

Constraint-based methods

Two reference algorithms

- Pearl et Verma : IC, IC*
- Spirtes, Glymour et Scheines : SGS, PC, CI, FCI

Common principle

- build an undirected graph describing direct dependences between variables (χ^2 tests)
 - by adding edges (Pearl et Verma)
 - by deleting edges (SGS)
- detect V-structures (from previous statistical tests)
- propagate some edge orientation (inferred edges) in order to obtain a CPDAG

Constraint-based methods

Some inconvenients

- reliability of CI test conditionally to several variables with a limited amount of data)
 - SGS heuristic : if $df < \frac{N}{10}$, then declare dependence
- Combinatorial explosion of the number of tests
 - PC heuristic : begin with order 0 ($X_A \perp X_B$) then order 1 ($X_A \perp X_B \mid X_C$), etc ...

Score-based methods

How to search a **good** BN ?

- Constraint-based methods
BN = independence model
⇒ find CI in data in order to build the DAG
- **Score-based methods**
BN = probabilistic model that must fit data as well as possible
⇒ search the DAG space in order to maximize a scoring function
- Hybrid methods

Notion of score

General principle : Occam razor

- *Pluralitas non est ponenda sine neccesitate*
(La pluralité (des notions) ne devrait pas être posée sans nécessité) plurality should not be posited without necessity
- *Frustra fit per plura quod potest fieri per pauciora*
(C'est en vain que l'on fait avec plusieurs ce que l'on peut faire avec un petit nombre) It is pointless to do with more what can be done with fewer

= Parcimony principle : find a model

● fitting the data \mathcal{D} :

likelihood : $L(\mathcal{D}|\theta, B)$

● the simplest possible :

dimension of B : $Dim(B)$

Score examples

AIC and BIC

- Compromise between likelihood and complexity
- Application of AIC (Akaike 70) and BIC (Schwartz 78) criteria

$$S_{AIC}(B, \mathcal{D}) = \log L(\mathcal{D} | \theta^{MV}, B) - \text{Dim}(B)$$

$$S_{BIC}(B, \mathcal{D}) = \log L(\mathcal{D} | \theta^{MV}, B) - \frac{1}{2} \text{Dim}(B) \log N$$

Bayesian scores : BD, BDe, BDeu

- $S_{BD}(B, \mathcal{D}) = P(B, \mathcal{D})$ (Cooper et Herskovits 92)
- $BDe = BD + \text{score equivalence}$ (Heckerman 94)

$$S_{BD}(B, \mathcal{D}) = P(B) \prod_{i=1}^n \prod_{j=1}^{q_i} \frac{\Gamma(\alpha_{ij})}{\Gamma(N_{ij} + \alpha_{ij})} \prod_{k=1}^{r_i} \frac{\Gamma(N_{ijk} + \alpha_{ijk})}{\Gamma(\alpha_{ijk})}$$

Score properties

Two important properties

Decomposability

$$(Global)Score(B, \mathcal{D}) = \sum_{i=1}^n (local)score(X_i, pa_i)$$

Score equivalence

If two BN B_1 and B_2 are Markov equivalent then
 $S(B_1, \mathcal{D}) = S(B_2, \mathcal{D})$

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Heuristic exploration of search space

Search space and heuristics

- espace \mathcal{B}
 - restriction to tree space : Chow&Liu, MWST
 - DAG with node ordering : K2 algorithm
 - greedy search
 - genetic algorithms, ...
- espace \mathcal{E}
 - greedy equivalence search

Restriction to tree space

Principle

- what is the best tree connecting all the nodes, i.e. maximizing a weight defined for each possible edge ?

Answer : maximal weighted spanning tree (MWST)

- (Chow et Liu 68) : weight = mutual information :

$$W(X_A, X_B) = \sum_{a,b} \frac{N_{ab}}{N} \log \frac{N_{ab}N}{N_a \cdot N_b}$$

- (Heckerman 94) : any local score :

$$W(X_A, X_B) = \text{score}(X_A, Pa(X_A) = X_B) - \text{score}(X_A, \emptyset)$$

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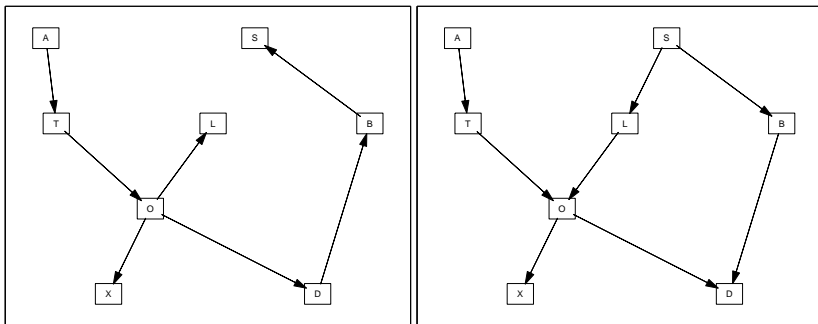
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Restriction to tree space

Remarks

- MWST returns an undirected tree
- this undirected tree = CPDAG of all the directed tree with this skeleton
- Obtain a directed tree by (randomly) choosing one root and orienting the edges with a depth first search over this tree

Example : obtained DAG vs. target one



MWST can not discover cycles neither V-structures (tree space !)

Heuristic exploration of search space

Search space and heuristics

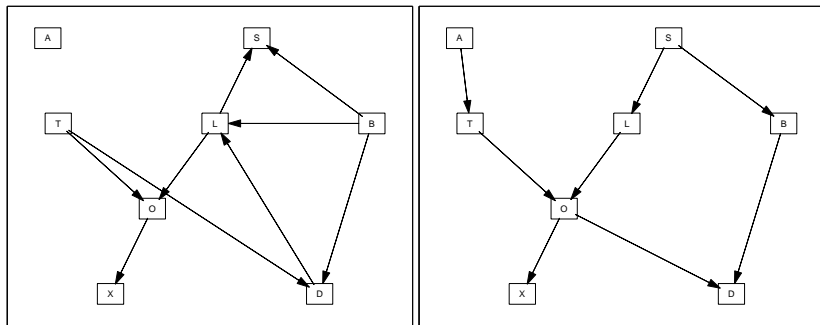
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Greedy search

Principle

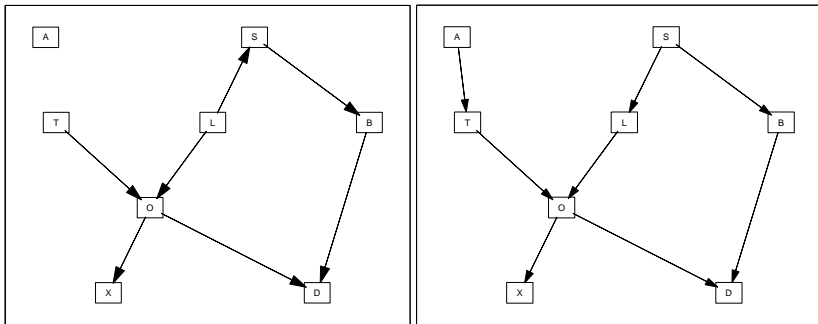
- Exploration of the search space with traversal operators
 - add edge
 - invert edge
 - delete edge
- and respect the DAG definition (no cycle)
- exploration can begin from any given DAG

Example : obtained DAG vs. target one



start = empty graph. GS result = local optimum :-)

Example : obtained DAG vs. target one



start = MWST result. GS result is better

Heuristic exploration of search space

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What about changing our search space

Preliminaries

- IC/PC result = CPDAG
- MWST result = CPDAG
- Score-based methods do not distinguish equivalent DAGs

Search in \mathcal{E}

- \mathcal{E} = CPDAG space
- Better properties : YES
 - 2 equivalent structures = 1 unique structure in \mathcal{E}
- Better size : NO
 - \mathcal{E} size is quasi similar to DAG space
(asymptotic ratio is 3,7 : Gillispie et Perlman 2001)

Greedy Equivalent Search

Principe (Chickering 2002)

- Greedy search in \mathcal{E}
- Phase 1 : add edges until convergence
- Phase 2 : delete edges until convergence

Score-based methods

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- Hybrid methods

Hybrid methods = local search methods

Local search and global learning

- search one local *neighborhood* for a given node T
- reiterate for each T
- learn the global structure with these local informations

which neighborhood ?

- $PC(T)$: Parents and Children T (without distinction)
- $MB(T)$: Markov Blanket of T - Parents, children and spouses

Local search identification

Identification of MB(T) or PC(T)

- IAMB (Aliferis 2002)
- MMPC (Tsamardinos et al. 2003), ...

Hybrid structure learning algorithms

- MMHC (Tsamardinos et al. 2006) = MMPC + Greedy search

A BN is not a causal model

- Usual BN
 - $A \rightarrow B$ does not imply direct causal relationship between A and B ,
 - only edges from the CPDAG represent causal relationships *

Confusion

- when the DAG is given by an expert, this graph is very often causal
 - when the DAG is learnt from data, no reason to be causal !
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- But, is it important ?
 - equivalent DAGs \Rightarrow same joint distribution so same result for (probabilistic) inference
 - \Rightarrow causality is not required for (probabilistic) inference

Motivations

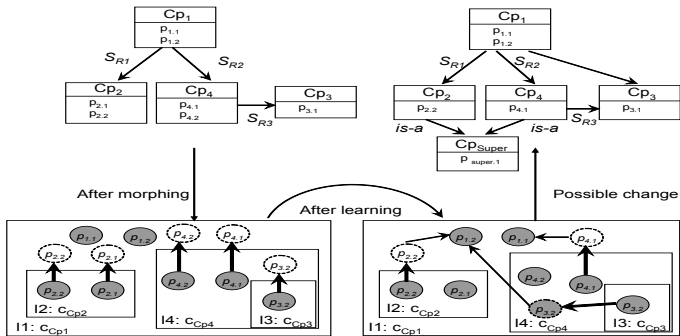
- Knowledge-based systems aim to make expertise available for decision making and information sharing.
- To resolve some complex problems, the combination of different knowledge-based systems can be very powerful.

Our idea

Combine Two KBSs : Bayesian Networks and Ontologies, to help both.

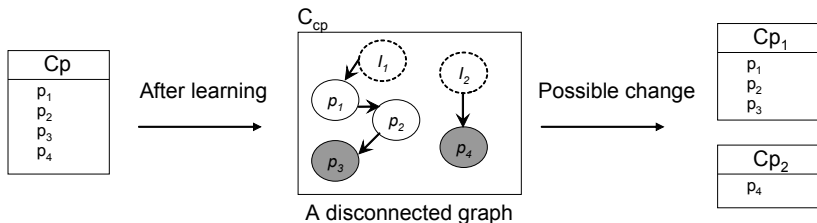
Example : add concepts / relations

- If c_{Cp} communicates with only one class \rightarrow Add relation.
- Otherwise, check classes similarities \rightarrow Add concepts / relations.



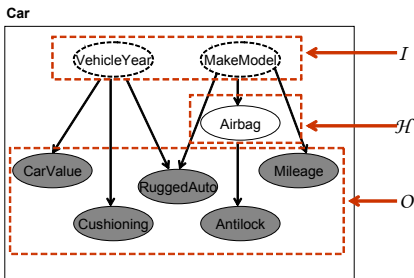
Example : concepts redefinition

- If the class contains more than one component, then The corresponding concept may be deconstructed into more refined ones.



OOBN definition

- Models the domain using fragments of a Bayesian network known as classes.
- Each class is a DAG over three sets of nodes ($\mathcal{I}, \mathcal{H}, \mathcal{O}$):

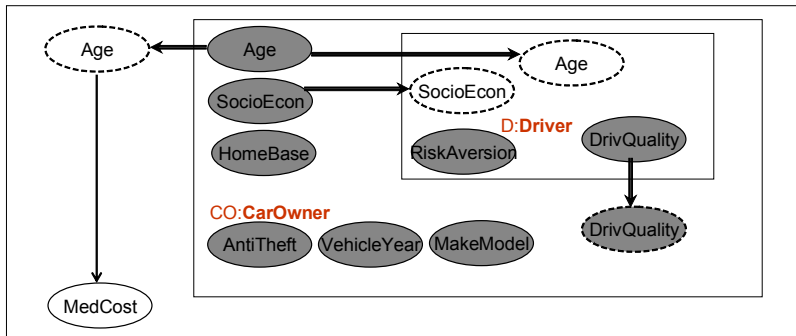


- $\mathcal{I} + \mathcal{O} =$ the class interface.

OOBN nodes

Instantiations: representing the instantiation of a class inside another class.

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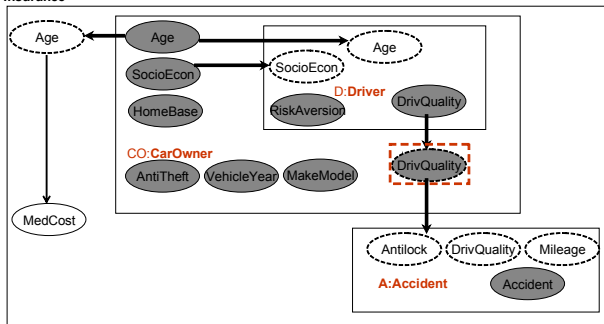


OOBN nodes

Simple nodes:

- **Reference nodes:** for specification of input and output nodes only.
- **Real nodes:** represent variables.

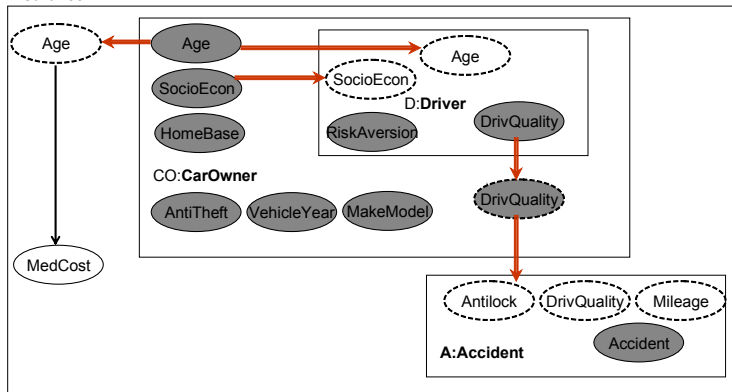
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OOBN links

Reference links: to link reference or real nodes to reference nodes.

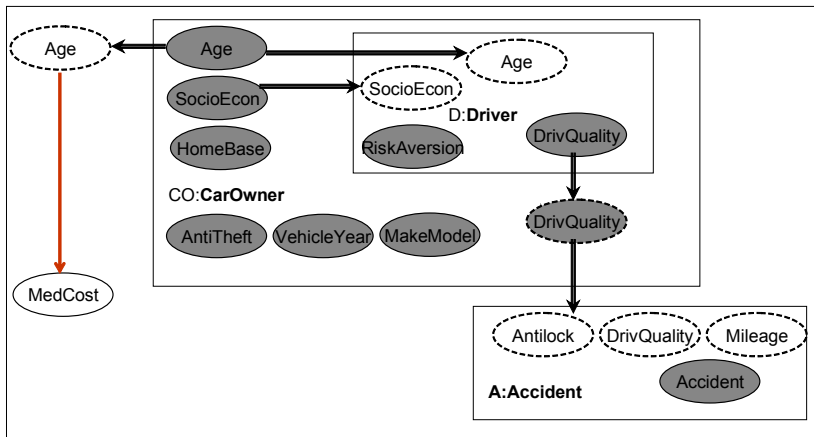
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OOBN links

Directed links: to link reference or real nodes to real nodes.

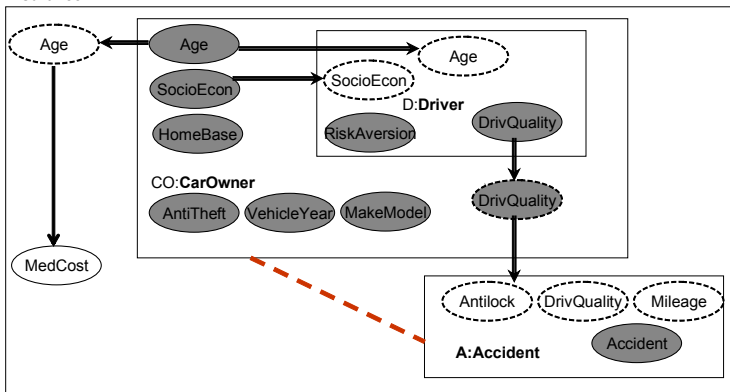
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OOBN links

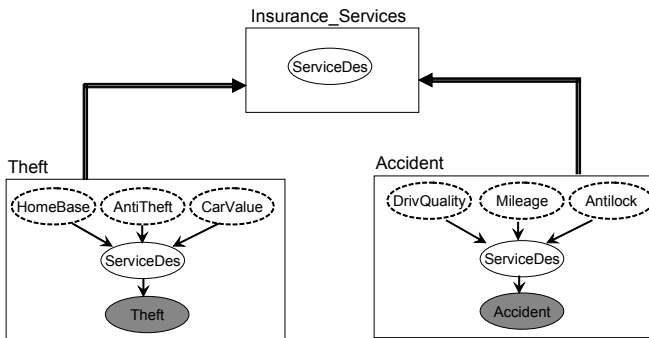
Construction links: to express that two nodes (or instantiations) are linked in some manner.

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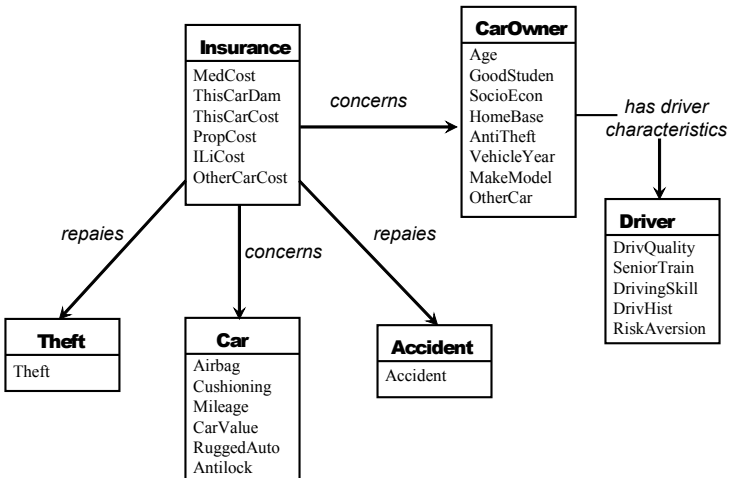


OOBN classes hierarchy

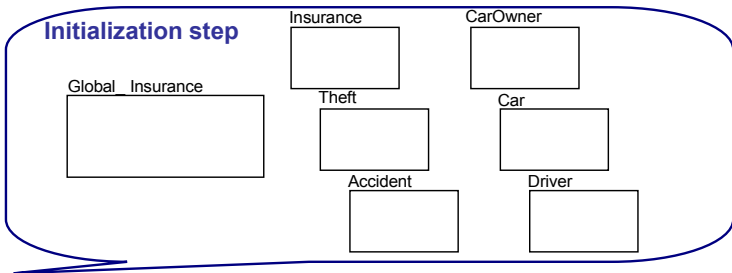
- Classes may have subclasses.
 - Subclass inherits all the superclass nodes.
 - Subclass may have additional nodes not yet represented in the superclass.



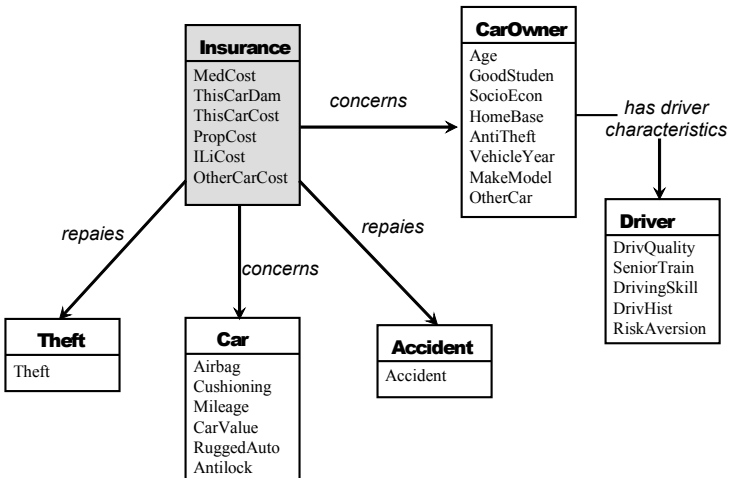
Illustrative example : more details



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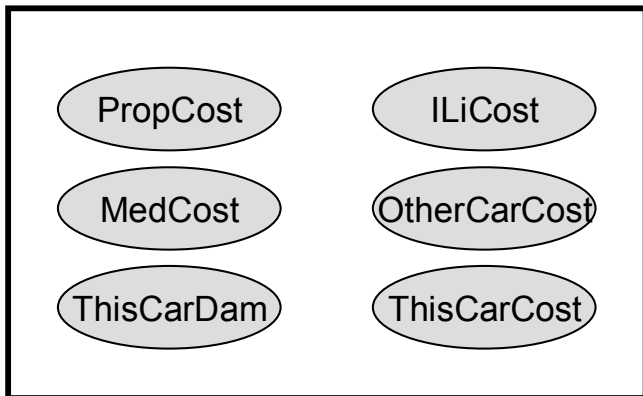


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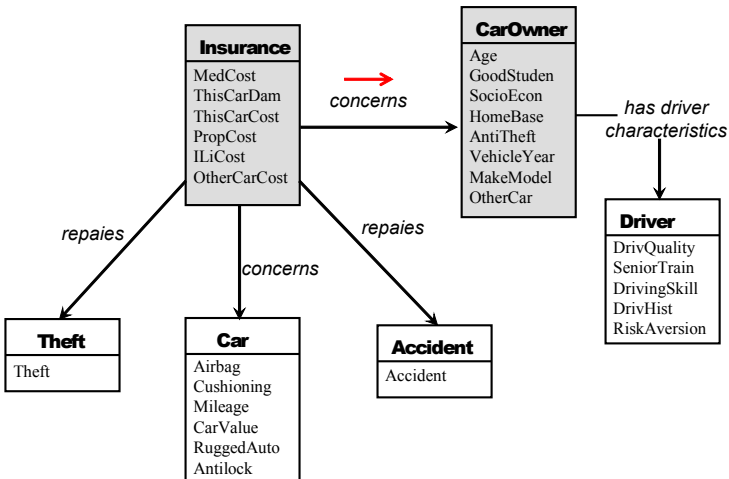


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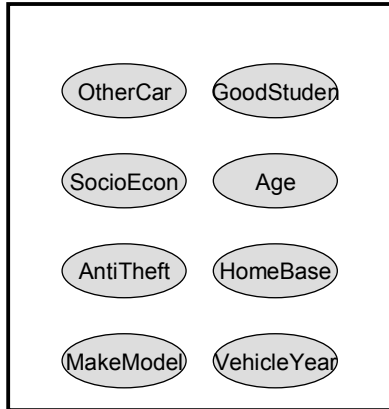


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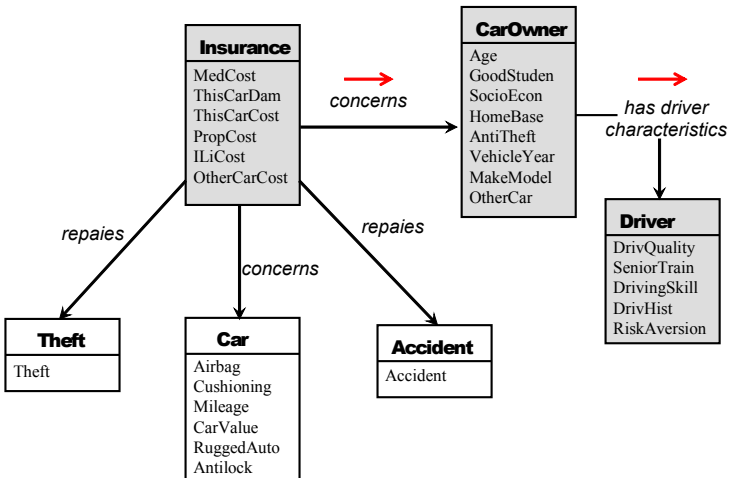


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CarOwner

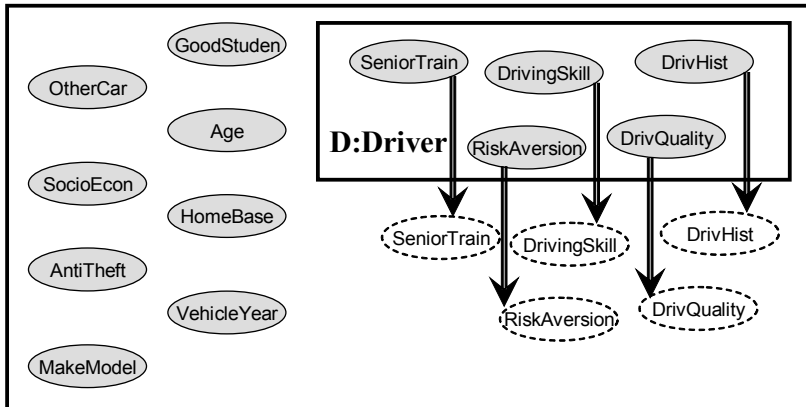


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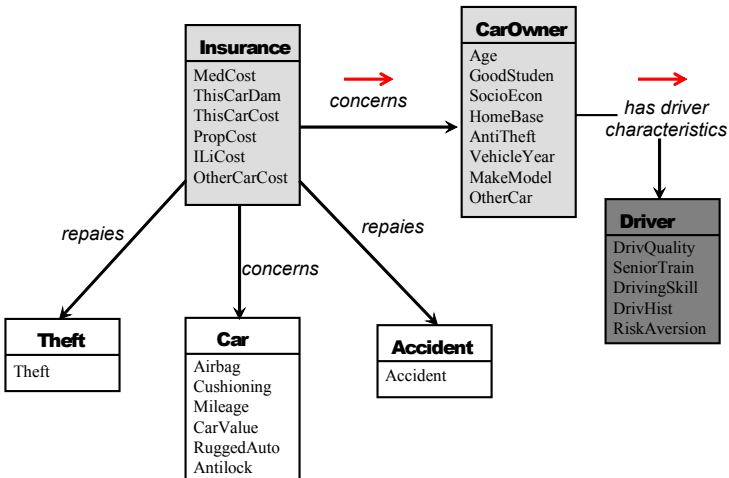


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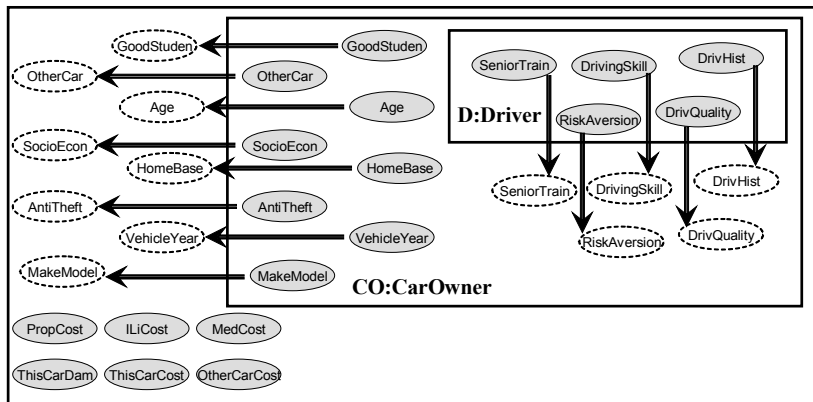


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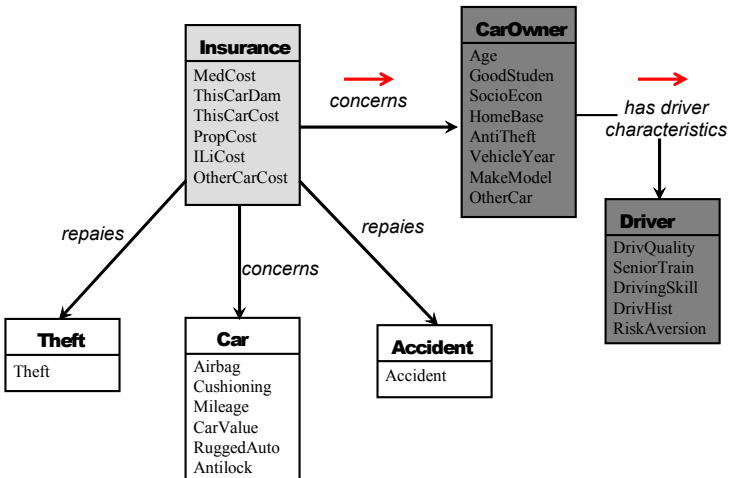


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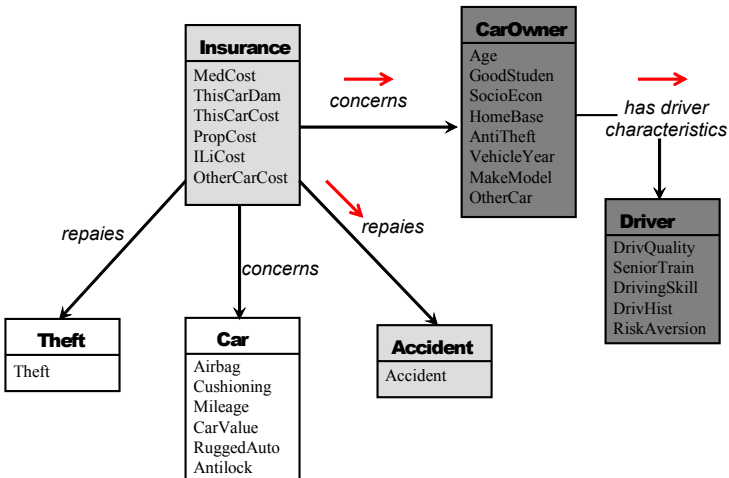
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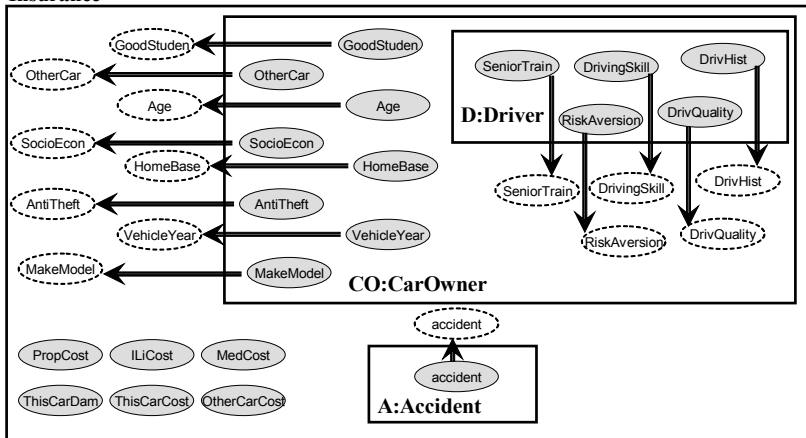


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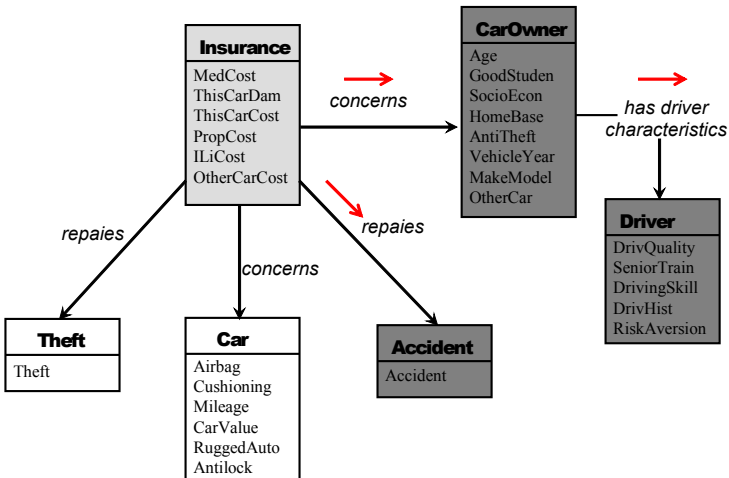


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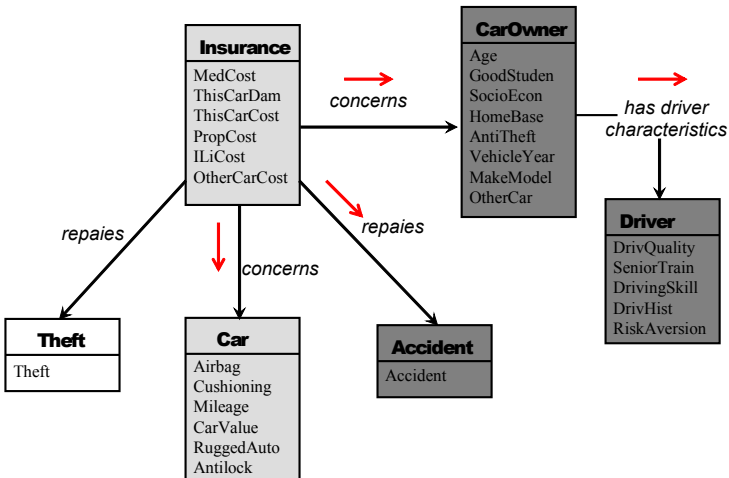
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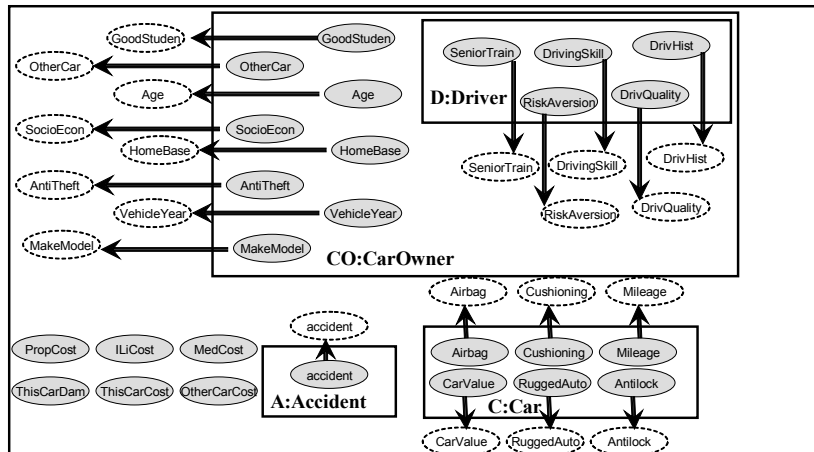


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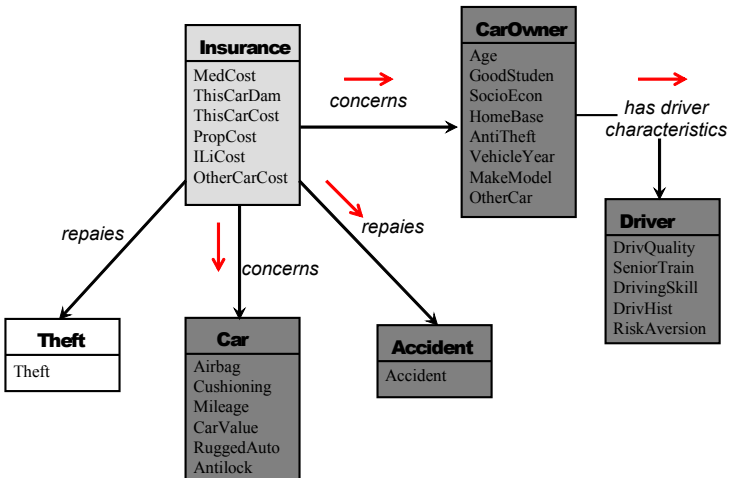


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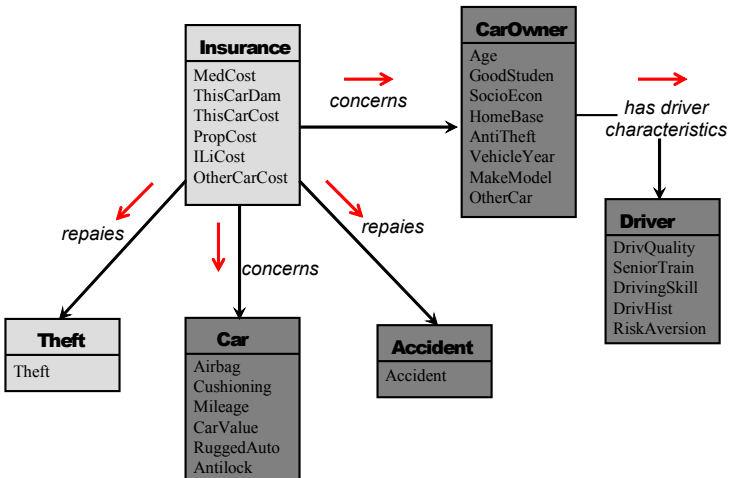
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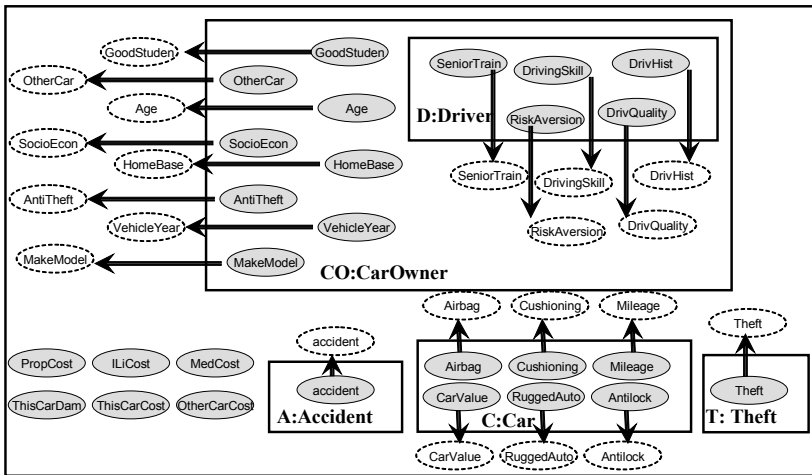


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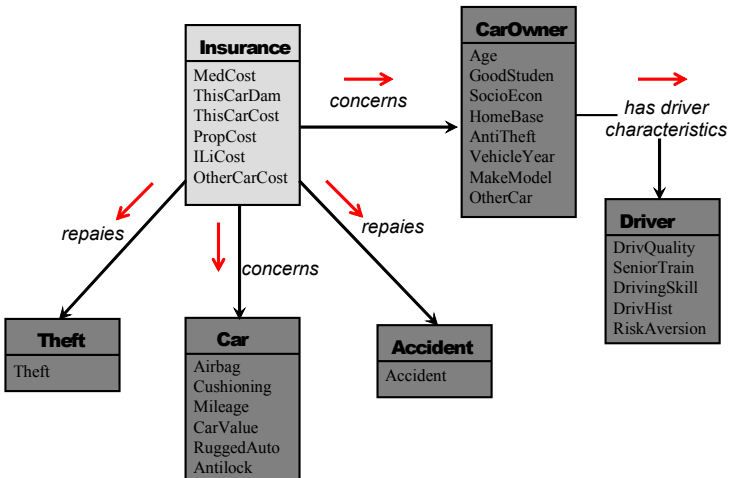


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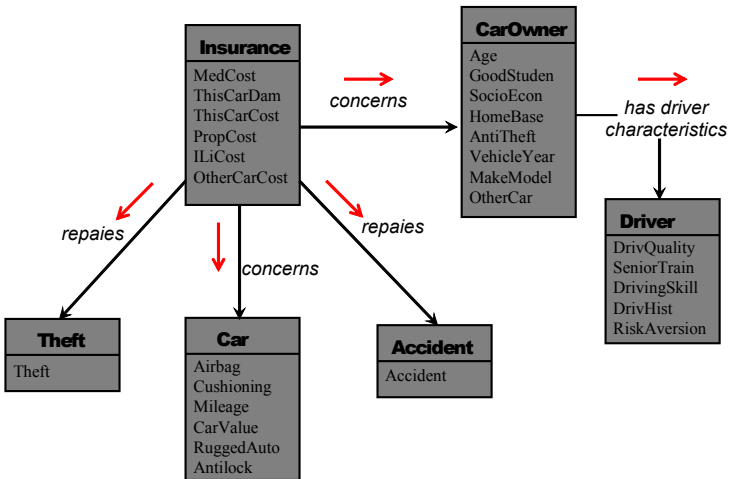
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