028(659) | 0.017(809) | 0.025(879) | 1.647 | 1.12 |
| pedigree25 | 169 | 5 | 0.108(2934) | 0.041(7242) | 0.038(9474) | 2.634 | 2.842 |
| pedigree30 | 296 | 5 | 1.204(15209) | 0.137(20579) | 0.092(29184) | 8.788 | 13.08 |
| pedigree31 | 261 | 4 | 1.797(32405) | 0.375(54345) | 0.254(74297) | 4.792 | 7.074 |
| pedigree33 | 176 | 4 | 0.077(1578) | 0.024(2179) | 0.034(2396) | 3.208 | 2.264 |
| pedigree34 | 224 | 4 | 0.306(6630) | 0.083(13778) | 0.064(17345) | 3.686 | 4.781 |
| pedigree37 | 143 | 4 | 0.029(494) | 0.020(1032) | 0.045(1407) | 1.45 | 0.644 |
| pedigree38 | 156 | 5 | 0.170(4356) | 0.053(8548) | 0.038(10740) | 3.207 | 4.473 |
| pedigree39 | 161 | 4 | 0.036(812) | 0.021(2391) | 0.025(2198) | 1.714 | 1.44 |
| pedigree40 | 274 | 6 | - | - | - | - | - |
| pedigree41 | 230 | 5 | 31.893(512625) | 5.620(745619) | 2.795(807495) | 5.674 | 11.41 |
| pedigree42 | 123 | 5 | 0.062(1556) | 0.026(2923) | 0.025(3529) | 2.384 | 2.48 |
| pedigree44 | 212 | 4 | 13.549(220064) | 1.650(230637) | 0.672(209425) | 8.211 | 20.16 |
| pedigree50 | 129 | 6 | 0.429(11124) | 0.087(15525) | 0.054(17360) | 4.931 | 7.944 |
| pedigree51 | 295 | 5 | 401.881(6133198) | 60.016(7803526) | 34.320(9420126) | 6.696 | 11.70 |
| pedigree7 | 244 | 4 | 0.876(16205) | 0.129(21818) | 0.099(33103) | 6.790 | 8.848 |
| pedigree9 | 232 | 4 | 1.490(24462) | 0.283(38086) | 0.209(59311) | 5.265 | 7.129 |

Table 5: Linkage benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

| Instance | n | d | HBFS-1 | HBFS-10 | HBFS-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| brock200_1 | 200 | 2 | 138.485(6629072) | 16.674(6368892) | 7.543(5824793) | 8.305 | 18.35 |
| brock200_2 | 200 | 2 | 3.202(111981) | 0.659(159608) | 0.678(209486) | 4.858 | 4.722 |
| brock200_3 | 200 | 2 | 9.466(370784) | 1.273(407930) | 0.926(487573) | 7.435 | 10.22 |
| brock200_4 | 200 | 2 | 26.018(1127989) | 3.429(1186235) | 1.630(1041930) | 7.587 | 15.96 |
| brock400_1 | 400 | 2 | - | - | - | - | - |
| brock400_2 | 400 | 2 | - | - | - | - | - |
| brock400_3 | 400 | 2 | - | - | - | - | - |
| brock400_4 | 400 | 2 | - | - | 2685.064(897034280) | - | - |
| brock800_1 | 800 | 2 | - | - | - | - | - |
| brock800_2 | 800 | 2 | - | - | - | - | - |
| brock800_3 | 800 | 2 | - | - | - | - | - |
| brock800_4 | 800 | 2 | - | - | - | - | - |
| c-fat200-1 | 200 | 2 | 0.225(475) | 0.254(630) | 0.303(665) | 0.885 | 0.742 |
| c-fat200-2 | 200 | 2 | 0.197(467) | 0.250(840) | 0.336(981) | 0.788 | 0.586 |
| c-fat200-5 | 200 | 2 | 0.115(499) | 0.141(1407) | 0.220(2110) | 0.815 | 0.522 |
| c-fat500-10 | 500 | 2 | 2.337(1599) | 2.201(4566) | 2.836(6458) | 1.061 | 0.824 |
| c-fat500-1 | 500 | 2 | 5.563(1065) | 5.354(1250) | 5.727(1320) | 1.039 | 0.971 |
| c-fat500-2 | 500 | 2 | 5.017(1197) | 4.712(1846) | 5.096(2156) | 1.064 | 0.984 |
| c-fat500-5 | 500 | 2 | 4.071(1588) | 3.510(3654) | 4.330(5263) | 1.159 | 0.940 |
| hamming10-2 | 1024 | 2 | 0.037(256) | 0.060(256) | 0.059(256) | 0.616 | 0.627 |
| hamming10-4 | 1024 | 2 | - | - | - | - | - |
| hamming6-2 | 64 | 2 | 0.001(16) | 0.022(16) | 0.736(16) | 0.045 | 0.001 |
| hamming6-4 | 64 | 2 | 0.024(1731) | 0.014(1874) | 0.025(1872) | 1.714 | 0.96 |
| hamming8-2 | 256 | 2 | 0.006(64) | 0.739(64) | 0.013(64) | 0.008 | 0.461 |
| hamming8-4 | 256 | 2 | 61.834(2225278) | 6.811(1922339) | 3.403(1695882) | 9.078 | 18.17 |
| johnson16-2-4 | 120 | 2 | 47.482(4446093) | 5.684(4264346) | 2.794(4372450) | 8.353 | 16.99 |
| johnson32-2-4 | 496 | 2 | - | - | - | - | - |
| johnson8-2-4 | 28 | 2 | 0.003(310) | 0.003(319) | 0.009(344) | 1.0 | 0.333 |
| johnson8-4-4 | 70 | 2 | 0.062(4360) | 0.017(5304) | 0.027(5825) | 3.647 | 2.296 |
| keller4 | 171 | 2 | 13.965(865323) | 1.833(880465) | 0.854(830158) | 7.618 | 16.35 |
| keller5 | 776 | 2 | - | - | - | - | - |
| MANN_a27 | 378 | 2 | 1.862(101564) | 0.113(39411) | 0.073(33430) | 16.47 | 25.50 |
| MANN_a45 | 1035 | 2 | 1268.360(51000545) | 94.127(31089322) | 35.951(24192525) | 13.47 | 35.28 |
| MANN_a81 | 3321 | 2 | - | - | - | - | - |
| MANN_a9 | 45 | 2 | 0.001(48) | 0.012(45) | 0.012(45) | 0.083 | 0.083 |
| p_hat1000-1 | 1000 | 2 | 534.817(4320505) | 162.431(6030224) | 64.857(4834827) | 3.292 | 8.246 |
| p_hat1000-2 | 1000 | 2 | - | - | - | - | - |
| p_hat1000-3 | 1000 | 2 | - | - | - | - | - |
| p_hat300-1 | 300 | 2 | 3.345(52496) | 1.022(60106) | 0.872(55076) | 3.272 | 3.836 |
| p_hat300-2 | 300 | 2 | 14.448(404290) | 1.680(249567) | 1.127(229423) | 8.6 | 12.81 |
| p_hat300-3 | 300 | 2 | 3271.164(98363142) | 248.686(64359457) | 77.053(38258155) | 13.15 | 42.45 |
| p_hat500-1 | 500 | 2 | 32.377(340431) | 6.042(294509) | 5.019(307260) | 5.358 | 6.450 |
| p_hat500-2 | 500 | 2 | 1591.568(24781134) | 222.485(23875099) | 134.999(25756543) | 7.153 | 11.78 |
| p_hat500-3 | 500 | 2 | - | - | - | - | - |
| p_hat700-1 | 700 | 2 | 137.259(1105721) | 40.209(1244364) | 19.056(965954) | 3.413 | 7.202 |
| p_hat700-2 | 700 | 2 | - | - | - | - | - |
| p_hat700-3 | 700 | 2 | - | - | - | - | - |
| san1000 | 1000 | 2 | - | - | - | - | - |
| san200_0.7_1 | 200 | 2 | 229.142(17257759) | 90.847(63189008) | 24.220(33634154) | 2.522 | 9.460 |
| san200_0.7_2 | 200 | 2 | - | - | - | - | - |
| san200_0.9_1 | 200 | 2 | 0.262(6698) | 0.042(6482) | 0.041(9639) | 6.238 | 6.390 |
| san200_0.9_2 | 200 | 2 | 6.948(203789) | 0.711(174967) | 0.288(135340) | 9.772 | 24.12 |
| san200_0.9_3 | 200 | 2 | 1936.883(86332248) | 177.737(62093525) | 87.931(64906983) | 10.89 | 22.02 |
| san400_0.5_1 | 400 | 2 | - | - | - | - | - |
| san400_0.7_1 | 400 | 2 | - | - | - | - | - |
| san400_0.7_2 | 400 | 2 | - | - | - | - | - |
| san400_0.7_3 | 400 | 2 | - | - | - | - | - |
| san400_0.9_1 | 400 | 2 | - | 3495.865(454884726) | 1935.818(517806883) | - | - |
| sanr200_0.7 | 200 | 2 | 47.497(2094971) | 6.266(2296979) | 3.058(2143047) | 7.580 | 15.53 |
| sanr200_0.9 | 200 | 2 | 3593.205(155684986) | 334.728(116112217) | 214.218(160719451) | 10.73 | 16.77 |
| sanr400_0.5 | 400 | 2 | 490.192(8894646) | 48.308(6129795) | 30.322(7233212) | 10.14 | 16.16 |
| sanr400_0.7 | 400 | 2 | - | - | 3200.752(1132310915) | - | - |



Table 6: MaxClique benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

| Instance | n | d | cplex-1 | cplex-10 | cplex-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| 1BK2 | 22 | 145 | 41.87(0) | 44.80(0) | 44.15(0) | 0.934 | 0.948 |
| 1BRS | 38 | 178 | - | - | - | - | - |
| 1C9O | 35 | 177 | 229.57(0) | 255.60(0) | 234.34(0) | 0.898 | 0.979 |
| 1CDL | 38 | 170 | - | - | - | - | - |
| 1CM1 | 42 | 167 | 3150.68(5) | - | - | - | - |
| 1CSE | 40 | 114 | 72.00(0) | 77.69(0) | 95.57(0) | 0.926 | 0.753 |
| 1CSK | 18 | 41 | 5.54(0) | 6.11(0) | 6.05(0) | 0.906 | 0.915 |
| 1CSP | 14 | 142 | 390.45(438) | 74.86(159) | 95.06(560) | 5.215 | 4.107 |
| 1CTF | 36 | 43 | 102.99(69) | 115.49(17) | 118.84(73) | 0.891 | 0.866 |
| 1DKT | 45 | 174 | 322.04(0) | 377.40(0) | 417.18(0) | 0.853 | 0.771 |
| 1ENH | 36 | 156 | - | - | - | - | - |
| 1FNA | 29 | 45 | 36.26(0) | 51.97(0) | 58.22(0) | 0.697 | 0.622 |
| 1FYN | 21 | 183 | 396.56(0) | 541.57(0) | 441.46(0) | 0.732 | 0.898 |
| 1GVP | 52 | 170 | - | - | - | - | - |
| 1HNG | 57 | 168 | 689.69(1) | 946.05(1) | 868.50(3) | 0.729 | 0.794 |
| 1L63 | 73 | 175 | 506.80(0) | 636.52(0) | 665.37(0) | 0.796 | 0.761 |
| 1LZ1 | 53 | 43 | 316.41(674) | 241.49(610) | 270.29(304) | 1.310 | 1.170 |
| 1MJC | 5 | 136 | 1.39(0) | 1.76(0) | 1.84(0) | 0.789 | 0.755 |
| 1NXB | 25 | 35 | 6.37(0) | 8.43(0) | 8.53(0) | 0.755 | 0.746 |
| 1PGB | 31 | 179 | - | - | - | - | - |
| 1PIN | 26 | 185 | 3433.63(2353) | 1038.07(1887) | 429.90(47) | 3.307 | 7.987 |
| 1POH | 31 | 168 | 11.97(0) | 16.66(0) | 17.05(0) | 0.718 | 0.702 |
| 1RIS | 55 | 165 | - | - | - | - | - |
| 1SHF | 20 | 47 | 3.69(0) | 4.85(0) | 4.92(0) | 0.760 | 0.75 |
| 1SHG | 18 | 50 | 13.97(54) | 13.72(138) | 14.94(25) | 1.018 | 0.935 |
| 1STN | 120 | 180 | - | - | - | - | - |
| 1TEN | 21 | 43 | 10.93(0) | 12.95(0) | 12.93(0) | 0.844 | 0.845 |
| 1UBI | 35 | 141 | 641.35(241) | 296.49(251) | 273.56(239) | 2.163 | 2.344 |
| 2CI2 | 48 | 180 | - | - | - | - | - |
| 2DRI | 34 | 179 | - | - | - | - | - |
| 2PCY | 32 | 44 | 20.26(0) | 26.85(0) | 27.76(0) | 0.754 | 0.729 |
| 2RN2 | 52 | 43 | 151.51(0) | 201.27(0) | 246.72(0) | 0.752 | 0.614 |
| 2TRX | 56 | 179 | 212.50(0) | 219.26(0) | 212.13(0) | 0.969 | 1.001 |
| 3CHY | 66 | 56 | - | 2947.51(7263) | 3077.19(7059) | - | - |
| 3HHR | 115 | 175 | - | - | - | - | - |

Table 7: CPD benchmark solving time in seconds (number of nodes in parentheses) and speed-up for cplex solver with 1, 10, or 20 threads. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

| Instance | n | d | cplex-1 | cplex-10 | cplex-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| capmo1 | 200 | 100 | 11.92(150) | 6.51(161) | 6.19(209) | 1.831 | 1.925 |
| capmo2 | 200 | 100 | 2.52(29) | 2.70(25) | 2.86(25) | 0.933 | 0.881 |
| capmo3 | 200 | 100 | 13.76(99) | 7.38(91) | 8.12(87) | 1.864 | 1.694 |
| capmo4 | 200 | 100 | 2.59(19) | 2.84(23) | 2.99(23) | 0.911 | 0.866 |
| capmo5 | 200 | 100 | 2.27(28) | 2.77(29) | 2.92(29) | 0.819 | 0.777 |
| capmp1 | 400 | 200 | 69.32(85) | 47.52(107) | 59.49(97) | 1.458 | 1.165 |
| capmp2 | 400 | 200 | 27.27(31) | 24.40(31) | 25.73(33) | 1.117 | 1.059 |
| capmp3 | 400 | 200 | 54.86(47) | 42.85(41) | 46.37(41) | 1.280 | 1.183 |
| capmp4 | 400 | 200 | 74.56(65) | 61.64(69) | 64.59(69) | 1.209 | 1.154 |
| capmp5 | 400 | 200 | 30.03(33) | 21.72(29) | 23.23(29) | 1.382 | 1.292 |
| capmq1 | 600 | 300 | 186.56(67) | 132.70(67) | 148.43(67) | 1.405 | 1.256 |
| capmq2 | 600 | 300 | 314.25(113) | 141.38(121) | 139.76(117) | 2.222 | 2.248 |
| capmq3 | 600 | 300 | 143.84(37) | 77.74(37) | 86.68(37) | 1.850 | 1.659 |
| capmq4 | 600 | 300 | 350.82(81) | 201.17(83) | 207.21(83) | 1.743 | 1.693 |
| capmq5 | 600 | 300 | 572.89(157) | 258.99(185) | 256.29(177) | 2.212 | 2.235 |

Table 8: Warehouses benchmark solving time in seconds (number of nodes in parentheses) and speed-up for cplex solver with 1, 10, or 20 threads. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

| Instance | n | d | cplex-1 | cplex-10 | cplex-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| pedigree13 | 274 | 3 | 0.90(191) | 0.57(0) | 0.54(0) | 1.578 | 1.666 |
| pedigree18 | 288 | 5 | 1.90(572) | 1.00(334) | 1.08(315) | 1.9 | 1.759 |
| pedigree19 | 259 | 5 | 56.62(7136) | 16.88(13084) | 12.41(13674) | 3.354 | 4.562 |
| pedigree1 | 80 | 4 | 0.17(0) | 0.17(0) | 0.37(0) | 1.0 | 0.459 |
| pedigree20 | 115 | 5 | 0.89(505) | 0.46(217) | 0.62(0) | 1.934 | 1.435 |
| pedigree23 | 83 | 5 | 0.21(0) | 0.18(0) | 0.39(0) | 1.166 | 0.538 |
| pedigree25 | 169 | 5 | 1.59(1021) | 0.68(489) | 0.80(366) | 2.338 | 1.987 |
| pedigree30 | 296 | 5 | 1.09(166) | 1.08(263) | 1.32(257) | 1.009 | 0.825 |
| pedigree31 | 261 | 4 | 7.55(1637) | 4.16(3509) | 2.72(2361) | 1.814 | 2.775 |
| pedigree33 | 176 | 4 | 1.07(702) | 0.55(881) | 0.63(1034) | 1.945 | 1.698 |
| pedigree34 | 224 | 4 | 0.96(149) | 0.59(0) | 0.59(0) | 1.627 | 1.627 |
| pedigree37 | 143 | 4 | 0.56(85) | 0.54(23) | 0.74(32) | 1.037 | 0.756 |
| pedigree38 | 156 | 5 | 0.46(0) | 0.44(0) | 0.46(0) | 1.045 | 1.0 |
| pedigree39 | 161 | 4 | 0.51(56) | 0.35(0) | 0.41(0) | 1.457 | 1.243 |
| pedigree40 | 274 | 6 | 86.24(8724) | 22.05(13031) | 20.22(20813) | 3.911 | 4.265 |
| pedigree41 | 230 | 5 | 4.02(1005) | 1.30(644) | 1.31(588) | 3.092 | 3.068 |
| pedigree42 | 123 | 5 | 0.29(0) | 0.29(0) | 0.28(0) | 1.0 | 1.035 |
| pedigree44 | 212 | 4 | 1.62(716) | 0.89(992) | 1.04(1377) | 1.820 | 1.557 |
| pedigree50 | 129 | 6 | 1.81(654) | 0.96(810) | 1.08(816) | 1.885 | 1.675 |
| pedigree51 | 295 | 5 | 6.61(2457) | 1.54(1348) | 1.62(1731) | 4.292 | 4.080 |
| pedigree7 | 244 | 4 | 0.63(141) | 0.62(0) | 0.65(0) | 1.016 | 0.969 |
| pedigree9 | 232 | 4 | 1.30(264) | 0.97(138) | 1.14(194) | 1.340 | 1.140 |

Table 9: Linkage benchmark solving time in seconds (number of nodes in parentheses) and speed-up for cplex solver with 1, 10, or 20 threads. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

| Instance | n | d | cplex-1 | cplex-10 | cplex-20 | Speed-10 | Speed-20 |
|---|---|---|---|---|---|---|---|
| brock200_1 | 200 | 2 | 694.09(276512) | 113.87(303894) | 39.35(152336) | 6.095 | 17.63 |
| brock200_2 | 200 | 2 | 57.82(9706) | 20.88(9614) | 9.31(5440) | 2.769 | 6.210 |
| brock200_3 | 200 | 2 | 224.39(56667) | 32.54(47830) | 18.92(24776) | 6.895 | 11.85 |
| brock200_4 | 200 | 2 | 185.73(54521) | 36.02(76408) | 24.40(54378) | 5.156 | 7.611 |
| brock400_1 | 400 | 2 | - | - | - | - | - |
| brock400_2 | 400 | 2 | - | - | - | - | - |
| brock400_3 | 400 | 2 | - | - | - | - | - |
| brock400_4 | 400 | 2 | - | - | - | - | - |
| brock800_1 | 800 | 2 | - | - | - | - | - |
| brock800_2 | 800 | 2 | - | - | - | - | - |
| brock800_3 | 800 | 2 | - | - | - | - | - |
| brock800_4 | 800 | 2 | - | - | - | - | - |
| c-fat200-1 | 200 | 2 | 6.60(0) | 6.46(0) | 6.44(0) | 1.021 | 1.024 |
| c-fat200-2 | 200 | 2 | 5.30(0) | 5.29(0) | 5.19(0) | 1.001 | 1.021 |
| c-fat200-5 | 200 | 2 | 5.01(5) | 2.29(0) | 2.26(0) | 2.187 | 2.216 |
| c-fat500-10 | 500 | 2 | 3.64(0) | 3.64(0) | 2.99(0) | 1.0 | 1.217 |
| c-fat500-1 | 500 | 2 | 1.92(0) | 2.52(0) | 2.44(0) | 0.761 | 0.786 |
| c-fat500-2 | 500 | 2 | 3.99(0) | 2.88(0) | 2.65(0) | 1.385 | 1.505 |
| c-fat500-5 | 500 | 2 | 4.82(0) | 4.50(0) | 2.88(0) | 1.071 | 1.673 |
| hamming10-2 | 1024 | 2 | 0.06(0) | 0.09(0) | 0.10(0) | 0.666 | 0.6 |
| hamming10-4 | 1024 | 2 | - | - | - | - | - |
| hamming6-2 | 64 | 2 | 0.00(0) | 0.01(0) | 0.01(0) | 0.0 | 0.0 |
| hamming6-4 | 64 | 2 | 0.08(33) | 0.08(0) | 0.22(0) | 1.0 | 0.363 |
| hamming8-2 | 256 | 2 | 0.01(0) | 0.03(0) | 0.03(0) | 0.333 | 0.333 |
| hamming8-4 | 256 | 2 | 0.69(0) | 0.26(0) | 0.26(0) | 2.653 | 2.653 |
| johnson16-2-4 | 120 | 2 | 0.00(0) | 0.02(0) | 0.01(0) | 0.0 | 0.0 |
| johnson32-2-4 | 496 | 2 | 0.02(0) | 0.03(0) | 0.04(0) | 0.666 | 0.5 |
| johnson8-2-4 | 28 | 2 | 0.00(0) | 0.01(0) | 0.01(0) | 0.0 | 0.0 |
| johnson8-4-4 | 70 | 2 | 0.00(0) | 0.01(0) | 0.01(0) | 0.0 | 0.0 |
| keller4 | 171 | 2 | 17.04(4879) | 1.96(4039) | 1.76(3479) | 8.693 | 9.681 |
| keller5 | 776 | 2 | - | - | - | - | - |
| MANN_a27 | 378 | 2 | 0.47(26) | 0.31(38) | 0.37(0) | 1.516 | 1.270 |
| MANN_a45 | 1035 | 2 | 28.42(31739) | 1.51(7080) | 9.26(58969) | 18.82 | 3.069 |
| MANN_a81 | 3321 | 2 | 2779.71(571243) | 804.89(1453615) | 1514.51(5980343) | 3.453 | 1.835 |
| MANN_a9 | 45 | 2 | 0.01(5) | 0.02(0) | 0.20(0) | 0.5 | 0.05 |
| p_hat1000-1 | 1000 | 2 | - | - | 3213.79(1222054) | - | - |
| p_hat1000-2 | 1000 | 2 | - | - | - | - | - |
| p_hat1000-3 | 1000 | 2 | - | - | - | - | - |
| p_hat300-1 | 300 | 2 | 237.06(4074) | 23.33(0) | 23.76(0) | 10.16 | 9.977 |
| p_hat300-2 | 300 | 2 | 160.25(6167) | 36.58(3177) | 31.71(2805) | 4.380 | 5.053 |
| p_hat300-3 | 300 | 2 | - | 435.40(280191) | 167.37(134542) | - | - |
| p_hat500-1 | 500 | 2 | 824.86(67226) | 119.48(34595) | 117.82(52937) | 6.903 | 7.001 |
| p_hat500-2 | 500 | 2 | - | 453.21(95388) | 152.69(77991) | - | - |
| p_hat500-3 | 500 | 2 | - | - | - | - | - |
| p_hat700-1 | 700 | 2 | 3166.67(155906) | 553.44(83394) | 322.35(139485) | 5.721 | 9.823 |
| p_hat700-2 | 700 | 2 | - | - | - | - | - |
| p_hat700-3 | 700 | 2 | - | - | - | - | - |
| san1000 | 1000 | 2 | 1444.64(5498) | 305.00(14121) | 604.09(43928) | 4.736 | 2.391 |
| san200_0.7_1 | 200 | 2 | 0.14(0) | 0.15(0) | 0.15(0) | 0.933 | 0.933 |
| san200_0.7_2 | 200 | 2 | 1.22(30) | 0.42(0) | 0.58(0) | 2.904 | 2.103 |
| san200_0.9_1 | 200 | 2 | 0.02(0) | 0.04(0) | 0.04(0) | 0.5 | 0.5 |
| san200_0.9_2 | 200 | 2 | 0.05(0) | 0.07(0) | 0.07(0) | 0.714 | 0.714 |
| san200_0.9_3 | 200 | 2 | 0.62(44) | 0.22(0) | 0.31(0) | 2.818 | 2.0 |
| san400_0.5_1 | 400 | 2 | 40.50(839) | 2.18(0) | 2.19(0) | 18.57 | 18.49 |
| san400_0.7_1 | 400 | 2 | 12.64(0) | 4.19(0) | 4.18(0) | 3.016 | 3.023 |
| san400_0.7_2 | 400 | 2 | 92.20(3294) | 9.54(0) | 21.80(8633) | 9.664 | 4.229 |
| san400_0.7_3 | 400 | 2 | 335.00(57914) | 84.05(93035) | 66.63(123128) | 3.985 | 5.027 |
| san400_0.9_1 | 400 | 2 | 4.20(0) | 1.57(0) | 1.55(0) | 2.675 | 2.709 |
| sanr200_0.7 | 200 | 2 | 413.30(130364) | 38.90(84111) | 34.06(74947) | 10.62 | 12.13 |
| sanr200_0.9 | 200 | 2 | 815.59(504637) | 110.19(527791) | 55.30(447549) | 7.401 | 14.74 |
| sanr400_0.5 | 400 | 2 | - | 2759.21(866709) | 2163.52(891225) | - | - |
| sanr400_0.7 | 400 | 2 | - | - | - | - | - |



Table 10: MaxClique benchmark solving time in seconds (number of nodes in parentheses) and speed-up for cplex solver with 1, 10, or 20 threads. '-': unsolved in 1h. Experiments were made on a medium-scale computer with cores running at 3 GHz.

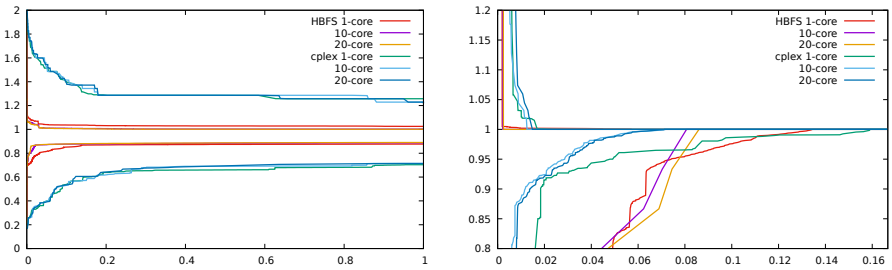


Figure 4: (Left fig.) Comparison on a medium-scale computer between toulbar2 using parallel HBFS (with burst mode) and cplex using multiple threads. The x-axis represents normalized time (with 1 corresponding to $3,600$ seconds). The y-axis corresponds to normalized lower and upper bounds on 35 CPD instances (with 1 corresponding to the optimum or best known cost). (Right fig.) Same comparison for 15 Warehouses instances.

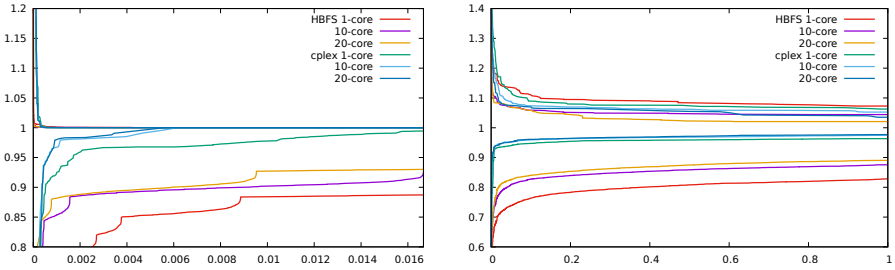


Figure 5: (Left fig.) Comparison on a medium-scale computer between toulbar2 using parallel HBFS (with burst mode) and cplex using multiple threads. The x-axis represents normalized time (with 1 corresponding to $3,600$ seconds). The y-axis corresponds to normalized lower and upper bounds on 22 Linkage instances (with 1 corresponding to the optimum or best known cost). (Right fig.) Same comparison for 62 MaxClique instances.

| Instance | n | d | HBFS-1 | HBFS-180 | Speed-180 |
|---|---|---|---|---|---|
| 1BK2 | 22 | 145 | 0.052(48) | 0.131(60) | 0.396 |
| 1BRS | 38 | 178 | 508.512(154898) | 37.541(211013) | 13.54 |
| 1C9O | 35 | 177 | 0.372(566) | 0.386(1462) | 0.963 |
| 1CDL | 38 | 170 | 368.057(70451) | 18.306(220318) | 20.10 |
| 1CM1 | 42 | 167 | 2.105(304) | 1.421(462) | 1.481 |
| 1CSE | 40 | 114 | 0.017(16) | 0.110(16) | 0.154 |
| 1CSK | 18 | 41 | 0.013(39) | 0.098(42) | 0.132 |
| 1CSP | 14 | 142 | 0.244(798) | 0.270(4294) | 0.903 |
| 1CTF | 36 | 43 | 0.205(974) | 0.208(6614) | 0.985 |
| 1DKT | 45 | 174 | 0.485(355) | 0.497(730) | 0.975 |
| 1ENH | 36 | 156 | - | - | - |
| 1FNA | 29 | 45 | 0.102(449) | 0.226(2120) | 0.451 |
| 1FYN | 21 | 183 | 0.636(106) | 0.604(200) | 1.052 |
| 1GVP | 52 | 170 | 771.199(193685) | 16.960(280594) | 45.47 |
| 1HNG | 57 | 168 | 0.664(1646) | 0.626(10219) | 1.060 |
| 1L63 | 73 | 175 | 0.179(199) | 0.266(216) | 0.672 |
| 1LZ1 | 53 | 43 | 0.897(5128) | 0.259(30140) | 3.463 |
| 1MJC | 5 | 136 | 0.006(2) | 0.094(2) | 0.063 |
| 1NXB | 25 | 35 | 0.011(17) | 0.094(17) | 0.117 |
| 1PGB | 31 | 179 | - | - | - |
| 1PIN | 26 | 185 | 1.260(992) | 0.984(7146) | 1.280 |
| 1POH | 31 | 168 | 0.030(39) | 0.112(39) | 0.267 |
| 1RIS | 55 | 165 | 101.082(25676) | 14.068(119841) | 7.185 |
| 1SHF | 20 | 47 | 0.016(16) | 0.105(16) | 0.152 |
| 1SHG | 18 | 50 | 0.053(536) | 0.144(3078) | 0.368 |
| 1STN | 120 | 180 | - | - | - |
| 1TEN | 21 | 43 | 0.015(109) | 0.115(101) | 0.130 |
| 1UBI | 35 | 141 | 1.079(3687) | 0.683(29429) | 1.579 |
| 2CI2 | 48 | 180 | - | - | - |
| 2DRI | 34 | 179 | 155.979(131753) | 10.400(360841) | 14.99 |
| 2PCY | 32 | 44 | 0.028(39) | 0.142(39) | 0.197 |
| 2RN2 | 52 | 43 | 0.149(272) | 0.186(427) | 0.801 |
| 2TRX | 56 | 179 | 0.238(328) | 0.323(310) | 0.736 |
| 3CHY | 66 | 56 | 58.544(82898) | 5.035(238679) | 11.62 |
| 3HHR | 115 | 175 | - | - | - |

Table 11: CPD benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 10h for HBFS-1 or 1h for HBFS-180. Experiments were made on a large-scale cluster with cores running at 2.3 GHz.

| Instance | n | d | HBFS-1 | HBFS-180 | Speed-180 |
|---|---|---|---|---|---|
| capmo1 | 200 | 100 | 14.091(7450) | 3.781(23832) | 3.726 |
| capmo2 | 200 | 100 | 3.876(3545) | 2.204(6070) | 1.758 |
| capmo3 | 200 | 100 | 12.549(5548) | 4.374(13565) | 2.868 |
| capmo4 | 200 | 100 | 3.171(2219) | 1.964(3345) | 1.614 |
| capmo5 | 200 | 100 | 2.359(2244) | 1.575(2873) | 1.497 |
| capmp1 | 400 | 200 | 247.903(22294) | 57.282(53136) | 4.327 |
| capmp2 | 400 | 200 | 103.542(14218) | 36.765(26063) | 2.816 |
| capmp3 | 400 | 200 | 109.467(14935) | 37.947(31034) | 2.884 |
| capmp4 | 400 | 200 | 213.391(18909) | 68.320(40794) | 3.123 |
| capmp5 | 400 | 200 | 70.143(13788) | 26.921(22084) | 2.605 |
| capmq1 | 600 | 300 | 752.794(41558) | 255.110(79859) | 2.950 |
| capmq2 | 600 | 300 | 1164.669(47832) | 392.648(113938) | 2.966 |
| capmq3 | 600 | 300 | 645.461(39463) | 214.766(81130) | 3.005 |
| capmq4 | 600 | 300 | 1187.680(45392) | 340.353(114189) | 3.489 |
| capmq5 | 600 | 300 | 1353.350(45824) | 446.052(130390) | 3.034 |

Table 12: Warehouses benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 10h for HBFS-1 or 1h for HBFS-180. Experiments were made on a large-scale cluster with cores running at 2.3 GHz.

| Instance | n | d | HBFS-1 | HBFS-180 | Speed-180 |
|---|---|---|---|---|---|
| pedigree13 | 274 | 3 | 12.154(155015) | 1.959(283712) | 6.204 |
| pedigree18 | 288 | 5 | 1.112(10672) | 0.237(37011) | 4.691 |
| pedigree19 | 259 | 5 | 8388.377(115302059) | 69.097(147212141) | 121.4 |
| pedigree1 | 80 | 4 | 0.021(184) | 0.118(805) | 0.177 |
| pedigree20 | 115 | 5 | 0.248(5273) | 0.159(15696) | 1.559 |
| pedigree23 | 83 | 5 | 0.043(659) | 0.122(1101) | 0.352 |
| pedigree25 | 169 | 5 | 0.144(2934) | 0.195(19647) | 0.738 |
| pedigree30 | 296 | 5 | 1.530(15209) | 0.431(71299) | 3.549 |
| pedigree31 | 261 | 4 | 2.258(32405) | 0.346(75517) | 6.526 |
| pedigree33 | 176 | 4 | 0.106(1578) | 0.168(6468) | 0.630 |
| pedigree34 | 224 | 4 | 0.385(6630) | 0.215(33920) | 1.790 |
| pedigree37 | 143 | 4 | 0.044(494) | 0.144(3320) | 0.305 |
| pedigree38 | 156 | 5 | 0.221(4356) | 0.189(23576) | 1.169 |
| pedigree39 | 161 | 4 | 0.051(812) | 0.134(2757) | 0.380 |
| pedigree40 | 274 | 6 | - | 1680.072(3485210910) | - |
| pedigree41 | 230 | 5 | 40.095(512625) | 5.861(540050) | 6.840 |
| pedigree42 | 123 | 5 | 0.086(1556) | 0.167(6717) | 0.514 |
| pedigree44 | 212 | 4 | 17.262(220064) | 1.164(196109) | 14.82 |
| pedigree50 | 129 | 6 | 0.563(11124) | 0.171(30764) | 3.292 |
| pedigree51 | 295 | 5 | 503.728(6133198) | 5.743(10956927) | 87.71 |
| pedigree7 | 244 | 4 | 1.095(16205) | 0.190(41103) | 5.763 |
| pedigree9 | 232 | 4 | 1.876(24462) | 0.304(86907) | 6.171 |

Table 13: Linkage benchmark solving time for Toulbar2 Burst version with 180 cores benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 10h for HBFS-1 or 1h for HBFS-180. Experiments were made on a large-scale cluster with cores running at 2.3 GHz.

| Instance | n | d | HBFS-1 | HBFS-180 | Speed-180 |
|---|---|---|---|---|---|
| brock200_1 | 200 | 2 | 192.680(6629072) | 1.247(4168113) | 154.5 |
| brock200_2 | 200 | 2 | 5.753(111981) | 0.353(236904) | 16.29 |
| brock200_3 | 200 | 2 | 14.996(370784) | 0.552(614202) | 27.16 |
| brock200_4 | 200 | 2 | 33.396(1127989) | 0.579(1045035) | 57.67 |
| brock400_1 | 400 | 2 | - | 1812.532(4913769406) | - |
| brock400_2 | 400 | 2 | - | 879.885(2461802722) | - |
| brock400_3 | 400 | 2 | - | 457.001(1218759299) | - |
| brock400_4 | 400 | 2 | - | 393.766(969943882) | - |
| brock800_1 | 800 | 2 | - | - | - |
| brock800_2 | 800 | 2 | - | - | - |
| brock800_3 | 800 | 2 | - | - | - |
| brock800_4 | 800 | 2 | - | - | - |
| c-fat200-1 | 200 | 2 | 0.581(475) | 0.472(665) | 1.230 |
| c-fat200-2 | 200 | 2 | 0.489(467) | 0.528(1063) | 0.926 |
| c-fat200-5 | 200 | 2 | 0.262(499) | 0.431(2589) | 0.607 |
| c-fat500-10 | 500 | 2 | 7.678(1599) | 5.833(10571) | 1.316 |
| c-fat500-1 | 500 | 2 | 13.585(1065) | 8.238(1320) | 1.649 |
| c-fat500-2 | 500 | 2 | 12.894(1197) | 7.675(2379) | 1.68 |
| c-fat500-5 | 500 | 2 | 11.044(1588) | 7.970(8526) | 1.385 |
| hamming10-2 | 1024 | 2 | 0.071(256) | 0.156(256) | 0.455 |
| hamming10-4 | 1024 | 2 | - | - | - |
| hamming6-2 | 64 | 2 | 0.003(16) | 0.119(16) | 0.025 |
| hamming6-4 | 64 | 2 | 0.032(1731) | 0.109(1942) | 0.293 |
| hamming8-2 | 256 | 2 | 0.009(64) | 0.125(64) | 0.072 |
| hamming8-4 | 256 | 2 | 81.407(2225278) | 1.980(1754776) | 41.11 |
| johnson16-2-4 | 120 | 2 | 63.304(4446093) | 0.983(3679681) | 64.39 |
| johnson32-2-4 | 496 | 2 | - | - | - |
| johnson8-2-4 | 28 | 2 | 0.005(310) | 0.107(350) | 0.046 |
| johnson8-4-4 | 70 | 2 | 0.083(4360) | 0.149(7429) | 0.557 |
| keller4 | 171 | 2 | 20.536(865323) | 1.006(658678) | 20.41 |
| keller5 | 776 | 2 | - | - | - |
| MANN_a27 | 378 | 2 | 2.492(101564) | 0.205(42097) | 12.15 |
| MANN_a45 | 1035 | 2 | 1653.564(51000545) | 5.494(19568286) | 300.9 |
| MANN_a81 | 3321 | 2 | - | - | - |
| MANN_a9 | 45 | 2 | 0.003(48) | 0.100(45) | 0.03 |
| p_hat1000-1 | 1000 | 2 | 1052.774(4320505) | 54.050(4220989) | 19.47 |
| p_hat1000-2 | 1000 | 2 | - | - | - |
| p_hat1000-3 | 1000 | 2 | - | - | - |
| p_hat300-1 | 300 | 2 | 9.781(52496) | 0.954(139252) | 10.25 |
| p_hat300-2 | 300 | 2 | 26.115(404290) | 0.833(493415) | 31.35 |
| p_hat300-3 | 300 | 2 | 4186.002(98363142) | 5.404(17026384) | 774.6 |
| p_hat500-1 | 500 | 2 | 73.752(340431) | 6.513(1741366) | 11.32 |
| p_hat500-2 | 500 | 2 | 2563.109(24781134) | 18.296(8553910) | 140.0 |
| p_hat500-3 | 500 | 2 | - | - | - |
| p_hat700-1 | 700 | 2 | 294.866(1105721) | 14.593(1130223) | 20.20 |
| p_hat700-2 | 700 | 2 | - | 751.873(768842909) | - |
| p_hat700-3 | 700 | 2 | - | - | - |
| san1000 | 1000 | 2 | - | - | - |
| san200_0.7_1 | 200 | 2 | 309.192(17257759) | 38.200(207958887) | 8.094 |
| san200_0.7_2 | 200 | 2 | - | - | - |
| san200_0.9_1 | 200 | 2 | 0.302(6698) | 0.184(40263) | 1.641 |
| san200_0.9_2 | 200 | 2 | 8.095(203789) | 0.320(138461) | 25.29 |
| san200_0.9_3 | 200 | 2 | 2489.096(86332248) | 18.066(71321410) | 137.7 |
| san400_0.5_1 | 400 | 2 | - | 1220.347(3960964343) | - |
| san400_0.7_1 | 400 | 2 | - | - | - |
| san400_0.7_2 | 400 | 2 | - | - | - |
| san400_0.7_3 | 400 | 2 | - | - | - |
| san400_0.9_1 | 400 | 2 | - | 88.830(194097616) | - |
| sanr200_0.7 | 200 | 2 | 66.691(2094971) | 1.353(1728672) | 49.29 |
| sanr200_0.9 | 200 | 2 | 4510.857(155684986) | 30.682(131733784) | 147.0 |
| sanr400_0.5 | 400 | 2 | 941.357(8894646) | 4.368(5817184) | 215.5 |
| sanr400_0.7 | 400 | 2 | - | 471.271(1203449249) | - |



Table 14: MaxClique benchmark solving time in seconds (number of nodes in parentheses) and speed-up for sequential versus parallel HBFS (with burst mode) in toulbar2. '-': unsolved in 10h for HBFS-1 or 1h for HBFS-180. Experiments were made on a large-scale cluster with cores running at 2.3 GHz.