

# ISC Operator for reconstructing Bayesian Network in gene networks context.

Jimmy Vandel & Simon de Givry

## Outlines:

- Biological motivation
- Bayesian Networks framework
- Learning Algorithms
- Local Operators
- Comet language
- Experimentation

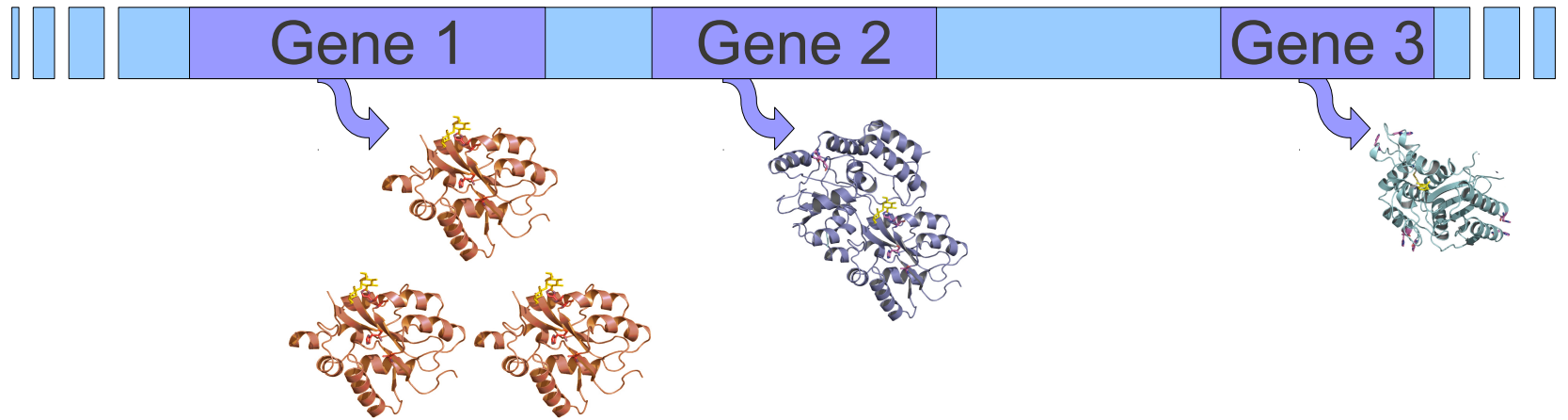
# Biological motivation

DNA



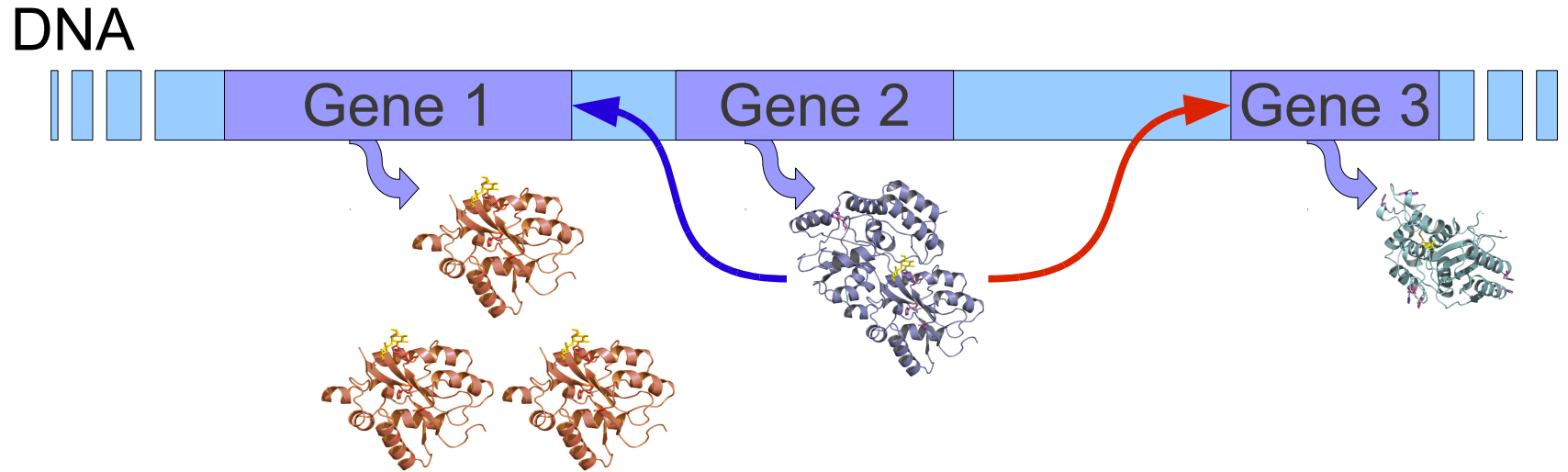
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→ gene expressions (mRNA concentrations)

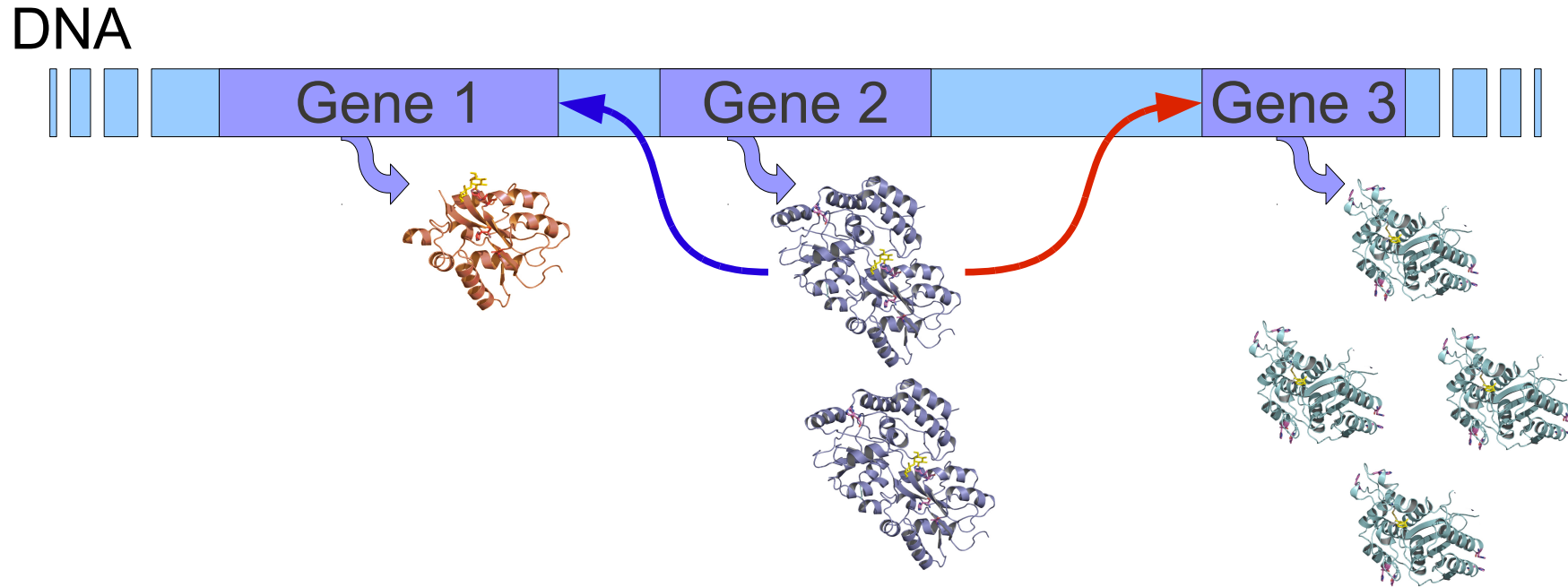
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→ gene expressions (mRNA concentrations)

→ gene regulations

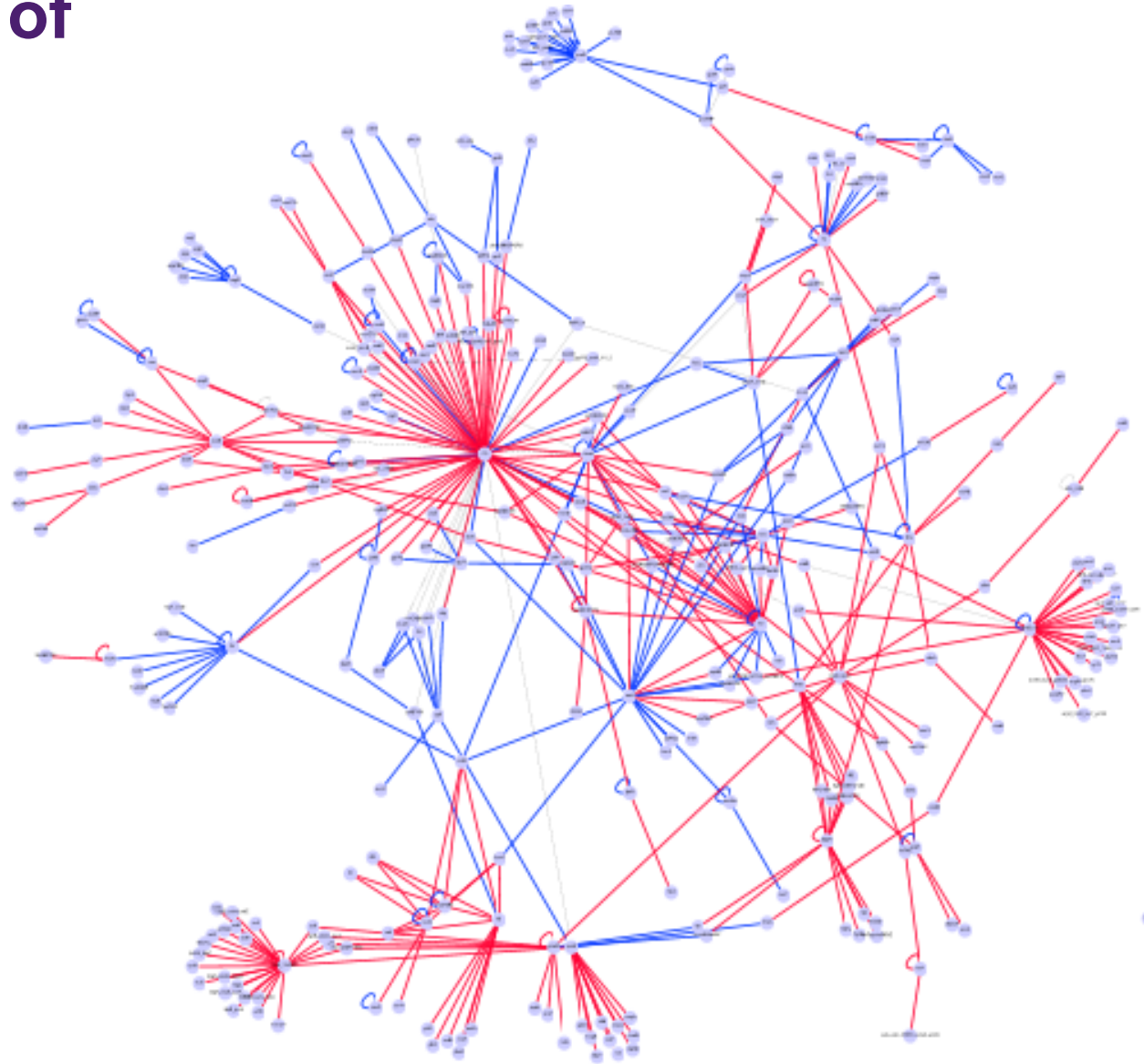
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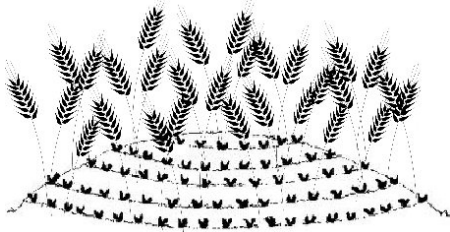
→ gene regulations

**Goal :**  
**Reconstruction of  
gene regulatory  
network.**



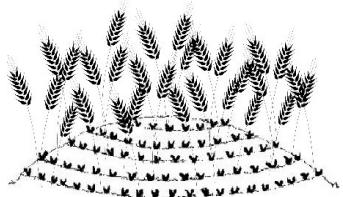
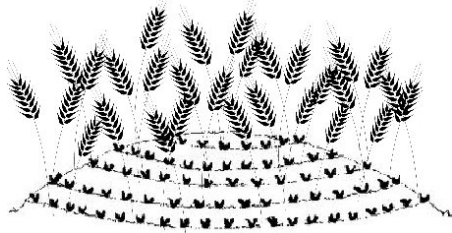
**Escherichia coli**  
(423 genes, 578 regulations)  
(SS. *Shen-Orr and al., 2002*)

# Polymorphism

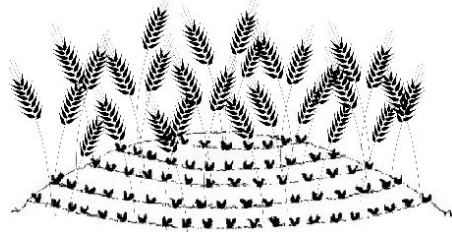




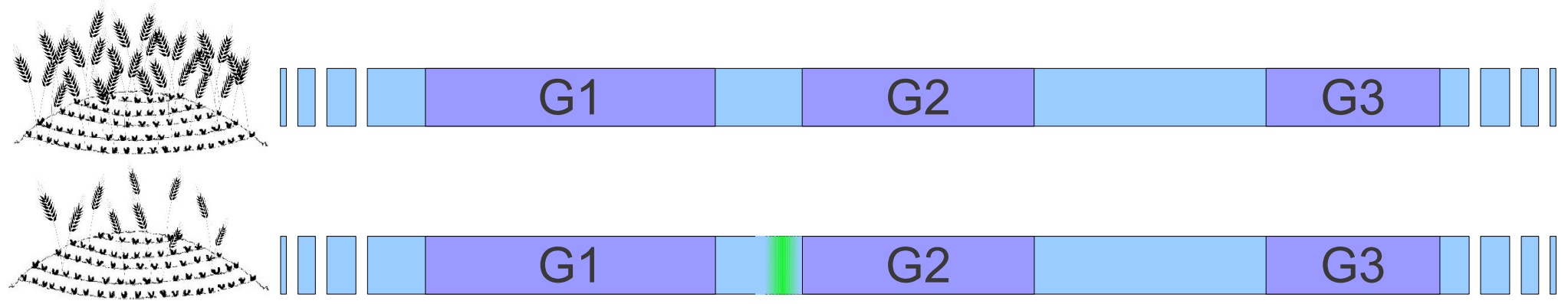
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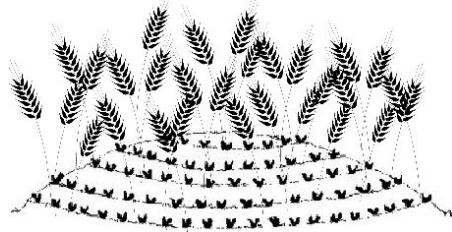


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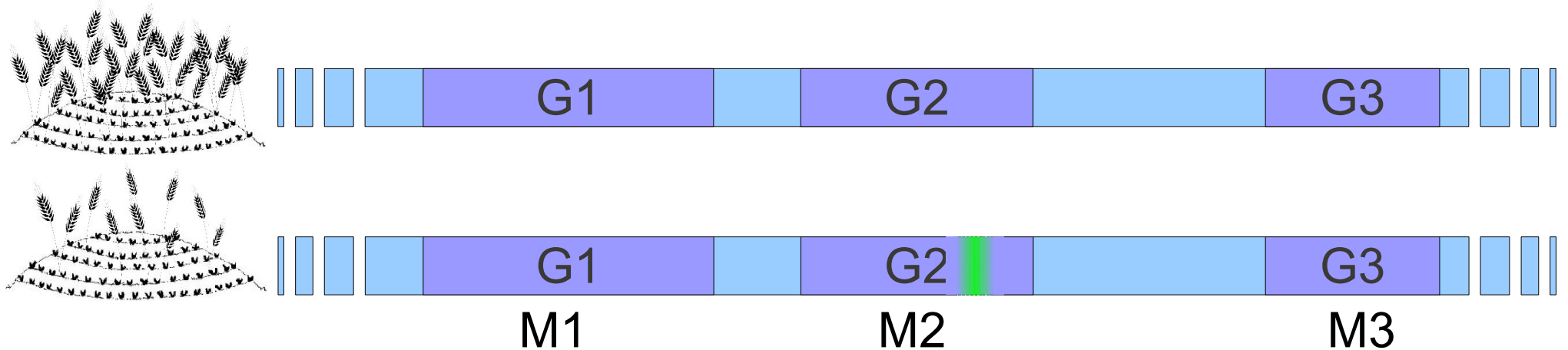
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DNA mutations in genes: in promoter region → impact on its gene activity  
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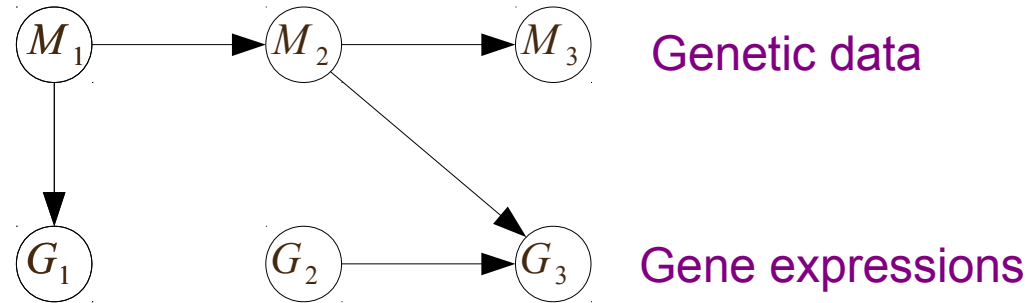


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Genetic data from one genetic marker (SNP) for each gene

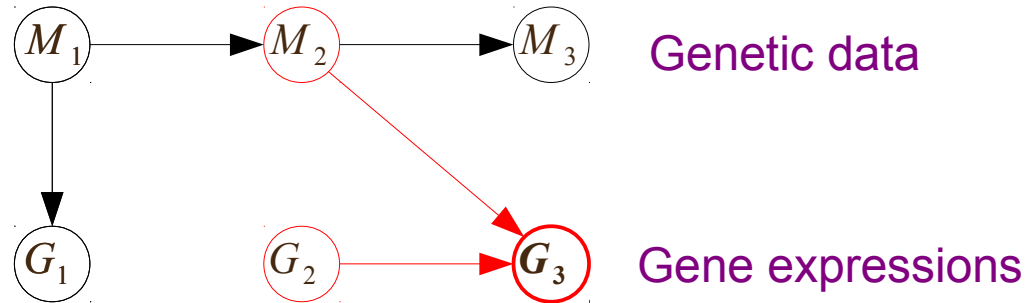
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Directed acyclic graph  $G$  composed of  $n$  variables  $X_i = \{G_i, M_i\}$



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Conditional distribution  $P_G(\mathbf{G}_3 / G_2, M_2)$

		$G_3$	$!G_3$
$G_2$	$M_2$	0.72	0.28
$G_2$	$!M_2$	0.59	0.41
$!G_2$	$M_2$	0.63	0.37
$!G_2$	$!M_2$	0.10	0.90

Graphic representation of a joint probability distribution

$$P_G(X) = \prod_{i=1}^n P_G(X_i / Pa_i)$$

# Learning strategy

We look for the graph  $G_{score} = \operatorname{argmax}_{G_i} P(G_i | D)$  with dataset  $D$ .

$$P(G_i | D) = \frac{P(D | G_i) P(G_i)}{P(D)}$$

$$\propto P(D | G_i) P(G_i)$$

- $P(D | G_i)$  : marginal likelihood of  $G_i$
- $P(G_i)$  : prior probability of the graph  $G_i$   
→ assumed to be uniform

Objective function easy to evaluate and avoids over-fitting

- decomposable and penalized scores
  - **BDe score** (**D.Heckerman** *Machine learning* 1995)
  - **BIC score** (**G.Schwartz** *Annals of statistics* 1978)



# Local search components

## 1. Search space

- **Directed Acyclic Graph**
- Partial DAG (PDAG)
  
- variable orders

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## 4. Meta-heuristics

- **hill climbing (with restarts)**
- tabu search
- simulated annealing
- MCMC
- genetic algorithms
- ...

# Local Operators

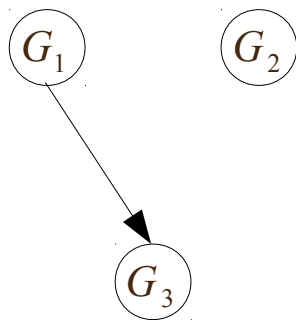
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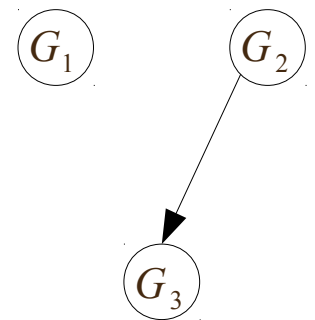
Example:

*Current situation*



$$\Delta_{score} Add(G_2, G_3) > \Delta_{score} Add(G_1, G_3) > 0$$

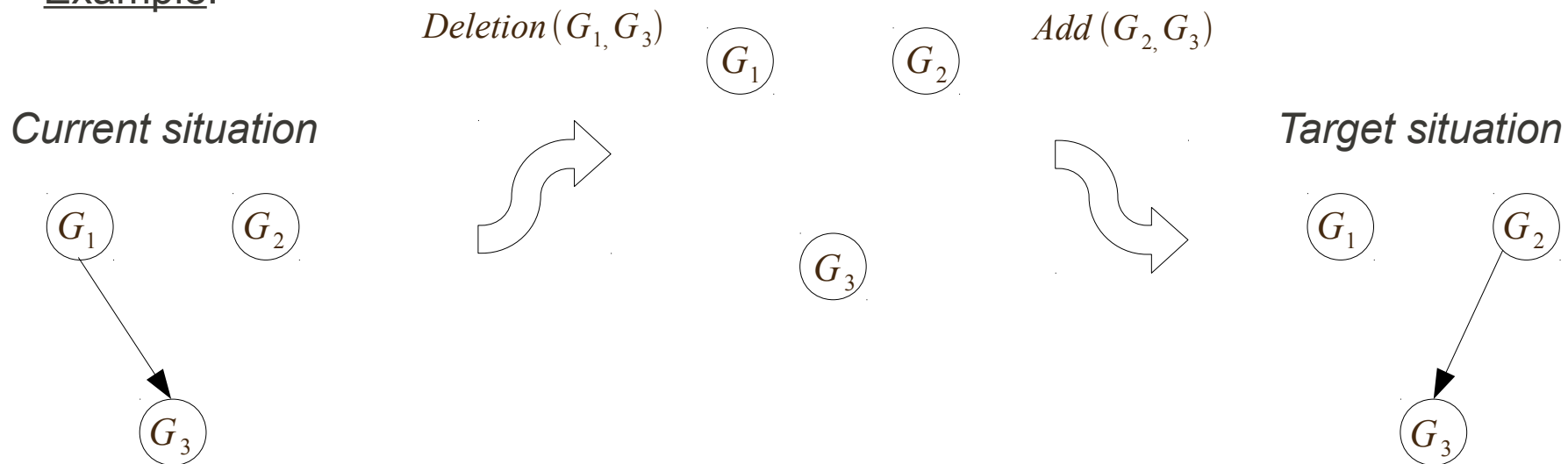
*Target situation*



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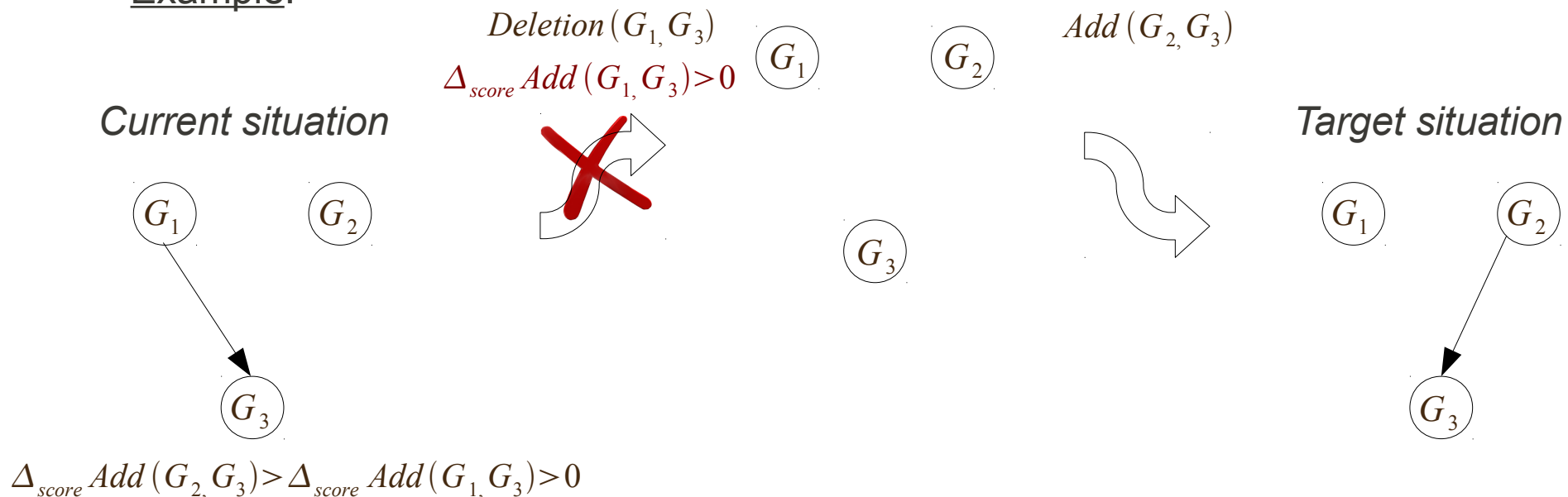


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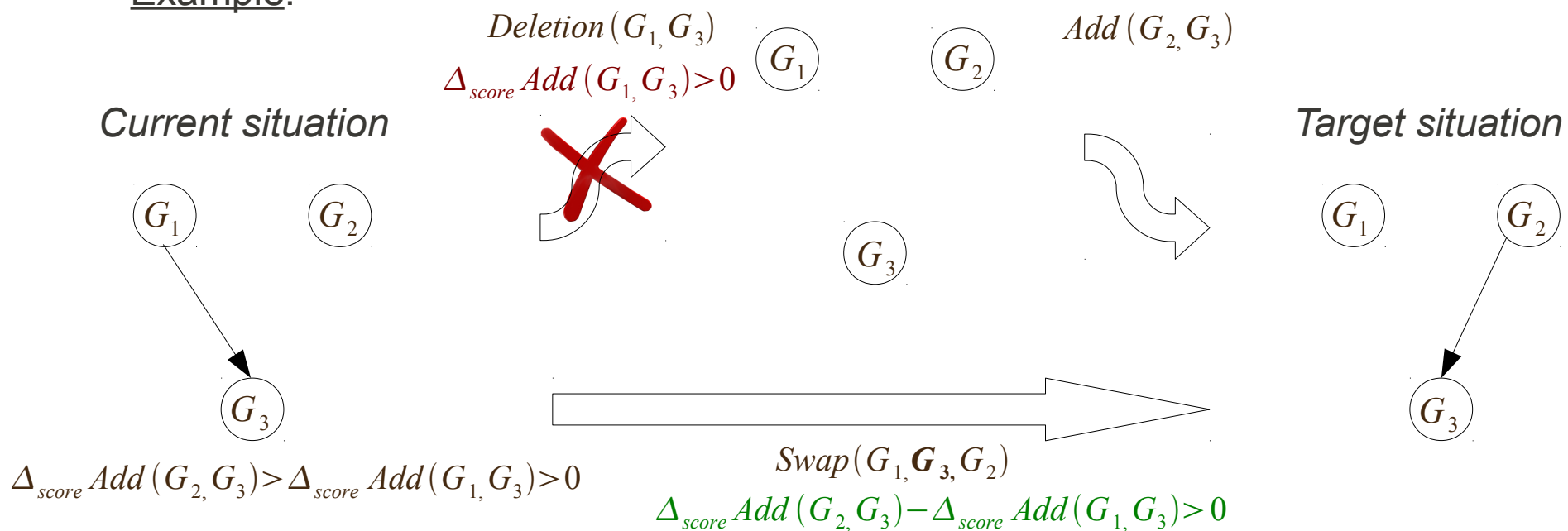




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Example:



→ escape from some local maxima

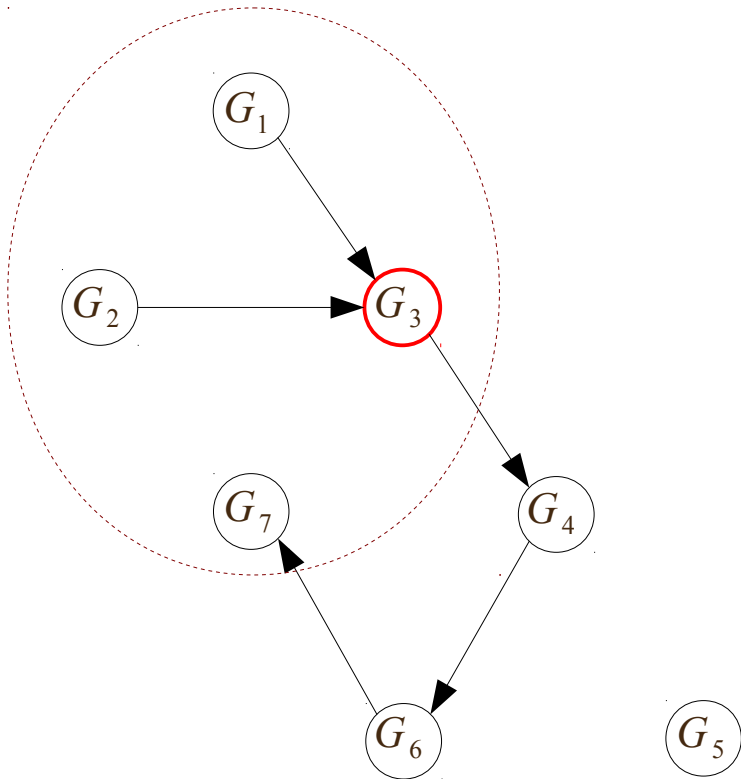
# ISC Operator

(Iterative Swap Operator)

$Swap(G_2, G_3, G_7)?$

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$$\Delta_{score} Add(G_7, G_3 | G_1) > \Delta_{score} Add(G_2, G_3 | G_1) > 0$$



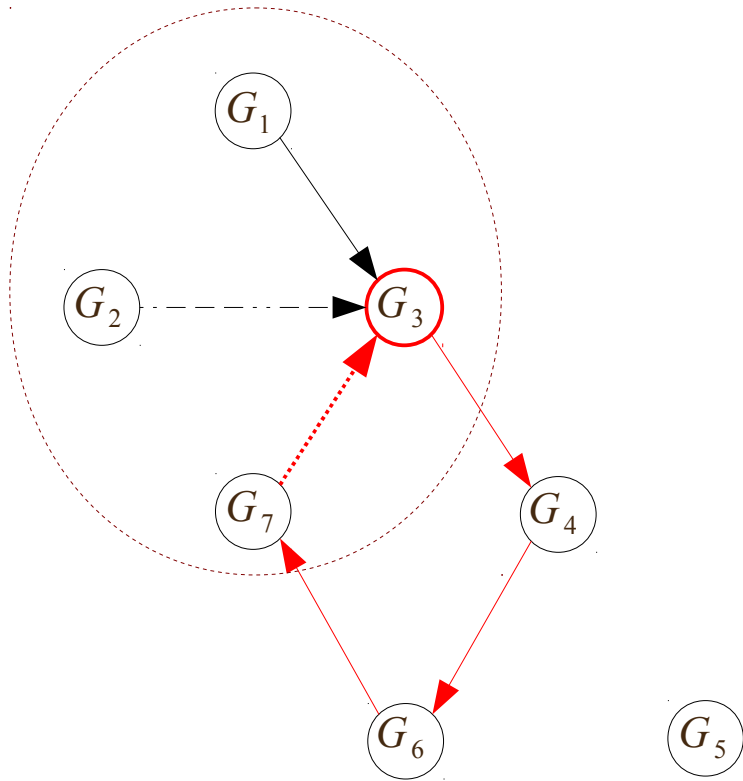
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$Swap(G_2, G_3, G_7)? \longrightarrow Cycle\{G_3, G_4, G_6, G_7\}$

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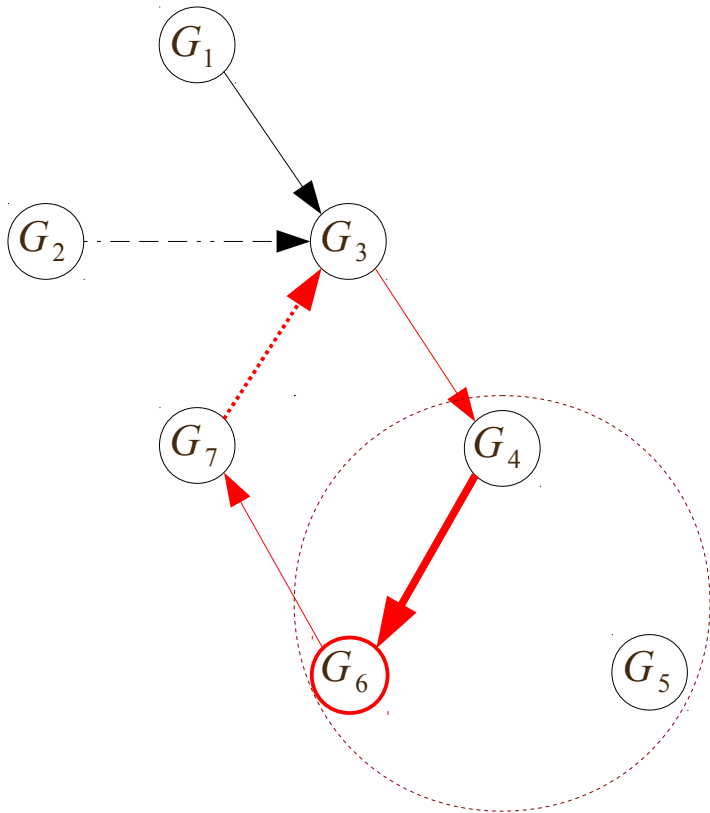
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Select the edge of the cycle minimizing  $\Delta_{score} Add$



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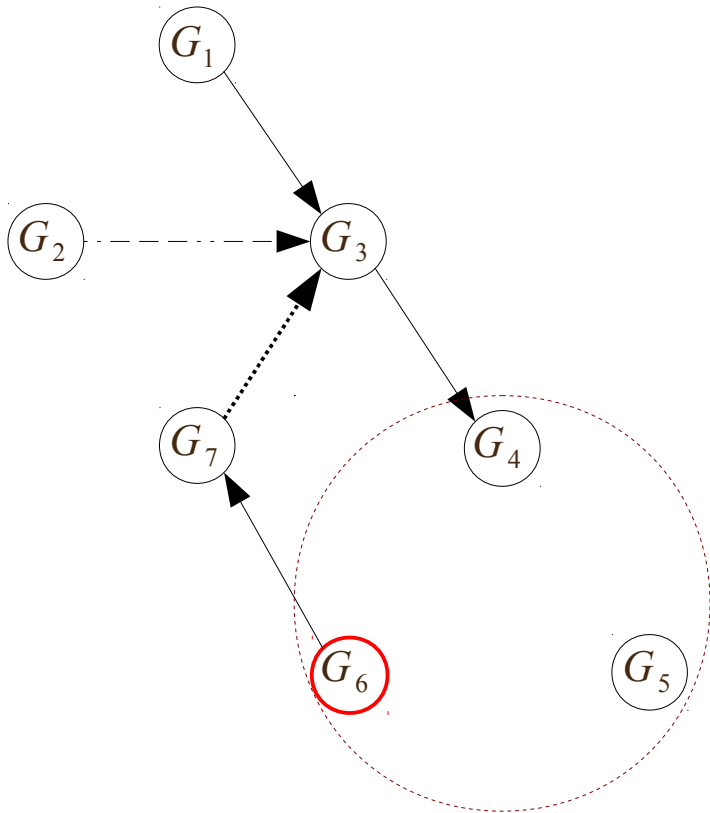
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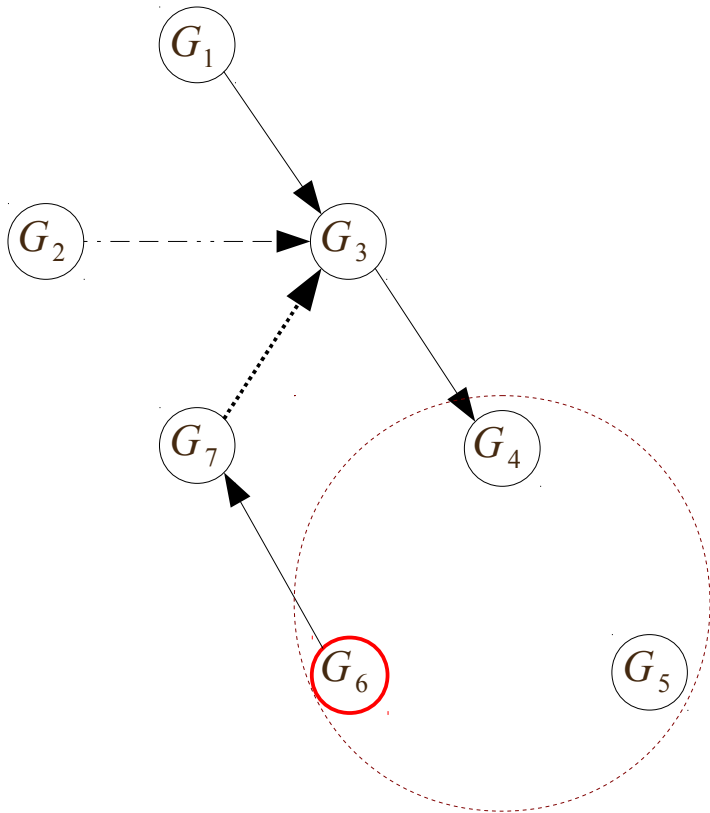
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If  $\Delta_{score} Swap(G_2, G_3, G_7) + \Delta_{score} Deletion(G_4, G_6) \leq 0$

Else

Record  $Deletion(G_4, G_6)$

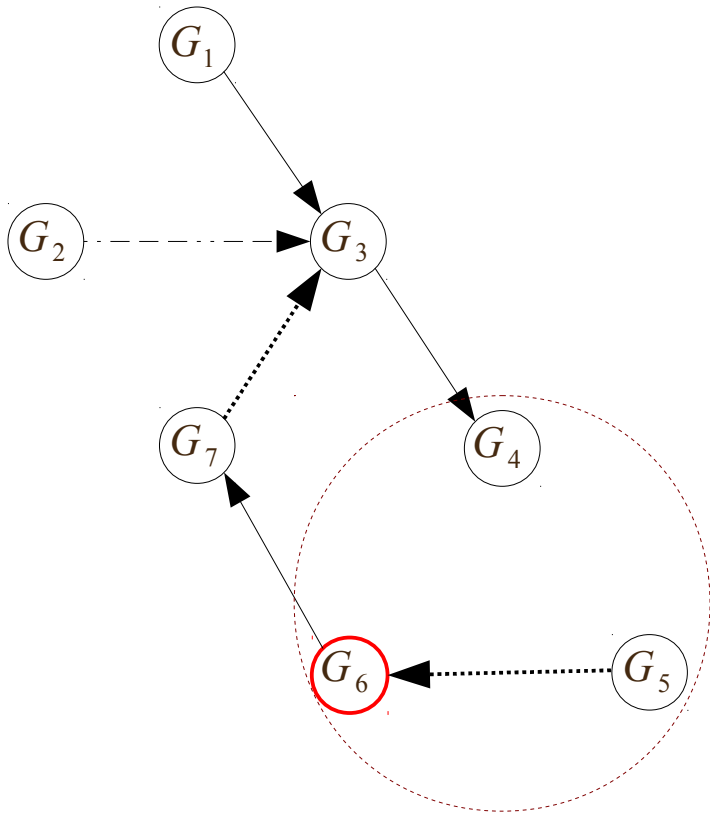
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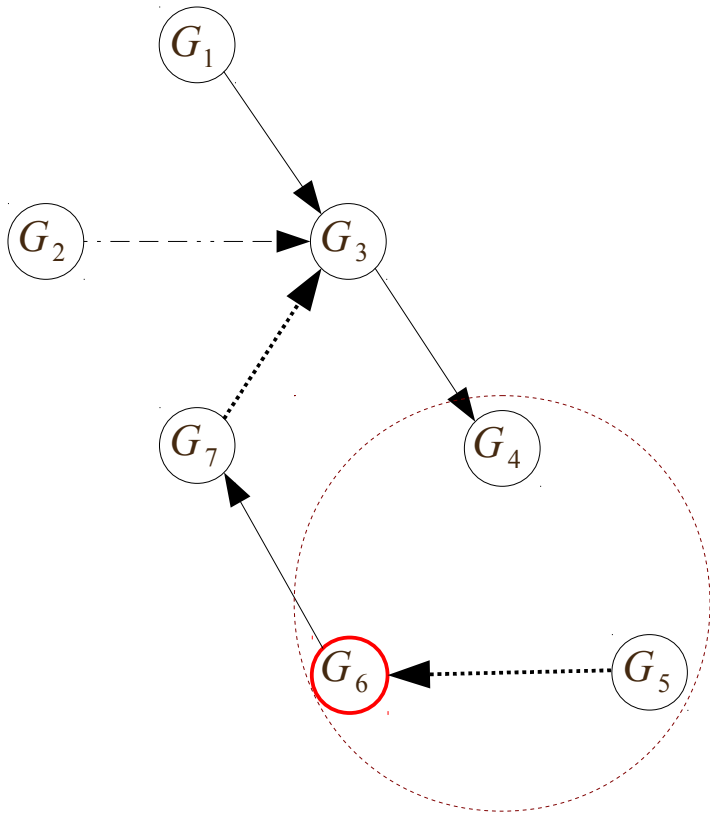
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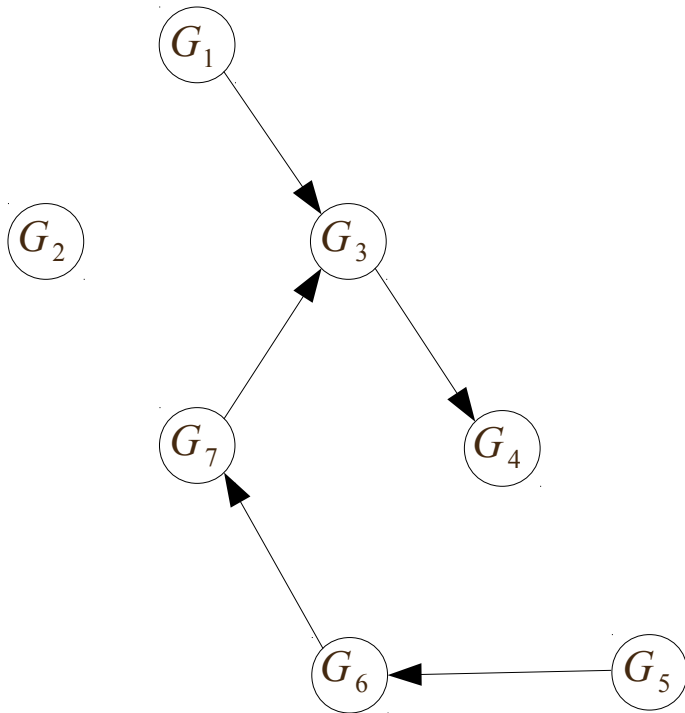
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Record  $Swap(G_4, G_6, G_5)$

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If ! STOP

Validate all recorded moves

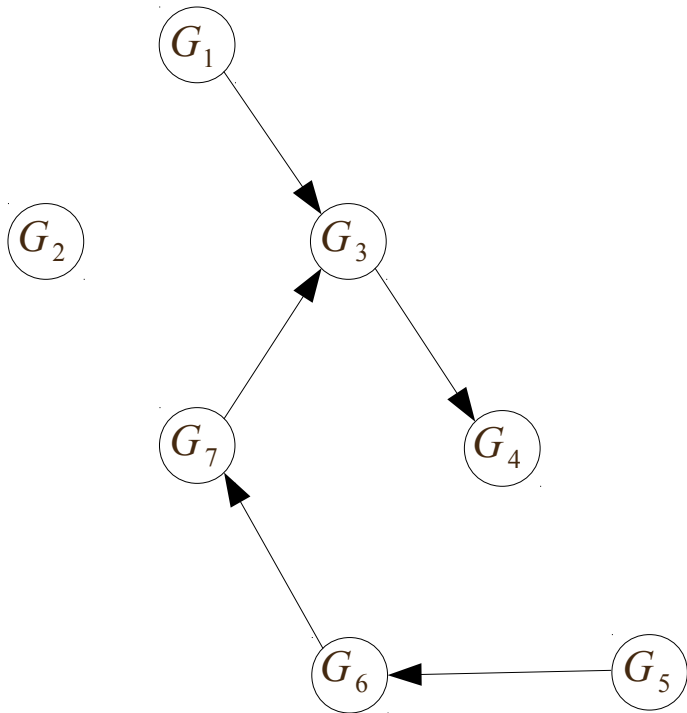
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~~While there exist a cycle and ! STOP~~ **nISC operator**

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**Is a**

High level programming language

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# Comet Language

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**Offering easy implementation for**

**Invariants**

Objective functions

Constraints definition

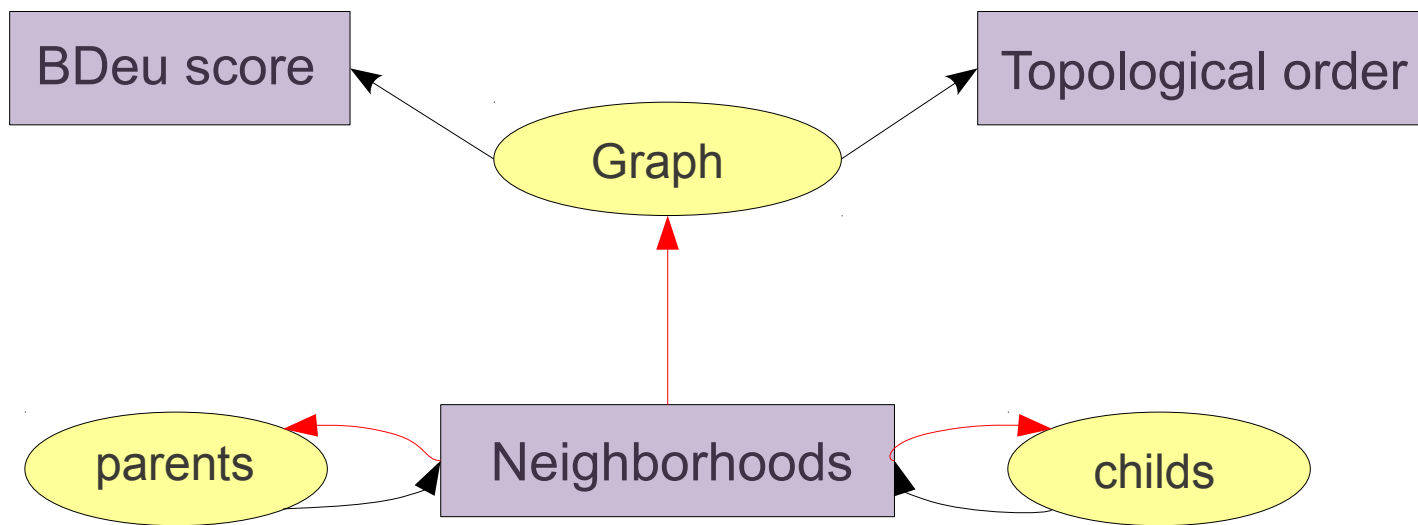
Parallel programming

...

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# Hill-climbing implementation in Comet



Invariant

Incremental variable

→ update when is modified

→ **modify**

# Experimentation

DREAM5 systems genetics challenge (November 2010, New York)

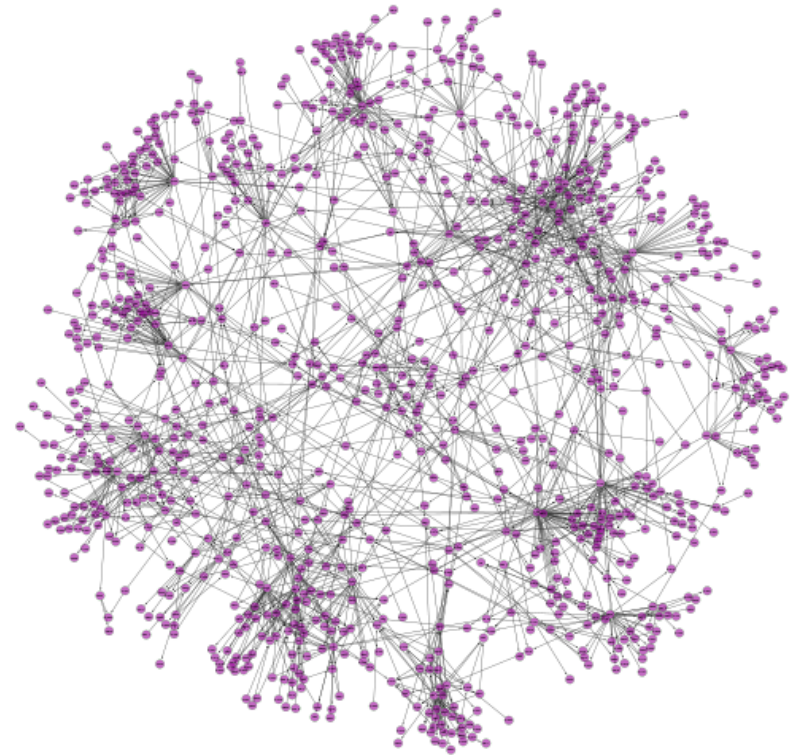
Objective: recover gene regulatory network from

- Gene expressions
- Genetic data

Our gold network

- 2000 nodes (1000 genes / 1000 genetic markers)
- 1983 edges

Simulated population of 300 individuals



*Gold standard network*



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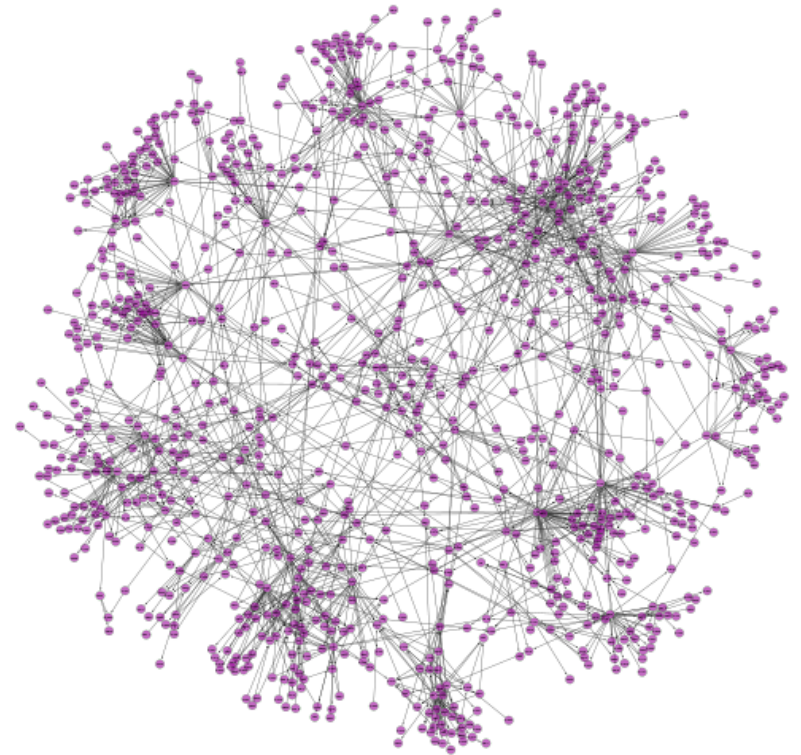
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Simulated population of 300 individuals

- Discretization of data (max. 4 classes)
- Pre-filtering candidate parents under condition  
 $\Delta Add(Parent, Target) > 0$
- Limit number of parents : 6



*Gold standard network*

# Results (1/4)

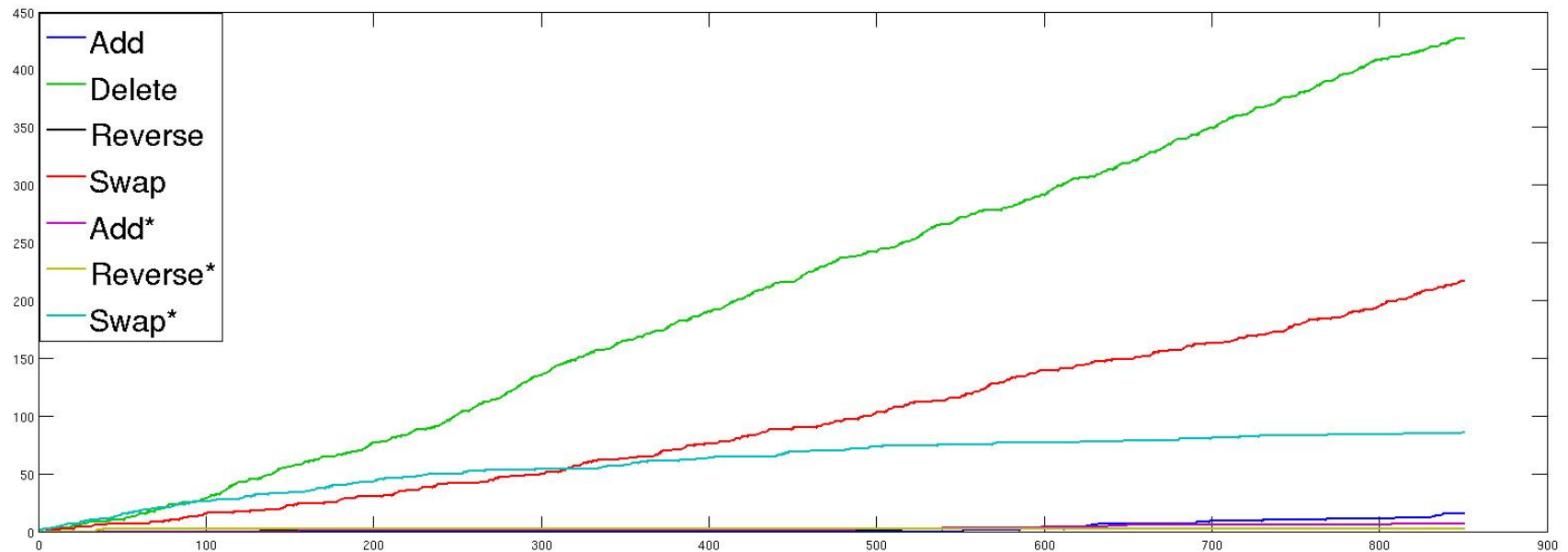
- **1000** runs of hill climbing algorithm
- Initialized with **random networks** (2 parents max)
- **5** operator configurations:
  - × Addition + Deletion
  - × Addition + Deletion + Reversal
  - × Addition + Deletion + Swap
  - × Addition + Deletion + Reversal + Swap
  - × Addition<sup>2</sup> + Deletion + Reversal<sup>2</sup> + Swap<sup>2</sup> (?:nISC)

	A+D	A+D+R	A+D+S	A+D+R+S	A <sup>2</sup> +D+R <sup>2</sup> +S <sup>2</sup>
BDeu scores					
➤ <i>mean</i>	-359 580	-359 430	<b>-357 990</b>	-357 850	<b>-357 460</b>
➤ <i>deviation</i>	169.3	168.5	<b>92.9</b>	91.0	<b>55.2</b>
Mean time (in seconds)	17.9	27.0	<b>27.6</b>	32.3	<b>149.2</b>

# Results (2/4)

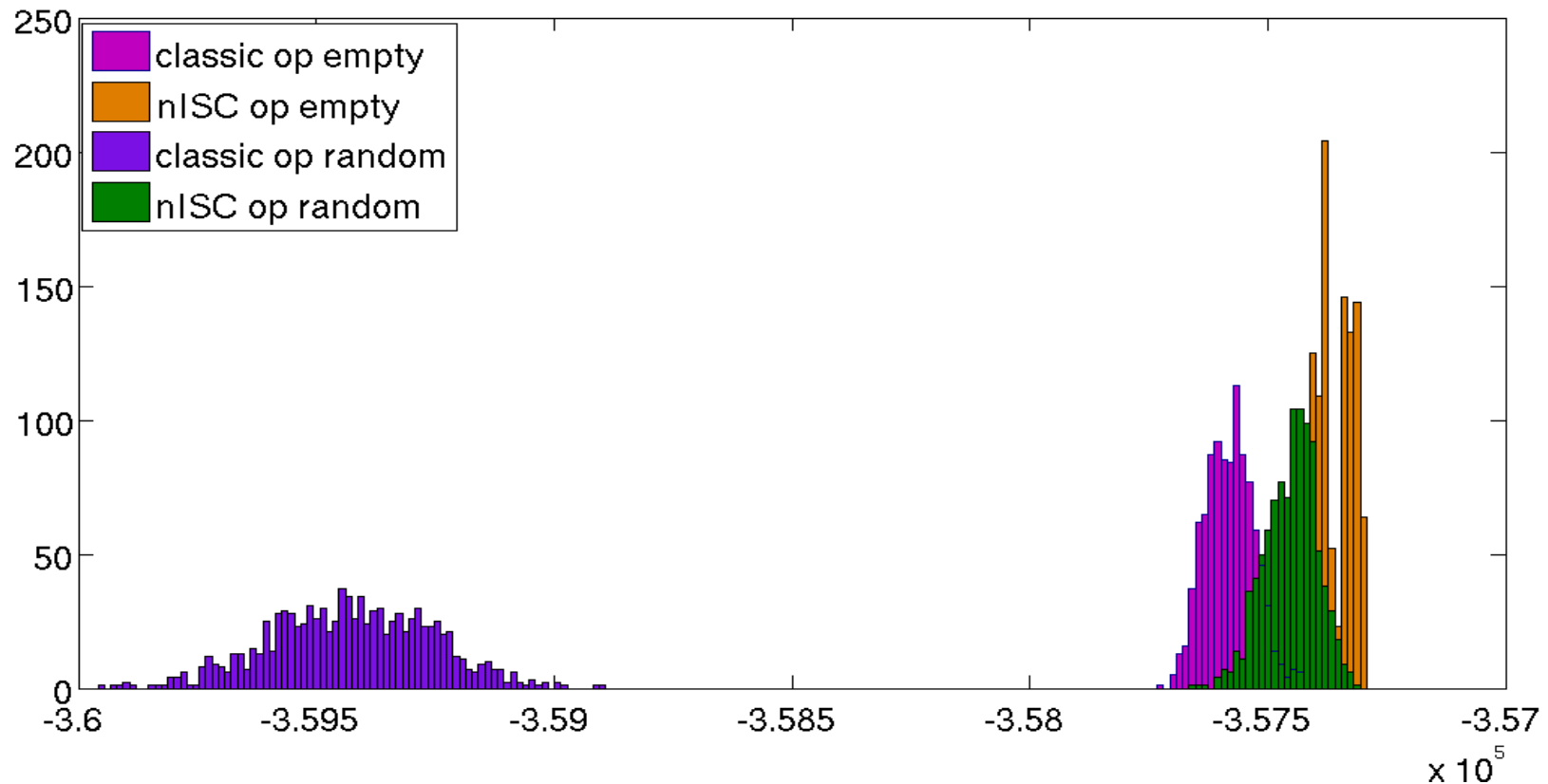
- 1 run of hill climbing algorithm
- Initialized with **random** networks (2 parents max)
- 1 operator configurations:  $\times$  Addition<sup>2</sup> + Deletion + Reversal<sup>2</sup> + Swap<sup>2</sup> (?:nISC)

Number of applied operators by type during the search



# Results (3/4)

- › **1000** runs of hill climbing algorithm
- › **2** starting configurations:
  - × empty network
  - × random networks (2 parents max)
- › **2** operator configurations:
  - × Addition + Deletion + Reversal
  - × Addition<sup>2</sup> + Deletion + Reversal<sup>2</sup> + Swap<sup>2</sup> (<sup>2</sup>:nISC)



# Results (4/4)

- **1000** runs of hill climbing algorithm
- Initialized with **random networks** (2 parents max)
- **5 configurations**:
  - × Addition + Deletion + Reversal
  - × Addition + Deletion + Swap
  - × Addition<sup>2</sup> + Deletion + Reversal<sup>2</sup> + Swap<sup>2</sup> (<sup>2</sup>:nISC)
  - × Addition\* + Deletion + Reversal\* + Swap\* (\*:ISC)
  - × **Tabu search** with Addition + Deletion + Reversal  
(10 000 operations, tabuu list size :100)

	A+D+R	A+D+S	A <sup>2</sup> +D+R <sup>2</sup> +S <sup>2</sup>	A*+D+R*+S*	Tabu
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➤ <i>deviation</i>	168.5	<b>92.9</b>	<b>55.2</b>	54.5	160.4
Mean time (in seconds)	27.0	<b>27.6</b>	<b>149.2</b>	373.1	291.5

# Conclusion & Perspectives

We

- › Propose a new Iterative Swap Operator breaking cycles
- › Improve BDeu scores of learned networks with this operator
- › Compare initial structure effect

TODO list:

- › try other meta-heuristics
- › tune Tabu parameters
- › improve time efficiency of ISC operator

**Question time !**