A Weighted CSP approach for solving spatio-temporal farm planning problems

Mahuna Akplogan, Jérome Dury, Simon de Givry,
Gauthier Quesnel, Alexandre Joannon, Arnaud Reynaud,
Jacques Éric Bergez, and Frédérick Garcia

INRA, F-31320 Castanet Tolosan, France
{makploga,jdury,degivry,gquesnel,joannon,
areynaud,jbergez,fgarcia}@toulouse.inra.fr

Abstract. Applications regarding the crop allocation problem (CAP) are required tools for agricultural advisors to design more efficient farming systems. Despite this issue has been extensively treated by agronomists in the past, few methods tackle the crop allocation problem considering both the spatial and the temporal aspects of the CAP. In this paper, we precisely propose an original approach based on weighted CSP (WCSP) to address the crop allocation planning problem while taking farmers' management choices into account. These are represented as hard and preference constraints. We illustrate our proposition by some results based on a virtual case study. This preliminary work foreshadows the development of a decision-aid tool for supporting farmers in their crop allocation strategies.

Key words: Weighted CSP, constraint satisfaction, optimization, spatio-temporal planning, crop allocation problem

1 Introduction

The design of a cropping plan is one of the first step in the process of crop production and is an important decision that farmers have to take. By cropping plan, we mean the *acreages* occupied by all the different crops every year and their *spatial allocation* within a farming land. The cropping plan decision can be summarized as (1) the choice of crops to be grown, (2) the determination of all crops' acreages, and (3) their allocation to plots. Despite the apparent simplicity of the decision problem, the cropping plan decisions depend on multiple spatial and temporal factors interacting at different levels of the farm management. The cropping plan decision-making combines long term planning activities, with managerial and operational activities to timely control the crop production process. Modelling a decision-making process to support such farmers' decisions therefore requires to consider the planning of crop allocation over a finite horizon, and to explicitly consider the sequence of problem-solving imposed by the changing context (e.g. weather, price).

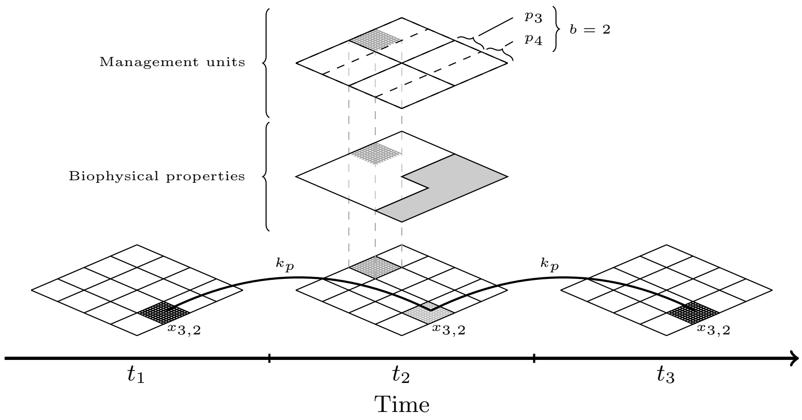


Fig. 1. Schematic representation of the spatial and temporal aspect of the decision-making problem (t_i: year, b: block, p_j: plot, $x_{b,i}$: landunit, k_p: preceding effect)

In this paper, we precisely focus on the activity of planning seen as a spatio-temporal crop allocation problem (CAP) whose relevance is assessed by a global objective function. In addition to many approaches based on optimization procedure, the objective of the work is to propose new directions to address crop allocation while taking farmers' decision factors into account. These factors are formalized as hard and preference constraints in the WCSP framework. The choice of constraints is based on a survey of farmers' processes taking into account annual working hours capacity restrictions [5]. However, designing cropping plans with such an approach is still an open question due to many other decision factors that could be taken into account to solve the crop allocation problem. This preliminary work foreshadows the implementation of a spatially explicit decision-aid tool, namely CRASH (Crop Rotation and Allocation Simulator using Heuristics), developed for supporting farmers in their crop allocation strategies.

This paper is organized as follows. In section 2, we describe the crop allocation problem. It introduces some specific definitions and emphasize crop allocation problem. Section 3 describes existing approaches used to design cropping plans with a focus on their main limitations. In section 4, we introduce the constraint model compliant with the weighted CSP framework. In section 5, we illustrate our modelling approach by a virtual case study in order to highlight the interests of the proposed approach. And finally in section 6 we discuss and conclude the relevance and limits of using WCSP to solve the CAP.

2 Crop allocation problem (CAP)

2.1 Global description of the problem

Let us consider a set of *landunits* defined as a piece of indivisible and homogeneous land whose historic and biophysical properties are identical. We define crop allocation as a spatio-temporal planning problem in which crops are assigned to landunits $x_{b,i}$ over a fixed horizon $\mathcal{H}$ of time (Fig. 1). These landunits are spatial sampling of the farmland where $x_{b,i}$ denotes the landunit i of *block* b.

The planning problem depends on multiple spatial and temporal factors. In space, these factors are organized in many different organizational levels called *management units* (Fig.1). These management units are decided by the farmer to organize his work and allocate resources. In order to simplify our example, we only considered the two main management units: *plot* (p_j) and *block* b. The first concerns the annual management of crops. A plot is a combination of landunits. Their delimitations are adapted over years in order to enforce the spatial balanced of crop acreages. As shown by Fig.1 *blocks* are subset of plots managed in a coherent way. Blocks are characterized by one cropping system defined by the same collection of crops and by the use of a coherent set of production techniques applied to these crops (e.g. fertilizer, irrigation water). The delimitation of blocks are not reshaped in the CAP considered in this work. They are mostly defined by the structural properties of the farm such as the availability of resources (e.g. access to irrigation water) and by the biophysical properties (eg. soil type, accessibility, topography). These *biophysical properties* are also used to define if a crop could not be produced in good condition on certain soil types.

In time, the sequence of crops on the same landunit is not allowed or not advisable without facing decrease in soil fertility, or increase in diseases or weeds infestation. We deal with these temporal factors by summarizing the assessment of crop sequence quality in two indicators: the *minimum return time (rt)* and the *Preceding effect(k_p)*. The *minimum return time (rt)* is defined as the minimum number of years before growing the same crop on a same landunit. On the figure 1, the minimum return time of the crop produced on $x_{3,2}$ (landunit 2 of block 3) at t_1 is equal to 2 years. More generally let t, t' be two different years ($t < t'$), $x_{b,i}$ a landunit and v a crop, $x_{b,i}^{t} = x_{b,i}^{t'} = v$ **if** $(t' - t) \geq rt(v)$.

The *preceding effect (k_p)* is an indicator representing the effect of the previous crop on the next one [12]. Based on k_p, some crop sequence can be ignored for their effects or recommended for their beneficial effects for production purposes. Further, some authors [4] have argued that the reproducibility of a cropping system over time is only ensured when crop allocation choices are derived from finite crop sequence which can be repeated over the time. We therefore introduce the concept of repeatability while looking for such a crop sequence. This means that the proposed crop sequence could be repeated over time without breaking the constraint rt. We introduce this concept, known as a *"crop rotation"*, because it is widely used by farmers as decision indicator.

2.2 Constraints description

Solving the crop allocation problem (CAP) is to assign crops to landunits $x_{b,i}$ over a fixed horizon $\mathcal{H}$ of time. An assignment of crops must satisfy a set of constraints.

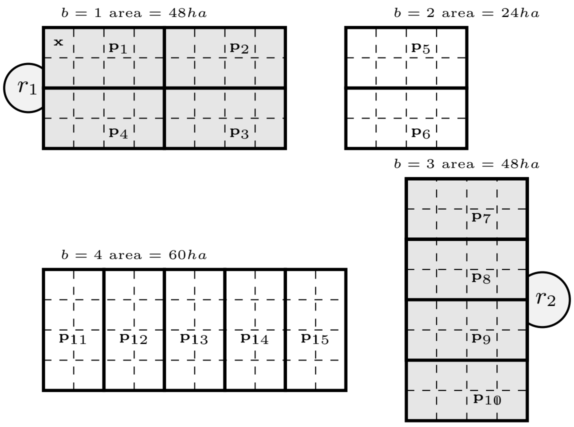
We retained as hard constraints the minimum *returned time* (rt), the *historic* of landunits and the *physical properties* (soil types, resource accessibility). Preference constraints are related to the *preceding effects* (k_p) and the spatio-temporal balance of crop acreages such that resources are efficiently used. Hard and preference constraints are defined either at:

- *plot level* to express for each plot (i) if they can be split/combined, (ii) if they must be fixed over the planning horizon in order to enforce the static aspect of the plot.
- *block level* to express for each landunit and crop the spatial compatibility of crop, the return time and the preceding effect.
- *farm level* to express preferences or the global use of resources.

Let us consider the crop allocation problem described in Fig. 2. In this problem, we consider 4 blocks and 15 plots sampled into 120 landunits. The size of the farmland (180 ha) and its sampling into landunits correspond to a middle real-world CAP. Four crops are produced over the all blocks: *winter wheat* (BH), *spring barley* (OP), *maize* (MA) and *winter rape* (CH). Each block has a fixed area (see Fig. 2). The blocks 1 and 3 have an access to irrigation equipments r_1 and r_2. The annual quota of irrigation water over the blocks is $6000m^3$ (respectively $4000m^3$) for r_1 (respectively r_2). Only the *maize* (MA) can be irrigated. There are two different types of soil: type 1 (block 1, 3) and type 2 (block 2, 4). The table on Fig. 2 shows the sequence of crops produced by each plot during the five previous years.

Spatio-temporal hard constraints

1. **h-SCC** - *spatial compatibility of crops*: for instance, the crop CH cannot be assigned to landunits whose soil type is 1 (block 1,3). This biophysical property is not suitable for the crop growing.
2. **h-EQU** - *landunit equality*: landunits on the plots p_7 (respectively p_9) and p_8 (respectively p_{10}) must have the same crop every year. Indeed, these landunits are decided by the farmer to be managed in the same manner.
3. **h-HST** - *landunit historic*: each landunit has defined historic values. The table in Fig. 2 defines the historic of each plot.
4. **h-TSC** - *temporal sequence of crop*: for each couple of crops and landunits, the minimum returned time rt must always be enforced. For instance in the CAP above, $rt(BH) = 2$, $rt(OP) = 3$, $rt(MA) = 2$ and $rt(CH) = 3$.
5. **h-CCS** - *cyclicity of crop sequence*: for each landunit, the crop sequence after the historic must be endlessly repeated by enforcing temporal sequence of crops.



Plots	t1	t2	t3	t4	t5
$\mathbf{p}_1$	MA	MA	BH	OP	MA
$\mathbf{p}_2$	OP	MA	MA	BH	OP
$\mathbf{p}_3$	BH	OP	MA	MA	BH
$\mathbf{p}_4$	MA	BH	OP	MA	MA
$\mathbf{p}_5$	BH	OP	BH	CH	BH
$\mathbf{p}_6$	OP	BH	CH	BH	OP
$\mathbf{p}_7$	MA	MA	MA	MA	MA
$\mathbf{p}_8$	MA	MA	MA	MA	MA
$\mathbf{p}_9$	MA	MA	MA	MA	MA
$\mathbf{p}_{10}$	MA	MA	MA	MA	MA
$\mathbf{p}_{11}$	BH	CH	BH	OP	BH
$\mathbf{p}_{12}$	CH	BH	OP	BH	CH
$\mathbf{p}_{13}$	BH	OP	BH	CH	BH
$\mathbf{p}_{14}$	OP	BH	CH	BH	OP
$\mathbf{p}_{15}$	BH	CH	BH	OP	BH

Fig. 2. A virtual farm with 4 blocks, 15 plots (12ha for each plot) split into 120 landunits. The grey blocks have their own irrigation equipment (r_1, r_2). The table contains the historic values for each plot

	previous crops			
	BH	OP	MA	CH
BH	4	1	1	0
OP	2	3	1	0
MA	0	0	3	0
CH	0	0	0	4

Fig. 3. Table of preceding effect

6. **h-RSC** - *resources capacity*: a fixed amount of resources are available. The quantities of resources accumulated on the landunits do not exceed some limits. For instance, in the CAP defined above, we have only one irrigated crop (*maize* - MA). Knowing that we need $165m^3$ of water by hectare, the annual production of MA on the blocks 1 cannot exceed $36, 36$ *ha*.
7. **h-SCA** - *same crops assigned*: over the time, the same subset of crops must be assigned to every landunit of the same block.

Spatio-temporal preferences

1. **s-TOP** - *Farm topology*: landunits where the same crops are assigned must be spatially grouped. By this we mean that it is preferable to group as most as possible the same crop on the same block. Thus, traveling time can be reduced as well as the time spend by the farmers on operational activities that control the crop production process. Therefore, every isolated landunit is penalized by a cost δ_1.
2. **s-SBC** - *Spatial balanced of crop acreages*: a defined acreage of some crops every year. For instance, in the CAP defined above, the acreage of MA should

be within the range $[24, 48]$ *ha* on block 1 and $[12, 24]$ *ha* on block 3. Any deviation is penalized by a cost δ_2.
3. **s-TBC** - *temporal balanced of crop acreages*: a defined acreage of some crops on each landunit over years. In the CAP defined above, between $[12, 24]$ *ha* of crop CH should be produced on every landunit. Any deviation is penalized by a cost δ_3.
4. **s-CSQ** - *Crop sequence quality*: each pair of successive crops is associated to a cost k_p that defines its preceding effect. Fig. 3 define all k_p values.

In practice, we suggest to define the costs k_p, δ_1, δ_2 and δ_3 such that $\sum k_p > \sum \delta_2 > \sum \delta_1 > \sum \delta_3$. By doing so, a realistic hierarchy can be introduced among the soft constraints. Indeed, first and foremost, the preceding effects k_p must be minimized because of their consequences on the next crops. The spatial balanced of crop acreages related to cost δ_2, implicitly defines the annual receipts of the farmer. It must be ensured as much as possible. Afterwards the working hours can be reduced by grouping the same crops together (δ_1). Lastly, the additional preferences related to the temporal balanced of crop acreages (δ_3) can be enforced.

3 Related work

Since Heady [7], the cropping plan decision was represented in most modelling approaches as the search of the best land-crop combination [11]. Objectives to achieve a suitable cropping plan were often based on complete rationality paradigm using a single monetary criteria optimization, multi-attribute optimization [1] or assessment procedures [2]. In these approaches, the cropping plan decision is mainly represented into models by one of the two concepts, i.e. the cropping acreage [13, 10, 18] or crop rotation [6, 4]. These two concepts are two sides of the cropping plan decision problem, i.e. the spatial and temporal aspects. The originality of our approach lies on the consideration of both dimensions, i.e. spatial and temporal while solving the CAP. In most of the modelling approaches, the cropping plan is not spatially represented and is summarized as simple crop acreage distributions across various land types. At the farm level, the heterogeneity of a farm territory is generally described using soil type as the sole criterion [5].

4 Weighted CSP model of crop allocation

4.1 Weighted CSP Formalism

According to the CAP definition, and assuming a purely CSP formalism cannot deal with preferences easily, we focus on the Weighted CSP (WCSP) formalism which is more appropriate for solving optimization problems. The WCSP formalism [14] extends the CSP formalism by associating cost functions (or preferences) to constraints. A WCSP is a triplet $\langle \mathcal{X}, \mathcal{D}, \mathcal{W} \rangle$ where:

- $\mathcal{X} = \{1, \cdots, n\}$ is a finite set of n variables.
- $\mathcal{D} = \{D_1, \cdots, D_n\}$ is a finite set of variables domain. Each variable $i \in \mathcal{X}$ has a finite domain $D_i \in \mathcal{D}$ of values.
- $\mathcal{W} = \{W_{S_1}, \cdots, W_{S_e}\}$ is a set of cost functions where $S_i \subset \mathcal{X}$ be a subset of variables (i.e., the scope). We denote $l(S_i)$ the set of tuples over S_i. Each cost function W_{S_i} is defined over a subset of variables S_i ($W_{S_i} : l(S_i) \rightarrow [0, m]$ where $m \in [1, \cdots, +\infty]$).

Solving a WCSP is to find a complete assignment $A \in l(\mathcal{X})$ that minimizes $min_{(A \in l(\mathcal{X}))} \left[\sum_{W_{S_i} \in \mathcal{W}} W_{S_i}(A[S_i]) \right]$, where $A[S_i]$ is the projection of a tuple on the set of variables S_i.

4.2 Crop allocation problem definition

The CAP is defined by a set of landunits and crops. The planning problem is defined over a finite horizon $\mathcal{H}$. We define the associated WCSP problem as follow.

$\mathcal{X}$ a set of variables $x_{b,i}^t$ that define the landunit i in block b ($i \in [1, \mathcal{N}_b]$, $b \in [1, \mathcal{B}]$ $\mathcal{B} = 4$ and $\mathcal{N}_1 = 32$ in the CAP described in Fig. 2) at year t ($t \in [1, \mathcal{H}]$). Thus, each landunit is described by $\mathcal{H}$ variables that represent the landunit occupation at every time. We define $[1, h]$ and $[h+1, \mathcal{H}]$ respectively the historic and the future times. For instance, following Fig. 2) and considering $\mathcal{H} = 9$ and $h = 5$, landunit i in block b will be represented by 9 variables where the first five variables (white nodes) are historic variables.

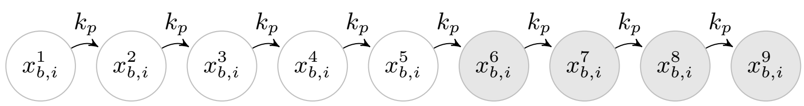


Fig. 4. A temporal sequence of variables over landunit i in block b

$\mathcal{D}$ the domains $D_{b,i}$ of variables $x_{b,i}^t$ is the set of possible crops over the landunit i in block b. Considering the problem in Fig. 2, $\forall b \in [1, \mathcal{B}], \forall i \in [1, \mathcal{N}_b], \quad D_{b,i} = \{1, 2, 3, 4\} = \{BH, OP, MA, CH\}$

$\mathcal{W}$ the cost functions are divided into five different types of hard and soft constraints: (1) simple tabular cost functions (arity up to 5), (2) same global constraint, (3) regular global constraint, (4) gcc global cardinality constraint, (5) soft-gcc soft global cardinality constraint. These cost functions are precisely defined in the next sections.

4.3 Simple cost functions

The hard and soft constraints h-SCC, h-EQU, h-HST, s-TOP and s-CSQ are defined by:

h-SCC : $\forall t \in [h+1, \mathcal{H}]$, $\forall b \in \mathcal{B}$, $\forall i \in \mathcal{N}_b$, let $W^{SCC}_{x^t_{b,i}}$ be a unary cost function associated to spatial compatibility of crops.

$$\forall a \in D_{b,i}, W^{SCC}_{x^t_{b,i}}(a) = \begin{cases} \infty \text{ } \textit{if } a \textit{ is forbidden for block } b, \textit{ landunit } i \\ 0 \quad \textit{otherwise} \end{cases} \tag{1}$$

h-EQU : $\forall t \in [h+1, \mathcal{H}]$, $\forall b \in \mathcal{B}$, for all couple of landunits $(i, j) \in \mathcal{N}_b \times \mathcal{N}_b$ that are decided by the farmer to be managed in the same manner, we define an equality constraint $W^{EQU}_{x^t_{b,i}, x^t_{b,j}}$ between the two landunits.

$$\forall a \in D_{b,i}, \forall a' \in D_{b,j}, W^{EQU}_{x^t_{b,i}, x^t_{b,j}}(a, a') = \begin{cases} 0 \quad \textit{if } a = a' \\ \infty \text{ } \textit{otherwise} \end{cases} \tag{2}$$

h-HST : $\forall b \in \mathcal{B}$, $\forall i \in \mathcal{N}_b$, $\forall t \in [1, h]$, let $W^{HST}_{x^t_{b,i}}$ be an unary cost function associated to the historic values of landunits.

$$\forall a \in D_{b,i}, W^{HST}_{x^t_{b,i}}(a) = \begin{cases} 0 \text{ if } a = \mathsf{historic}(x^t_{b,i}) \\ \infty \quad \textit{otherwise} \end{cases} \tag{3}$$

where $\mathsf{historic}(x^t_{b,i})$ returns the historic value of landunit i in block b at time t.

s-TOP : $\forall t \in [1, \mathcal{H}]$, $\forall b \in \mathcal{B}$, $\forall i \in \mathcal{N}_b$, let W^{TOP}_S be an n-ary cost function associated to the farm land topology. We define a neighborhood function $\mathsf{neighbor}(i)$ which returns the landunits $j \in \mathcal{N}_b$ spatially close to i. For instance, in the CAP presented on Fig. 2, we consider the 4 nearest neighbors, the so-called von Neumann neighborhood. Here, the scope S is equal to $\{x^t_{b,i}, x^t_{b,n}, x^t_{b,s}, x^t_{b,e}, x^t_{b,w}\}$ where landunits n, s, e, w are the 4 nearest neighbors respectively at the North, South, East and West of i. $\forall a \in D_{b,i}, \forall a_n \in D_{b,n}, \forall a_s \in D_{b,s}, \forall a_e \in D_{b,e}, \forall a_w \in D_{b,w}$

$$W^{TOP}_S(a, a_n, a_s, a_e, a_w) = \begin{cases} 0 \text{ if } a = a_n = a_s = a_e = a_w \\ \delta_1 \quad \text{otherwise} \end{cases} \tag{4}$$

According to the position of i in its block, the arity of W^{TOP}_S could be reduced to 4 or 3.

s-CSQ : $\forall t \in [1, \mathcal{H}]$, $\forall b \in \mathcal{B}$, $\forall i \in \mathcal{N}_b$, let $W^{CSQ}_{x^t_{b,i}, x^{t+1}_{b,i}}$ be a binary cost function associated to the preceding effect k_p.

We define a function $\mathsf{KP}(a, a')$ that returns the preceding effect k_p of doing the crop a' after a.

$$\forall a \in D_{b,i}, \forall a' \in D_{b,i}, W^{CSQ}_{x^t_{b,i}, x^{t+1}_{b,i}}(a, a') = \mathsf{KP}(a, a') \tag{5}$$

4.4 Crop collection over a block using same constraints

h-SCA : considering a block b, the subset of $(\mathcal{H} - h)$ future variables $x^t_{b,i}$ (with $t \in [h+1, \mathcal{H}]$) associated to each landunit i in b must be assigned to the same crop collection. Thus, $\forall (i, j) \in \mathcal{N}_b \times \mathcal{N}_b$ (with $i \neq j$), the set of values assigned to the temporal sequence of variables defining i is a permutation of those of j. By using the same constraint introduced in [3] we define h-SCA. For each block b, we choose a leading landunit i. We then define a $2 * (\mathcal{H} - h)$-ary cost function W^{SCA}_S associated to each pair of sequence of variables that defines $x^t_{b,i}$ and $x^t_{b,j}$ $(i \neq j)$. Thus, the scope S is $\{x^{h+1}_{b,i}, \cdots, x^{\mathcal{H}}_{b,i}, x^{h+1}_{b,j}, \cdots, x^{\mathcal{H}}_{b,j}\}$. Let $A[x^{h+1}_{b,i}, \cdots, x^{\mathcal{H}}_{b,i}]$ and $A[x^{h+1}_{b,j}, \cdots, x^{\mathcal{H}}_{b,j}]$ denote the two sub-assignments of the variables in S. The constraint W^{SCA}_S requires that $A[x^{h+1}_{b,i}, \cdots, x^{\mathcal{H}}_{b,i}]$ is a permutation of $A[x^{h+1}_{b,j}, \cdots, x^{\mathcal{H}}_{b,j}]$.

$$W^{SCA}_S = \mathsf{same}(\underbrace{x^{h+1}_{b,i}, \cdots, x^{\mathcal{H}}_{b,i}}_{i}, \underbrace{x^{h+1}_{b,j}, \cdots, x^{\mathcal{H}}_{b,j}}_{j}) \quad (6)$$

4.5 Crop sequence using regular global constraints

The constraints h-TSC and h-CCS are related to temporal crop sequences. We represent them by using the regular constraint [16]. $\forall t \in [1, \mathcal{H}]$, $\forall b \in \mathcal{B}$, $\forall i \in \mathcal{N}_b$, $\forall a \in D_{b,i}$, let $M^a_{b,i}$ be a non deterministic finite automaton (NFA), $\mathcal{L}(M^a_{b,i})$ the language defined by $M^a_{b,i}$, and $S_{b,i}$ a temporal sequence of $\mathcal{H}$ variables that describes landunit i of block b over the horizon. Solving a $\mathsf{regular}(S_{b,i}, M^a_{b,i})$ constraint is to find an assignment $A[S_{b,i}]$ such that $A[S_{b,i}] \in \mathcal{L}(M^a_{b,i})$.

h-TSC : considering each landunit $x_{b,i}$, the crop sequence is enforced by defining for each crop $a \in D_{b,i}$ a language $\mathcal{L}(M^a_{b,i})$ such that the same value a is assigned to $(x^t_{b,i}$ and $x^{t'}_{b,i})$ *iff* $x^{t'}_{b,i}$ enforces the minimum returned time $rt(a)$ i.e., $\forall t' \neq t, \quad t' \geq t + rt(a)$. We define $\mathsf{regular}(S_{b,i}, M^a_{b,i})$ where $M^a_{b,i}$ is described as in Fig. 5 for crop $a = CH$ the minimum return time of which is $rt(CH) = 3$ years. Here, the initial state is 0 while final states are $4, 5, 6$. Arcs are labelled with crop values.

As shown by the NFA in Fig. 5, the historic variables are used to enforce the minimum return time over the future variables. We then define an $\mathcal{H}$-ary cost function $W^{TSC^a}_{S_{b,i}}$ associated to each pair of landunit i in block b and each crop a such that:

$$\forall b \in \mathcal{B}, \forall i \in \mathcal{N}_b, \forall a \in D_{b,i}, W^{TSC^a}_{S_{b,i}} = \mathsf{regular}(x^1_{b,i}, \cdots, x^t_{b,i}, \cdots, x^{\mathcal{H}}_{b,i}, M^a_{b,i}) \quad (7)$$

h-CCS : considering each landunit $x_{b,i}$, we combine h-TSC with a repeatability constraint also defined by a set of regular constraints. The constraint h-CCS ensures that any crop sequence assignment after the historic can be endlessly

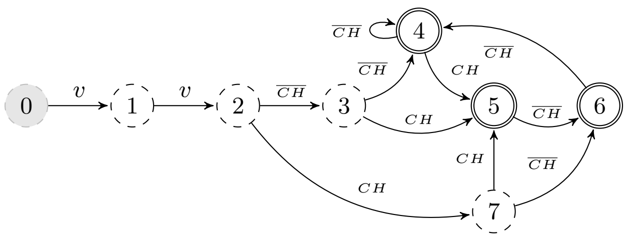


Fig. 5. Automaton for crop CH with $rt(CH) = 3$ and $h = 4$. v denotes any value in $D_{b,i}$. The notation $\overline{CH}$ corresponds to $D_{b,i} \setminus \{CH\}$. The associated language accepts every pattern over the historic variables and only the patterns that enforce the minimum return time in the future variables (e.g., OP-CH-OP-CH-BH-OP-CH-BH).

repeated without violating the minimum return time constraint h-TSC. Fig. 6 describes a cyclic NFA for crop CH. The initial state is 0 while final states are $3, 6, 9, 12$. The scope of the cost function $W^{CCS^a}_{S_{b,i}}$ is restricted to future variables.

$$\forall b \in \mathcal{B}, \forall i \in \mathcal{N}_b, \forall a \in D_{b,i}, W^{CCS^a}_{S_{b,i}} = \mathsf{regular}(x^{h+1}_{b,i}, \cdots, x^{\mathcal{H}}_{b,i}, M^a_{b,i}) \qquad (8)$$

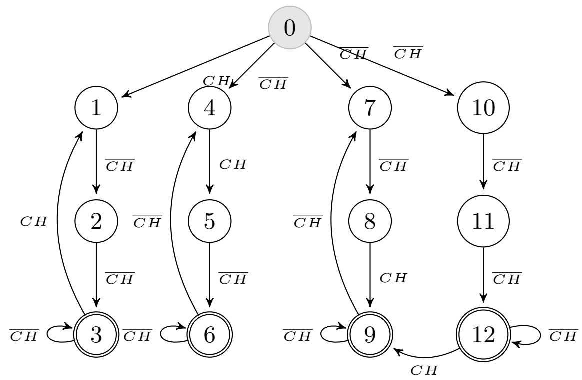


Fig. 6. Cyclic automaton for the crop CH with $rt(CH) = 3$ and $\mathcal{H} - h = 4$.

4.6 Resource capacity constraints using global cardinality constraints

In CAP, each landunit consumes a fixed amount of resources according to some structural (crop type, the area of landunits, etc.) and numerical (the irrigation dose) requirements. For instance, the maize (MA) is an irrigated crop whereas winter wheat (BH) does not need irrigation. A classical approach to deal with resources is to solve a shortest path problem with resource constraints [9]. The problem is NP-hard if the path needed is elementary. Loosely, solving a resource allocation problem involves both sequencing and counting reasoning. We assume in the CAP that this problem can be reduced to a counting problem under hypothesis 1 and 2.

Hypothesis 1 *: Resources are supposed to be usable and systematically renewed every year without doing anything (e.g. annual quota of irrigation water).*

This hypothesis is closed to a real CAP because farmers usually have a fixed quota of irrigation water. That can be exactly the case for the working hours capacity in a year if work regulations is taken into account.

Hypothesis 2 *: $\forall t \in [1, \mathcal{H}]$, $\forall (b, b') \in \mathcal{B} \times \mathcal{B}$ a couple of blocks, $\forall (i, j) \in \mathcal{N}_b \times \mathcal{N}'_b$ a couple of landunits. The areas of landunits i and j of block b (respectively b') can be considered equivalent according to the problem size.*

We make the assumption that the spatial sampling of the farm land into landunits is homogeneous. Under these hypothesis the annual resource allocation is seen as a counting problem at every time $t \in [h + 1, \mathcal{H}]$. Thus, given annual resources capacities for a CAP, we define for each time $t \in [h + 1, \mathcal{H}]$ an upper and lower bound to the number of variables $x^t_{i,b}$ that are assigned to a given crop according to both structural and numerical requirements.

h-RSC : to enforce resource capacity constraints h-RSC, we use the global cardinality constraint gcc [17] over the assignments of crops to landunits.

$\forall t \in [h+1, \mathcal{H}]$, let $W^{RSC}_{S^t_b}$ be a $\mathcal{N}_b$-ary global constraint associated to resource capacities.

Given $S^t_b = (x^t_{b,1}, \cdots, x^t_{b,\mathcal{N}_b})$ the global cardinality constraint (gcc) specifies, for each value $a \in \bigcup D_{b,i}$, an upper bound $ub(a)$ and a lower bound $lb(a)$ to the number of variables $x^t_{b,i}$ that are assigned to a.

$$W^{RSC}_{S^t_b} = \mathsf{gcc}(S^t_b, lb, ub) \tag{9}$$

has a solution if there exists an assignment of S^t_b such that

$$\forall a \in \bigcup D_{b,i},\ lb(a) \leq |\{x^t_{b,i} \in S^t_b | x^t_{b,i} = a\}| \leq ub(a) \tag{10}$$

4.7 Spatio-temporal balance of crops using soft-gcc

Preferences related to the spatio-temporal balance of crops (s-SBC and s-TBC) are defined as soft global cardinality constraints (soft-gcc) that allow the violation of both lower and upper bounds of the associated hard constraint gcc.

$$\text{soft-gcc}(S, lb, ub, z, \mu) = \{(A[S], a_z) | A[S] \in l(S), a_z \in D_z, \mu(A[S]) \leq a_z\} \quad (11)$$

where lb and ub are respectively the lower and upper bounds, z a cost variable with finite domain D_z, μ the violation measure for the global constraint soft-gcc. In this work, we use the variable-based violation measure (see [8]) which is the minimum number of variables whose values must be changed in order to satisfy the associated gcc constraint. Thus soft-gcc(S, lb, ub, z, μ) has a solution if $\exists\ A[S]$ such that $min(D_z) \leq \mu(A[S]) \leq max(D_z)$. Based on this definition the constraints s-SBC and s-TBC are formalized as follow.

s-SBC : $\forall t \in [h+1, \mathcal{H}]$, $\forall b \in \mathcal{B}' \subseteq \mathcal{B}$. Let $W^{SBC}_{S^t_b}$ be a $|\mathcal{B}'|$-ary soft-gcc constraint associated to block b at time t. The scope $S^t_b = \{x^t_{b,i} | i \in [1 \cdots \mathcal{N}_b]\}$.

$$W^{SBC}_{S^t_b} = \text{soft-gcc}(S^t_b, lb, ub, z, \mu) \quad (12)$$

s-TBC : $\forall b \in \mathcal{B}' \subseteq \mathcal{B}$, $\forall i \in \mathcal{N}_b$. Let $W^{TBC}_{S_{b,i}}$ be a $(\mathcal{H} - h)$-ary soft-gcc constraint associated to each landunit i. The scope $S_{b,i} = \{x^{h+1}_{b,i}, \cdots, x^{\mathcal{H}}_{b,i}\}$. Excepted the scope, $W^{TBC}_{S_{b,i}}$ is exactly defined as the global soft cardinality constraint defined for s-SBC.

5 Implementation

5.1 CAP instances description

We performed the experimentations by using four instances of the virtual farm presented in Fig. 2. Each instance corresponds to a new sampling of landunits. The number of landunits is increased from 15 to 120 (15, 30, 60, 120). For the CAP instance with 15 landunits, $\mathcal{N}_1 = \mathcal{N}_3 = 4, \mathcal{N}_2 = 2$ and $\mathcal{N}_4 = 5$ where $\mathcal{N}_i$ denotes the number of landunits in the block i. In this problem, sampling is done such that the plots (see Fig. 2) are also the landunits (12 *ha* per landunit). These landunits are gradually refined by splitting them into 2, 4 and 8 smaller ones, to respectively build the instances with 30, 60 and 120 landunits. These sampling are chosen to be representative of different farm sizes. The planning horizon is nine years. According to the minimum return time (*winter wheat* $rt(BH) = 2$, *spring barley* $rt(OP) = 3$, *maize* $rt(MA) = 2$ and *winter rape* $rt(CH) = 3$) the four last years are dedicated to the future while the five first are historic ones. We use the historic values presented in Fig. 2.

We should emphasis that there is no constraints or preferences between blocks as described in Section 2.2. Thus, we first focus on solving each block independently. The instances associated to the block 1 are B1-LU4, B1-LU8, B1-LU16, B1-LU32 respectively for 4, 8, 16, 32 landunits. For all these experimentations the costs associated to s-TOP, s-SBC and s-TBC are respectively $\delta_1 = 2$, $\delta_2 = 100$ and $\delta_3 = 10$. By doing so, we implicitly introduce a hierarchy among the soft constraints according to the criterion defined in the last paragraph of section 2.2. To fine-tune the weight of preceding effects k_p in the global cost function, we introduced a factor $\delta_4 = 10$ such that k_p are set to $\delta_4 * KP$. By doing so, the crop sequences that minimize the preceding effects are desired to be satisfied as much as possible.

Secondly, we add a new preference over all blocks in our original model. We define a new cost function $W^{SBC}_{S^t}$, extending the previous $W^{SBC}_{S^t_b}$ described in section 2.2 such that the annual global acreage of MA and BH over all blocks should be respectively within the range $[40, 72]$ ha and $[70, 100]$ ha. The CAP instances B1[1-4]-LU15(*), B[1-4]-LU30(*), B[1-4]-LU60(*) and B[1-4]-LU120(*) are associated to these new problems. The blocks are now interdependent and consequently the maximum arity of soft global cardinality constraints is equal to the total number of landunits. All of these instances are available in the cost function benchmark[1].

For each instance, the number of constraints depends on the domain size d ($d = 4$), the number of blocks b (from 1 to 4), the number $\mathcal{N}_i$ of landunits in the block i and the planning horizon h ($h = 9$). Let h^- and h^+ respectively be the size of historic and future. According to the instance, the number of global cardinality constraints (h-RSC, s-SBC, s-TBC) increase from $(2h^+ + 2)$ to $(6h^+ + 56)$. There are more than half of these constraints whose arity is $\mathcal{N}_i$ while the maximum can be $\sum \mathcal{N}_i$. The number of h-ary and h^+-ary regular constraints (h-TSC, h-CCS) is $2d \sum \mathcal{N}_i$. There are $\sum \mathcal{N}_i - b$ $2h^+$-ary same constraint (h-SCA). In addition to these constraints we can enumerate at least $h^- \sum \mathcal{N}_i$ unary cost functions (h-SCC, h-HST), $(h-1)\sum \mathcal{N}_i$ binary cost functions (s-CSQ) and about $\sum \mathcal{N}_i$ cost functions whose arity is more than two (s-TOP, h-EQU).

5.2 Analysis of the results

For solving the CAP, we use a Depth-First Branch and Bound (DFBB) algorithm implemented in the **Toulbar2** solver[2] (version 0.9.1) using default options. Columns $|\mathcal{X}|$ and $|\mathcal{W}|$ of Tab. 1 shows the number of variables and constraints for each instance.

The results presented in Tab. 1 are performed on a 2.27GHz Intel(R) Xeon(R) processor. Total CPU times are in seconds. We measure total times to find and prove optimality (column Time(s) of One optimal (DFBB)) starting with a relatively good upper bound (column UB). The initial upper bound has an important impact on performance. We chose its value empirically. Based on optimal values, we also measure total times to find all the optimal solutions (column Time(s) of All optimal (DFBB)) by setting the initial upper bound to the optimum (column Opt.) plus one.

[1] http://www.costfunction.org/benchmark?task=browseAnonymous&idb=33
[2] http://mulcyber.toulouse.inra.fr/projects/toulbar2

Table 1. An 0ptimal and all optimal solutions using DFBB

Instance of CAP	$\|\mathcal{X}\|$	UB	$\|\mathcal{W}\|$	Opt.	One optimal (DFBB)			All optimal DFBB			
					Time(s)	Nodes	BT	Time(s)	Nodes	BT	Nb.Sol
B1-LU4	36	1000	91	92	0.39	17	10	0.08	8	4	5
B1-LU8	72	2000	175	184	2.96	94	49	0.21	32	16	17
B1-LU16	144	4000	343	368	21.47	413	209	2.64	256	512	257
B1-LU32	288	6000	679	640	228	285	147	6.19	38	19	17
B2-LU2	18	1000	47	38	0.08	2	2	0.06	2	1	1
B2-LU4	36	2000	95	116	0.22	8	4	0.22	8	4	1
B2-LU8	72	4000	191	392	4.19	6	5	0.36	2	1	1
B2-LU16	144	6000	383	752	7.9	10	9	0.78	2	1	1
B3-LU4	36	1000	99	328	0.3	14	7	0.29	16	8	2
B3-LU8	72	2000	199	656	0.64	14	7	0.6	16	8	2
B3-LU16	144	4000	367	1312	1.51	18	9	1.37	16	8	2
B3-LU32	288	6000	703	2592	4.1	20	10	3.79	18	9	2
B4-LU5	45	1000	119	46	0.53	4	4	0.08	0	0	1
B4-LU10	90	2000	239	192	11.64	5	4	0.57	0	0	1
B4-LU20	180	4000	479	752	12.32	12	10	0.73	0	0	1
B4-LU40	360	6000	959	1504	39.33	23	19	1.97	2	1	1
B[1-4]-LU15(*)	135	2000	360	704	21.02	257	131	7.87	96	48	2
B[1-4]-LU30(*)	270	4000	712	1560	323.02	1029	521	155.9	498	249	12
B[1-4]-LU60(*)	540	4000	1384	3852	2412.97	1297	658	3697.23	2228	1114	136
B[1-4]-LU120(*)	1080	8000	2728	-	-	-	-	-	-	-	-

While focusing on independent blocks, the best solution is got in less than a minute excepte for B1-LU32. The optimum is found and proved for all the instances. The differences between CPU times to find one optimal and all the optimal solutions is mainly due to the quality of the initial upper bound. The results found while introducing interdependence between blocks are also acceptable compared to the problem size. Indeed, the scope of some gcc and soft-gcc constraints is equal to the number of landunits (120 variables in the worse case). This may explain why the instance B[1-4]-LU120(*) is not closed after 48 hours.

6 Conclusion

In this paper, we have modelled the crop allocation problem (CAP) using the Weighted CSP formalism. Contrary to existing approaches for solving such a problem, our proposition combines both the spatial and the temporal aspects of crop allocation. We explicitly described how the farmers' hard and soft constraints can be addressed as a global objective function optimization problem. The results have shown that on small and middle CAP, the *Toulbar2* solver can deliver relevant solutions in acceptable computational time. In the future, we will investigate the CUMULATIVE constraint for expressing more complex resource management and the COSTREGULAR constraint for mixing the return time and preceding effects, taking inspiration from the work done by [15].

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