Modèles Probabilistes Complexes

Models et Algorithms

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Inference in Bayesian networks

Variable eliminitation

Input : a set of CPTs $\mathbf{P}$ and a set of variables $\mathbf{X}$

Output : $P(\mathbf{X})$

$\mathbf{W} \leftarrow$ all the variables of the CPTs of $\mathbf{P}$ except $\mathbf{X}$

while $\mathbf{W} \neq \emptyset$ **do**

  let X_j be a variable in $\mathbf{W}$; remove X_j from $\mathbf{W}$

  let $\mathbf{Q}$ be the set of tables in $\mathbf{P}$ containing X_j

  compute table $q = \sum_{X_j} \prod_{f \in \mathbf{Q}} f$

  $\mathbf{P} \leftarrow (\mathbf{P} \backslash \mathbf{Q}) \cup \{q\}$

return table $\prod_{f \in \mathbf{P}} f$

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Bayesian network that models a dynamic system (or a repetition of patterns) :

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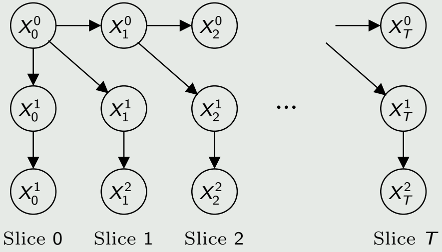

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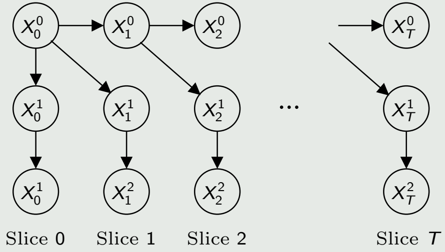

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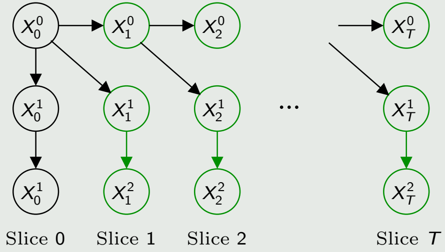

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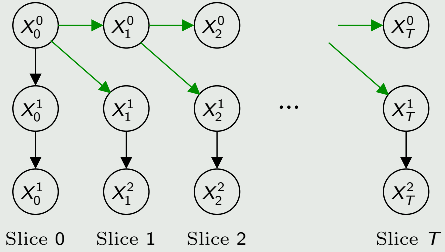

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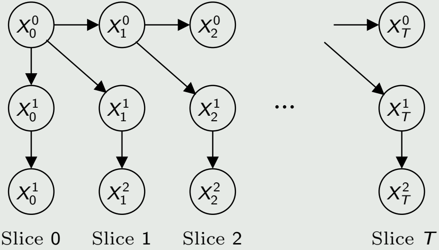

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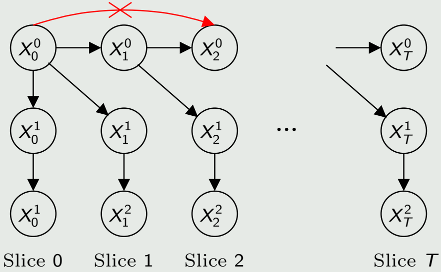

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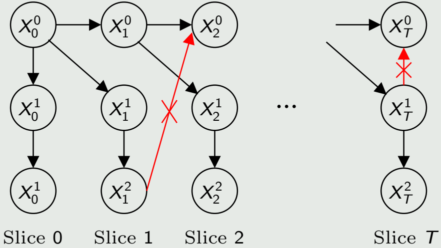

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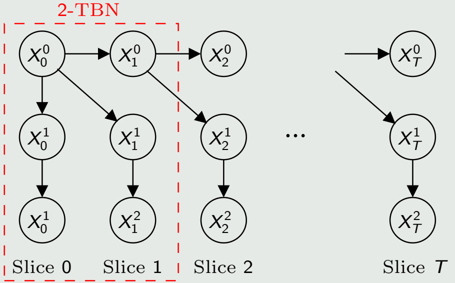

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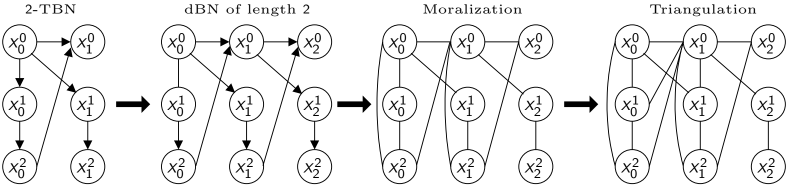

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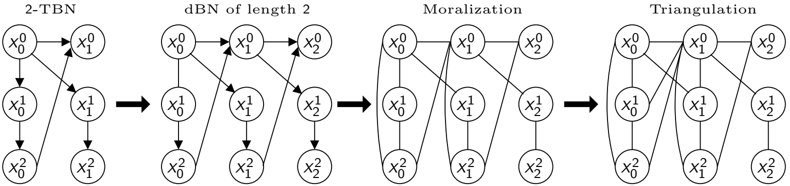

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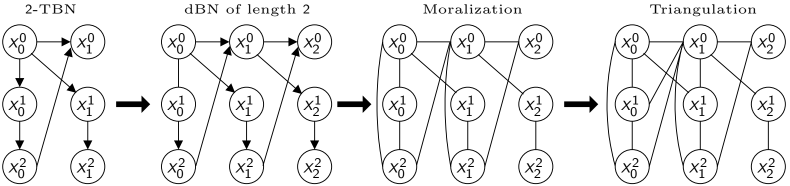

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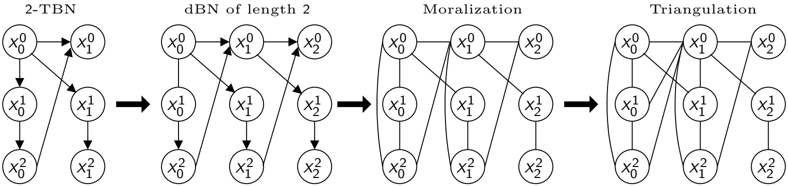

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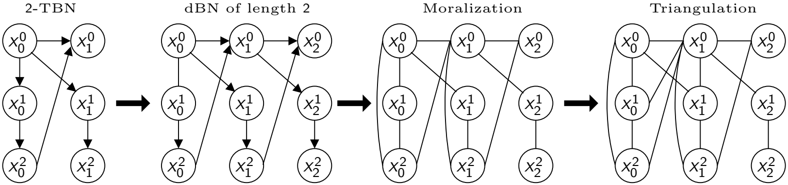

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3. The size of some cliques may be a function of T.

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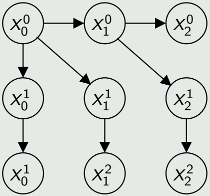

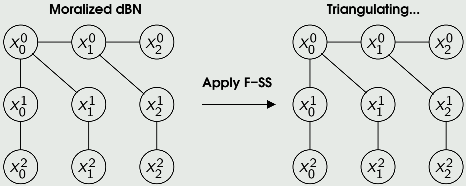

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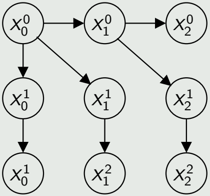

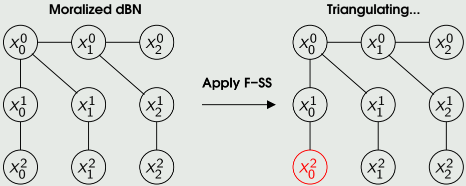

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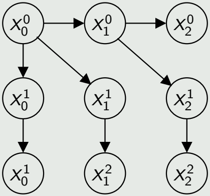

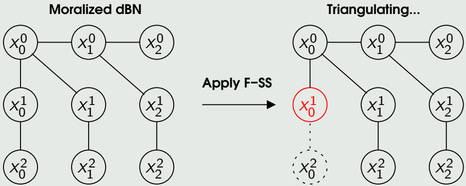

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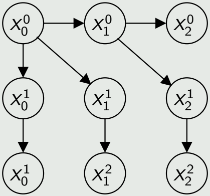

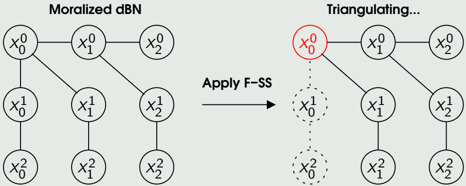

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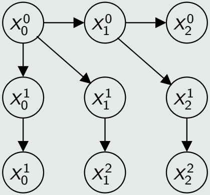

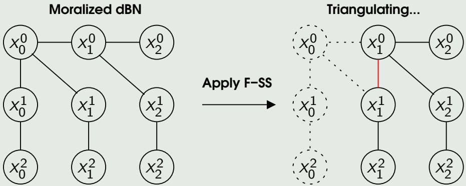

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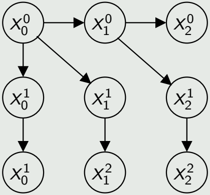

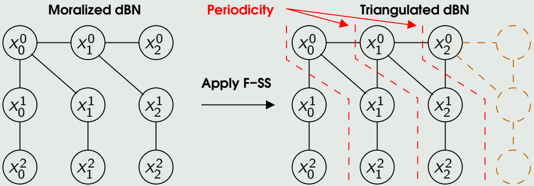

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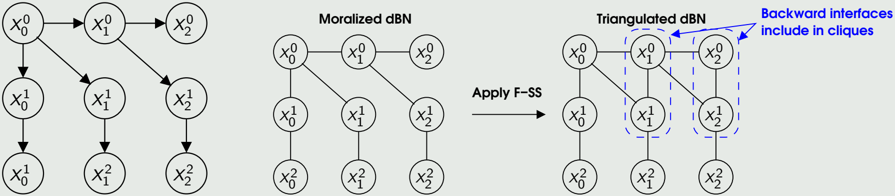

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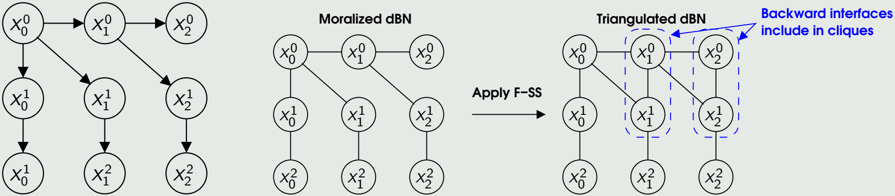

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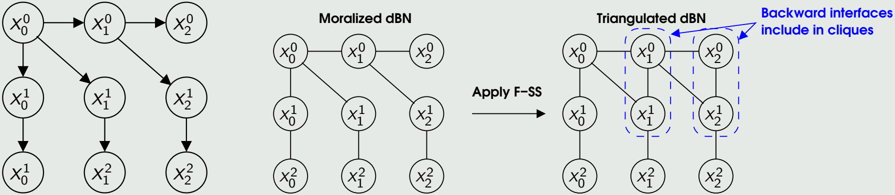

$\hookrightarrow$ *Backward interface* : nodes with parents or spouses in the previous slice.

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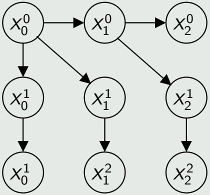

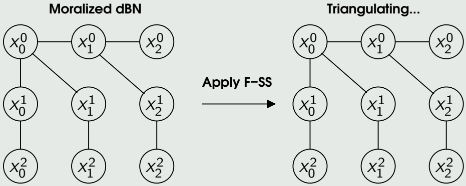

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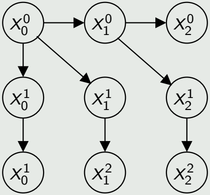

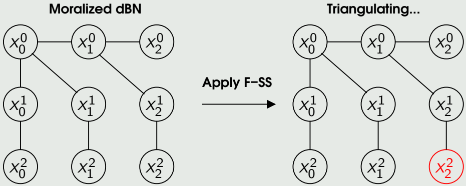

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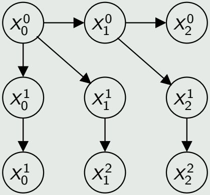

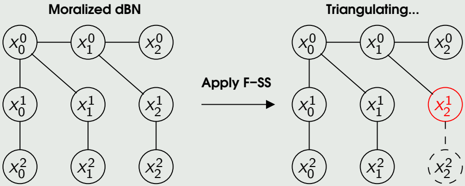

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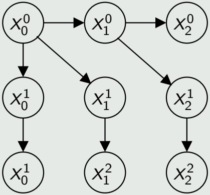

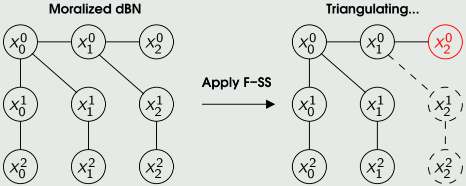

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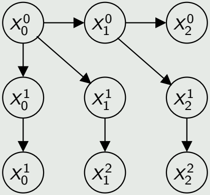

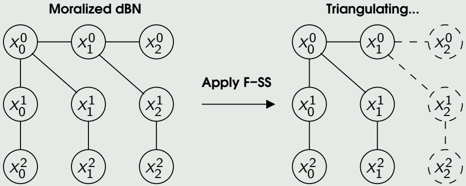

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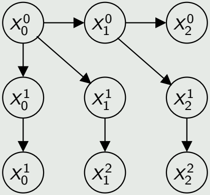

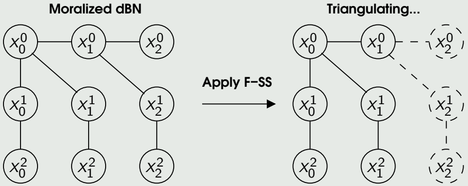

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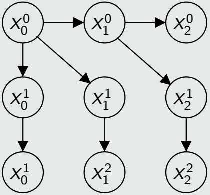
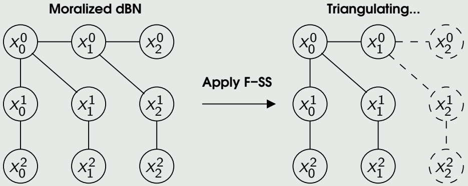
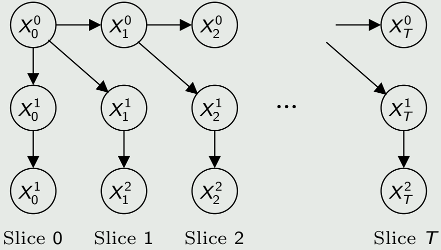




↪ *Forward interface* : nodes with children in the next slice.

General interface



- *Transition* model :
$$p(\mathbf{X}_t|\mathbf{X}_{t-1}) = p(\mathbf{X}_1|\mathbf{X}_0) = \prod_{i=0}^{n-1} p(X_1^i|pa(X_1^i))$$

- *Prior* distribution :
$$p(\mathbf{X}_0) = \prod_{i=0}^{n-1} p(X_0^i|pa(X_0^i))$$

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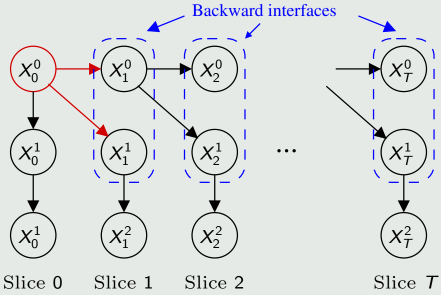

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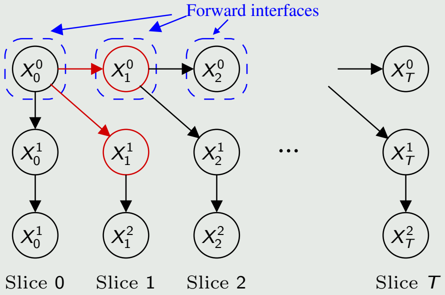

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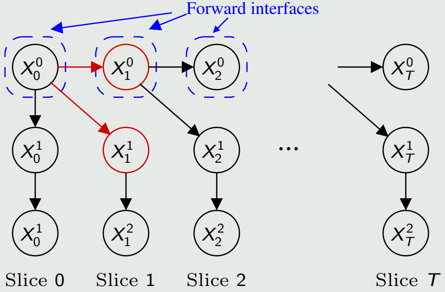

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Interface

Subset of nodes such that if they were removed they will disconnect the past from the future in the moral graph of a dBN.

Complexity for interface based ceo

Interface based Constrained elimination order (ceo)

Elimination order such that we enforce nodes to be eliminated with respect to a partial order given by an interface.

1. Provide a way to avoid re-triangulating the dBN,

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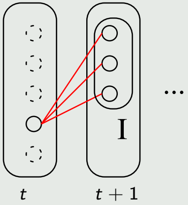

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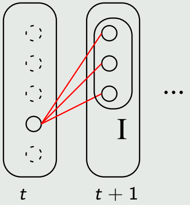

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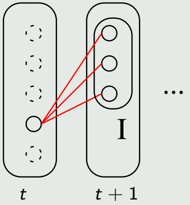

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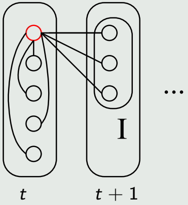
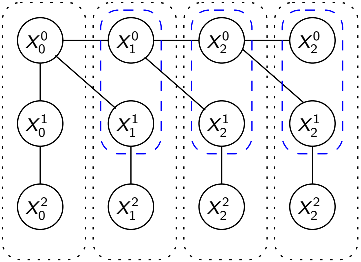
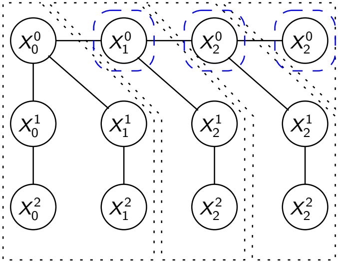


Worst case : clique of size $|I| + S$ (S : $\#$ nodes in a slice)

How to improve the bounds ?



$|I| + 1 = 3 \leq \omega = 3 \leq |I| + N = 5$



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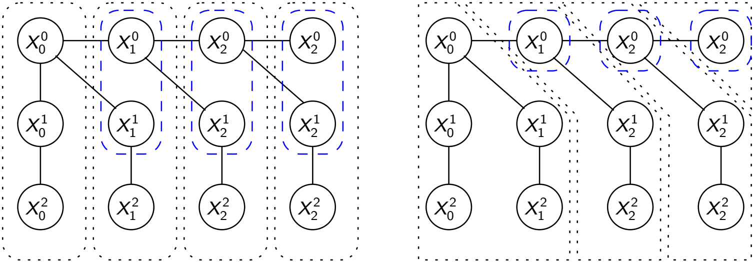

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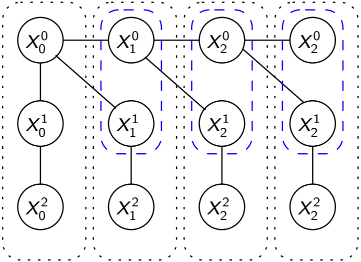

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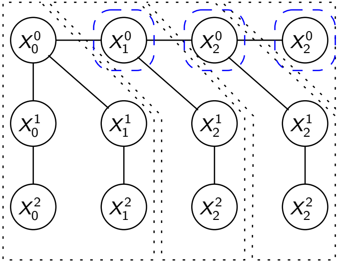

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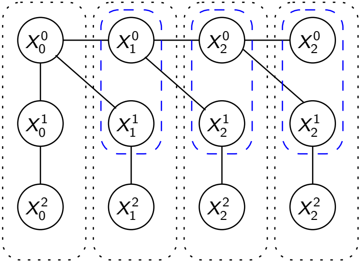

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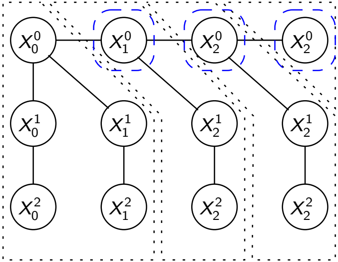

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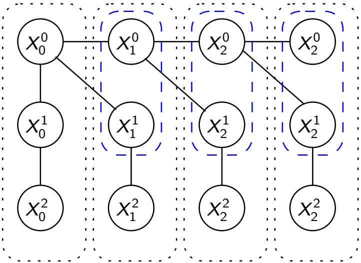

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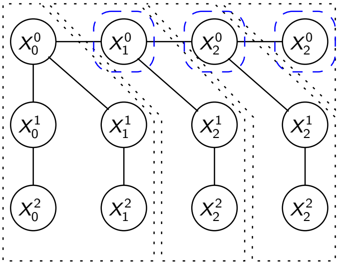

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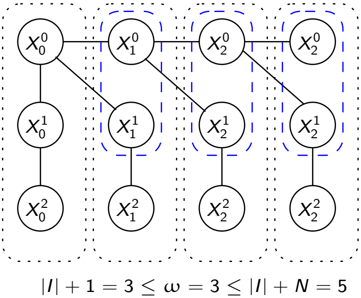

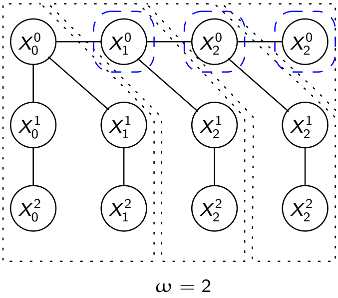

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Relation to the MINIMUM s–t CUT problem

Theorem

Given a dBN of length T, finding a minimum interface is equivalent to solve the minimum s–t cut problem

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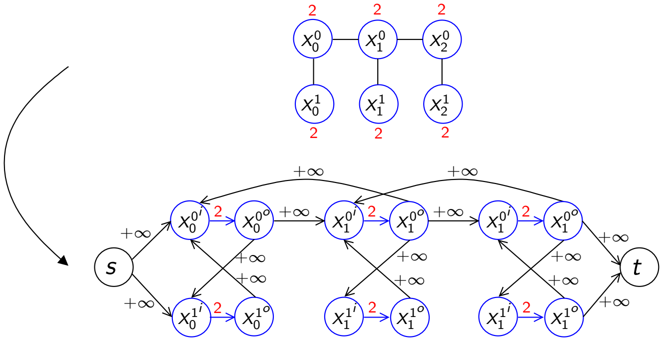

Relation to the MINIMUM s–t CUT problem

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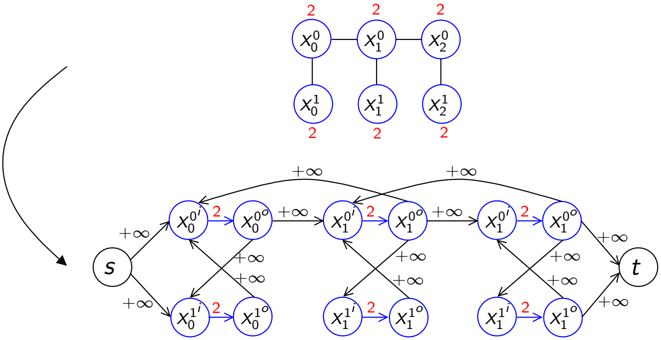
Given a dBN of length T, finding a minimum interface is equivalent to solve the minimum s–t cut problem



$\Rightarrow$ Polynomial time algorithm to find the minimum interface in a dBN of length T

Building the optimal elimination order

For wich value of T do we have the minimum interface ?

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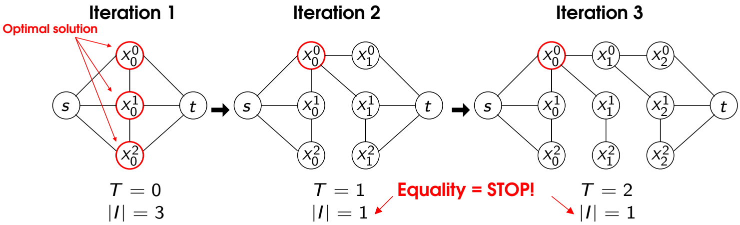

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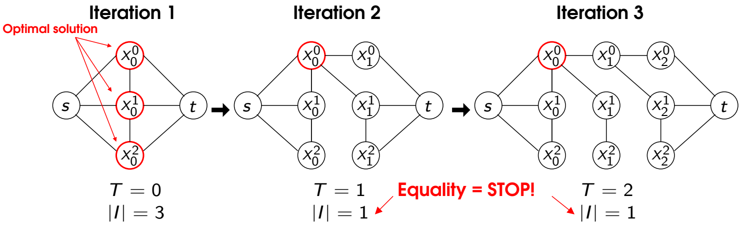
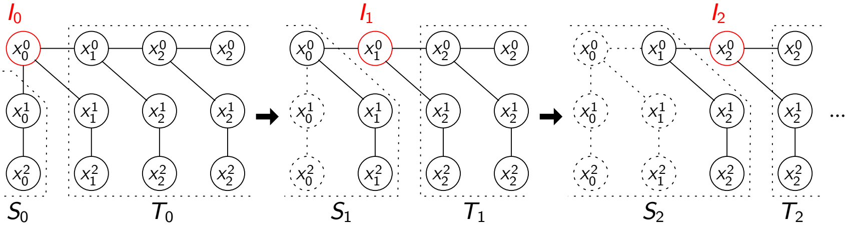


Theorem

At most $O(h)$ iterations where h is the number of nodes in a slice.

Building the elimination order



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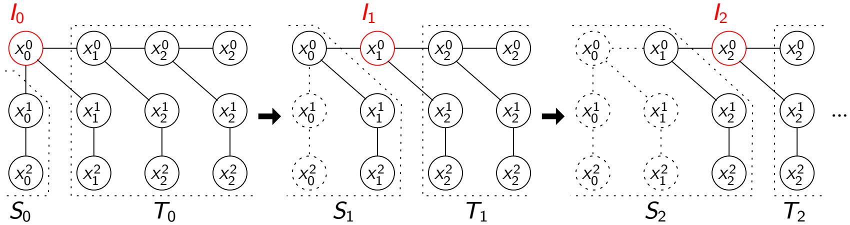

$$\textsc{Min-Elim} : \sigma = \mathbf{S}_0, \mathbf{S}_1 - \mathbf{S}_0, \mathbf{S}_2 - \mathbf{S}_1, ..., \mathbf{I}_{n-1} \cup \mathbf{T}_{n-1}$$

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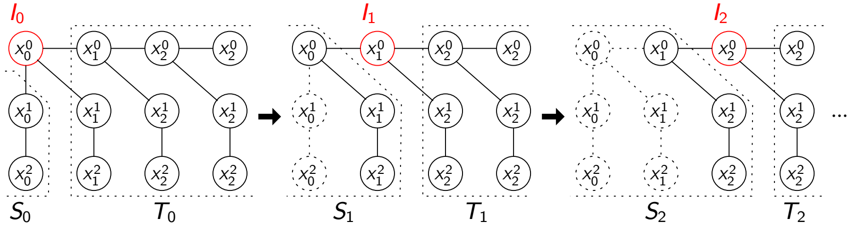

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Better lower bound

Best case : clique of size $|I^*| + 1$

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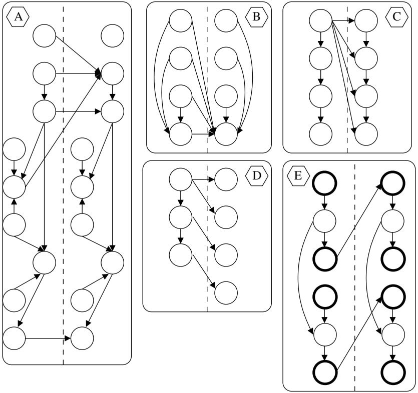
Experimental results – Classical dBNs



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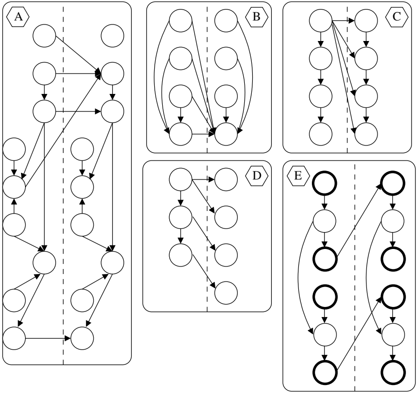

Maximum clique size :

Fig.	B-SS	F-SS	Min-Elim
C	3.16	**3.03**	**3.03**
B	**5.54**	**5.54**	**5.54**
D	**1.38**	3.00	**1.38**
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E	3.46	3.23	**3.00**

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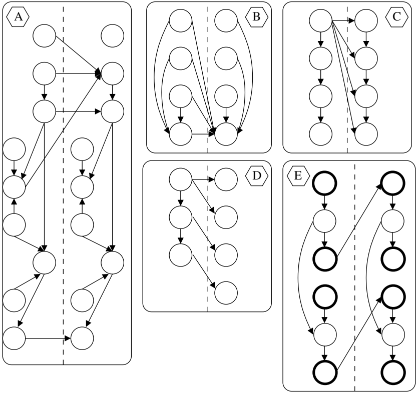

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Mean clique size :

Fig.	B-SS	F-SS	Min-Elim
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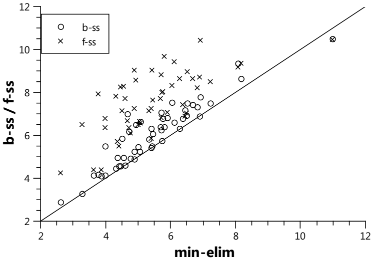
B	**5.54**	**5.54**	**5.54**
D	**1.39**	3.46	**1.39**
A	**2.08**	3.46	**2.08**
E	4.16	**3.46**	**3.46**

Experimental results – Random dBNs

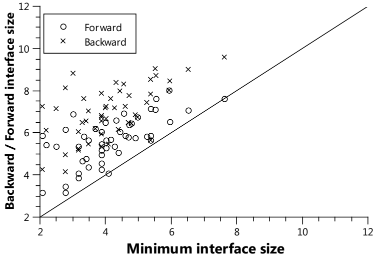
Results on randomly generated dBNs

1. 5 variables per slices
2. Cardinalities chosen uniformly between 2 and 8
3. DBNs unrolled 500 time steps

Mean clique size



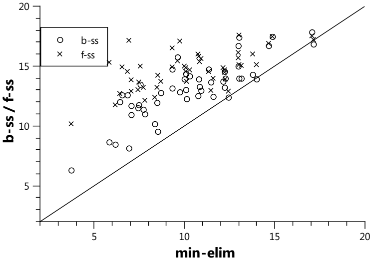
Interface size



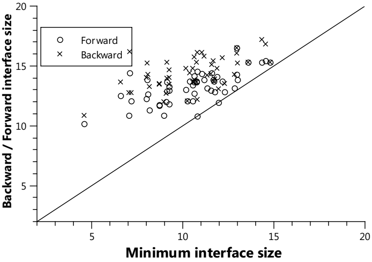
Results on randomly generated dBNs

1. $\mathbf{10}$ variables per slices
2. Cardinalities chosen uniformly between 2 and 8
3. DBNs unrolled 500 time steps

Mean clique size



Interface size



Results on randomly generated dBNs

1. $\mathbf{15}$ variables per slices
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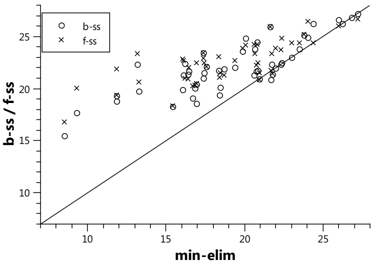
Mean clique size



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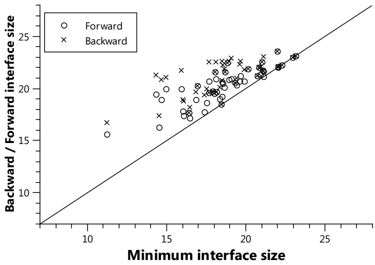
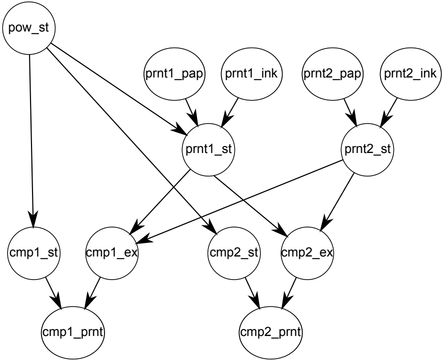


Motivation


pow_st
prnt1_pap
prnt1_ink
prnt2_pap
prnt2_ink
prnt1_st
prnt2_st
cmp1_st
cmp1_ex
cmp2_st
cmp2_ex
cmp1_prnt
cmp2_prnt

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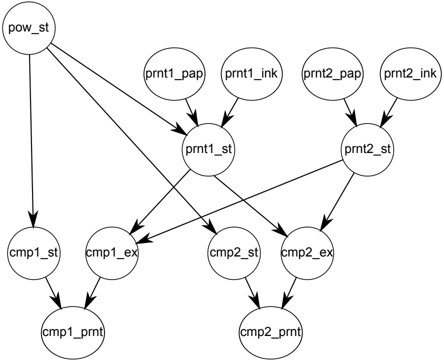
pow_st
prnt1_pap
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prnt2_ink
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cmp1_st
cmp1_ex
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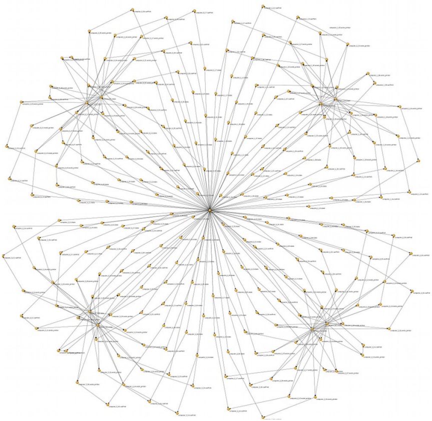
cmp1_prnt
cmp2_prnt



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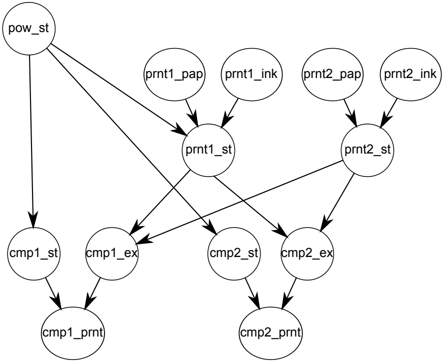
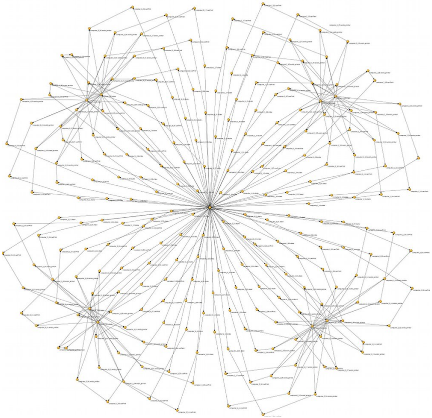
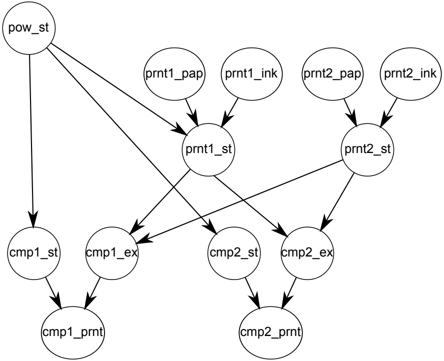
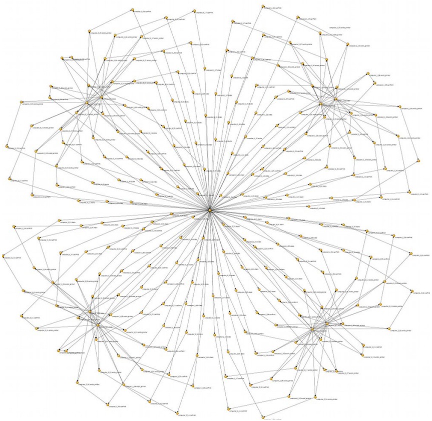




- Large-scale Bayesian networks "undesirable" features :

Motivation





- Large-scale Bayesian networks “undesirable” features :
    - design cost

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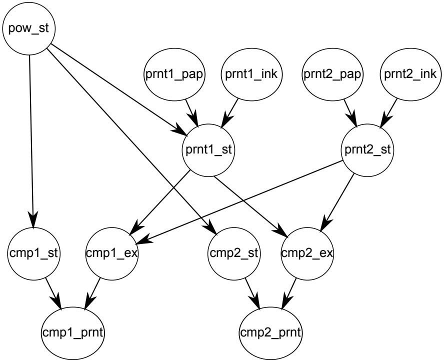

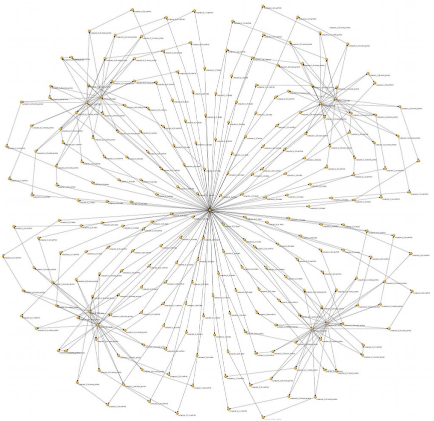

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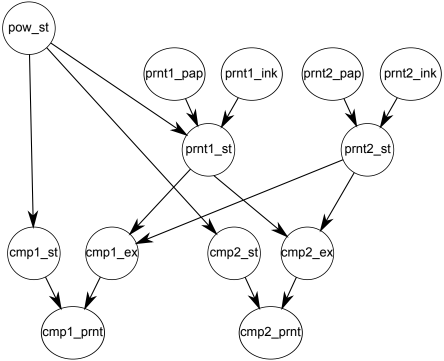

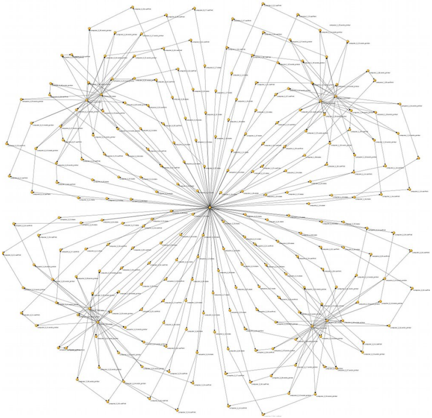

- Large-scale Bayesian networks “undesirable” features :
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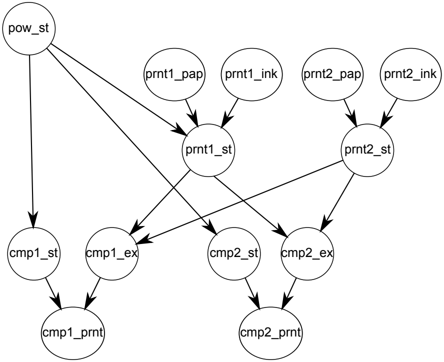
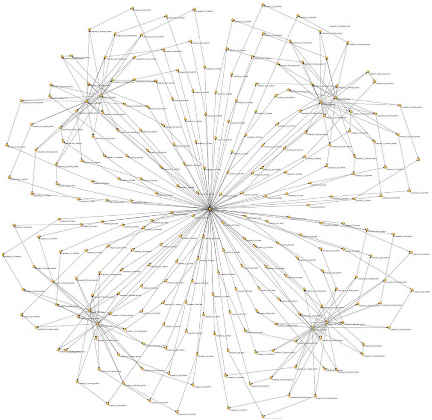
Motivation





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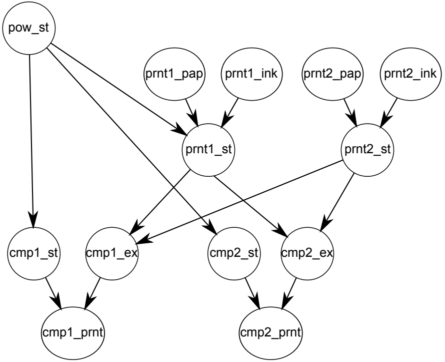
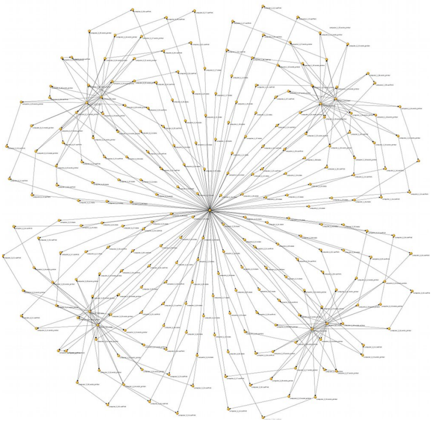
Motivation





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our goal : design complex PGM and speed-up inference in PRMs

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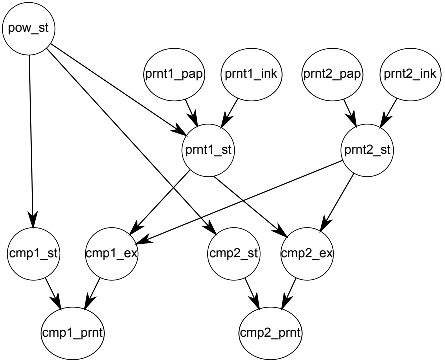

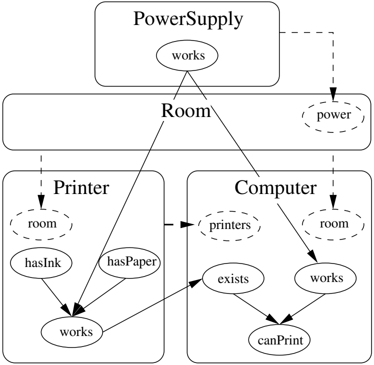

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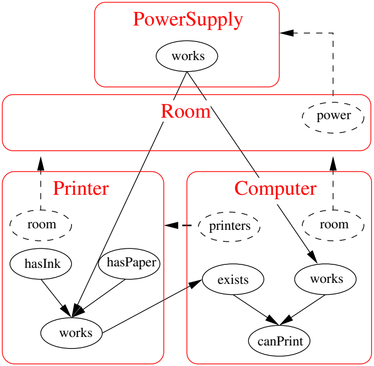

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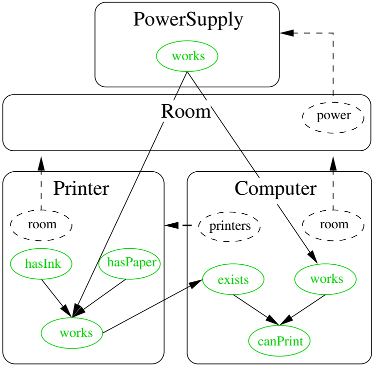

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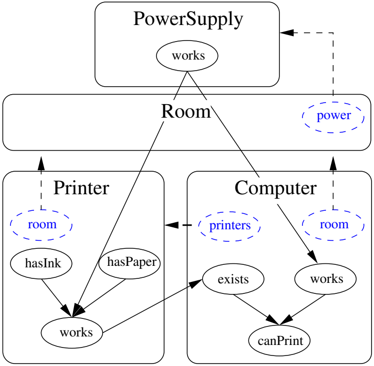

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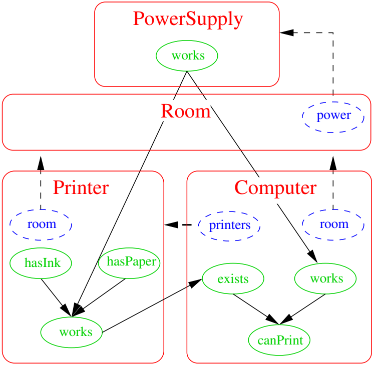
- Classes are instantiated in a system.



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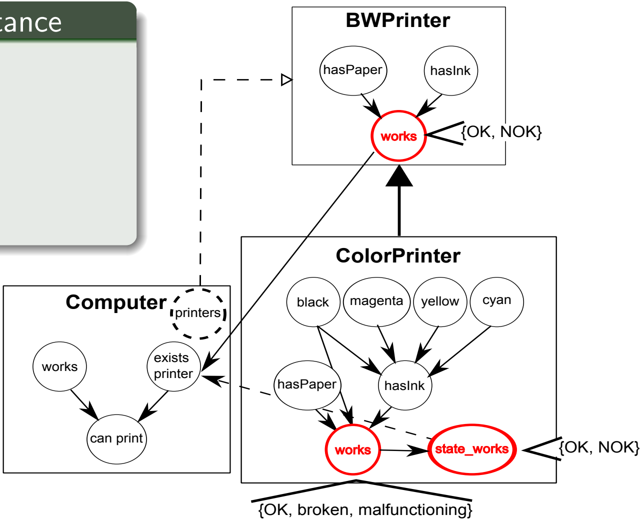
Enhanced class inheritance



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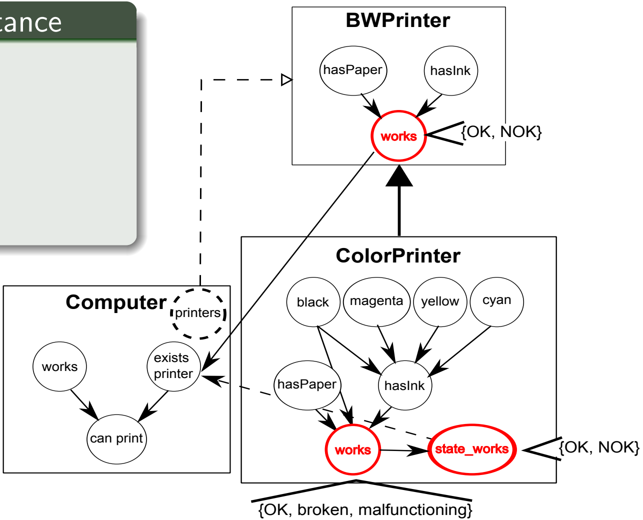

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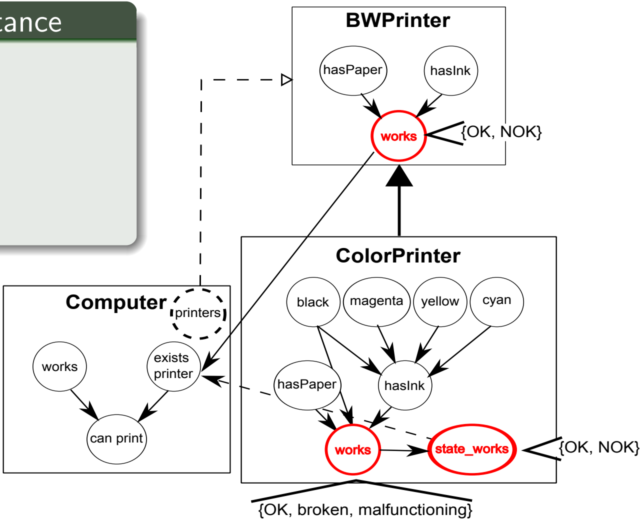

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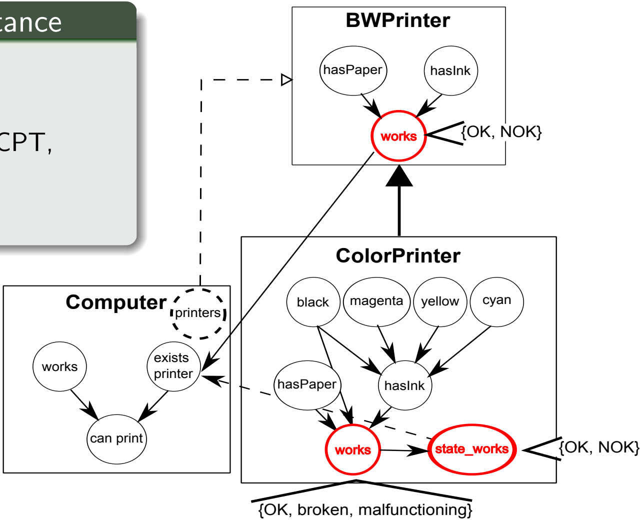

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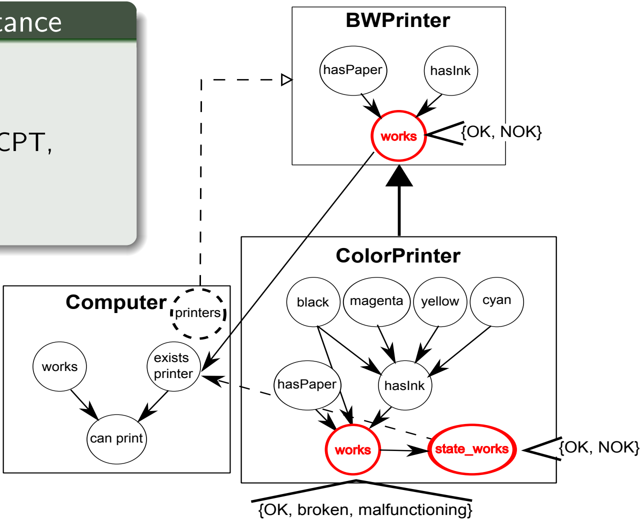

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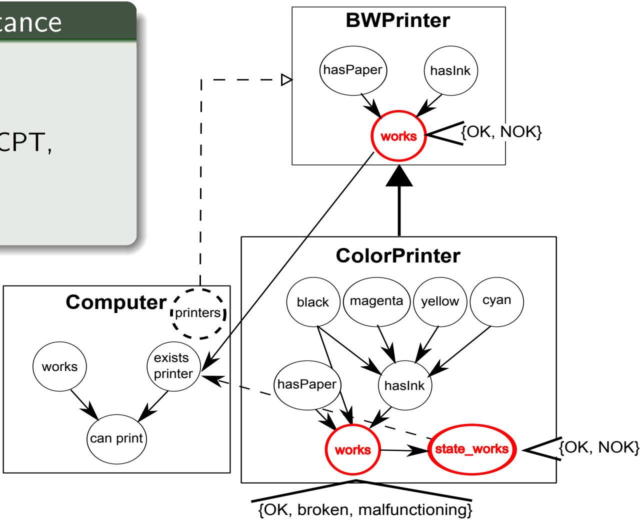

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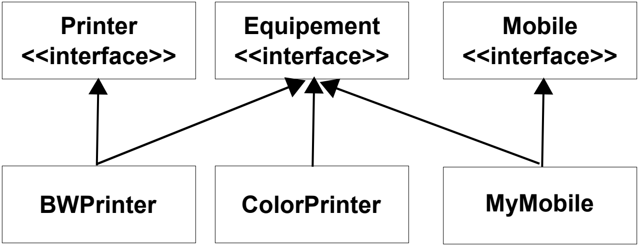

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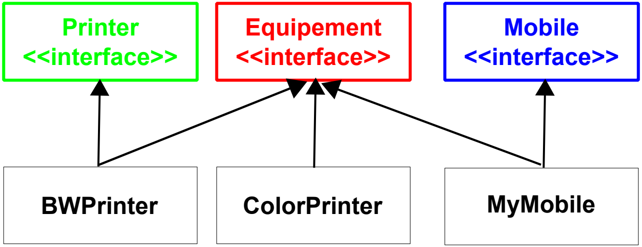
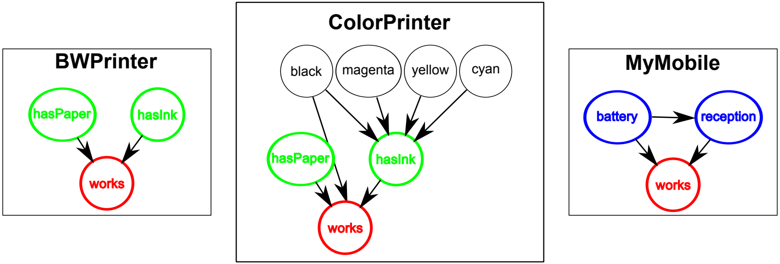
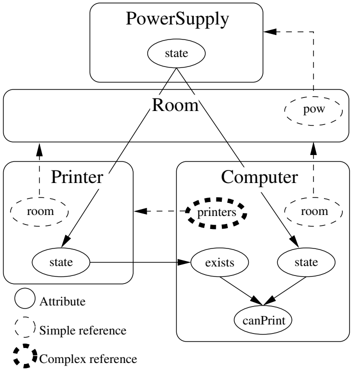




System : the printer example

A Class Dependency Graph.



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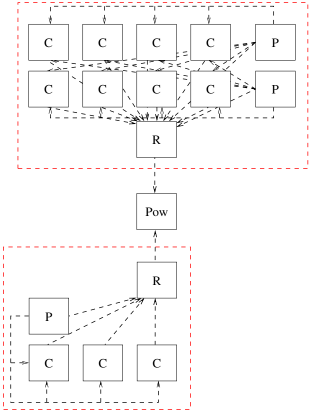

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- The models we created required the modification of the PRM framework presented here.

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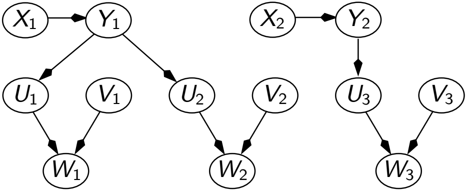

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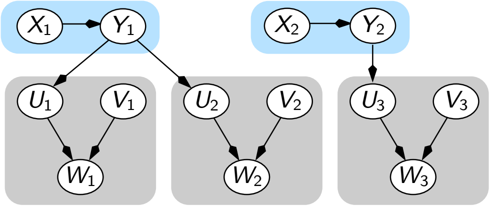

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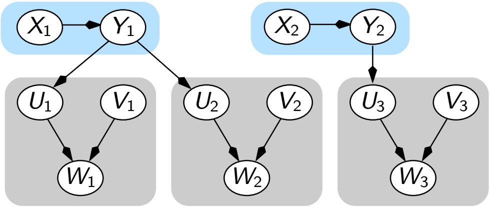

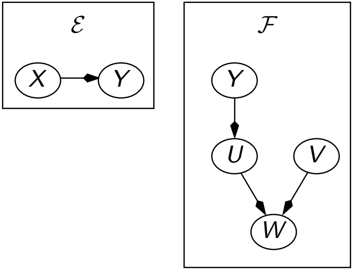

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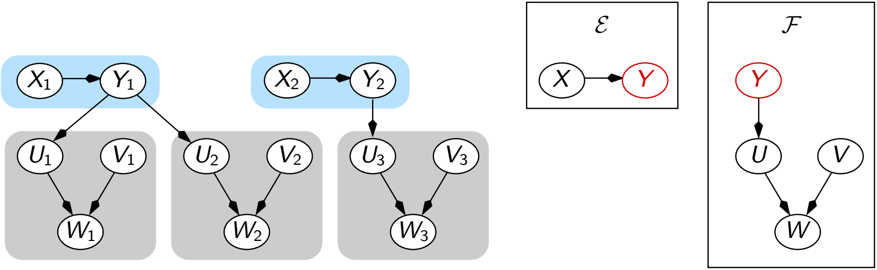

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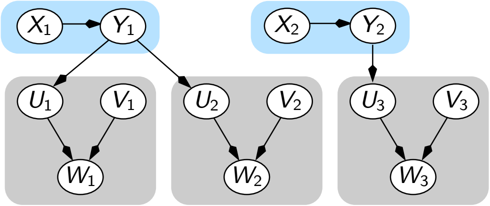

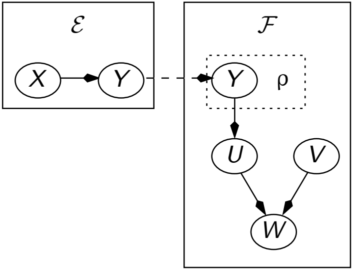

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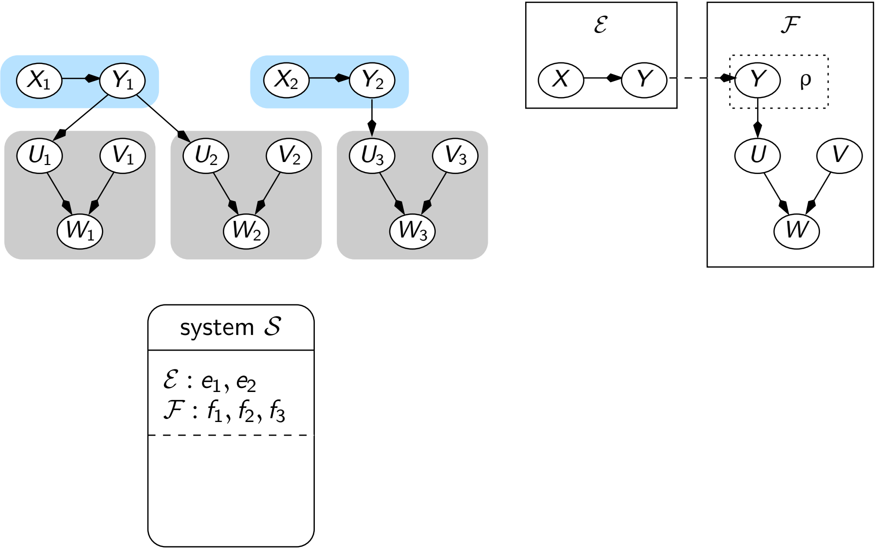

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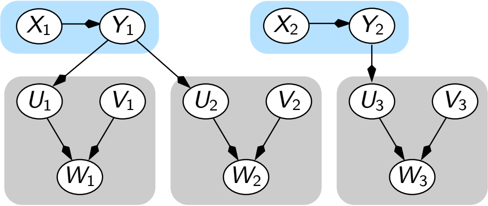

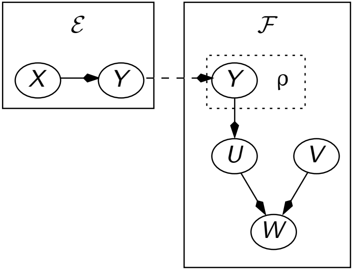

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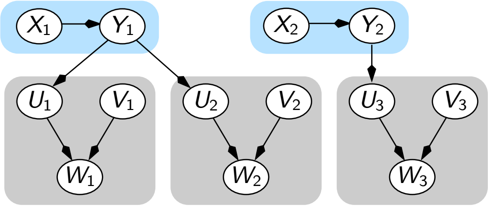

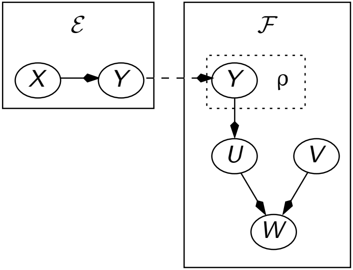

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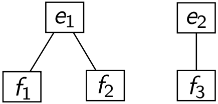

Relational skeleton

Structured Value Elimination [Pfeffer, 2000]

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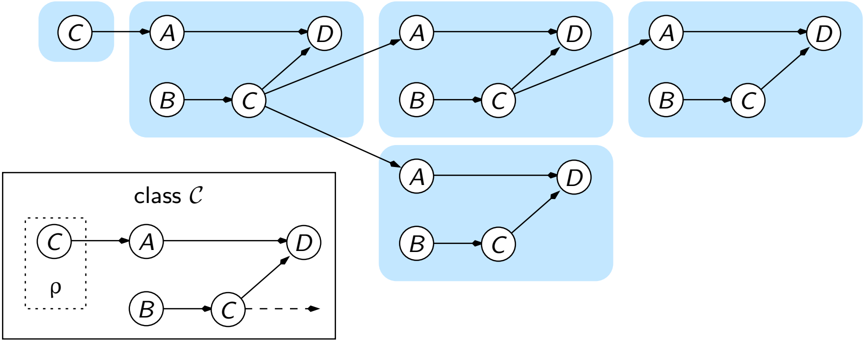
Once inner attributes are eliminated, SVE proceeds with a **bottom-up** elimination of each instance.

Structured Inference

Probability $P(C)$?



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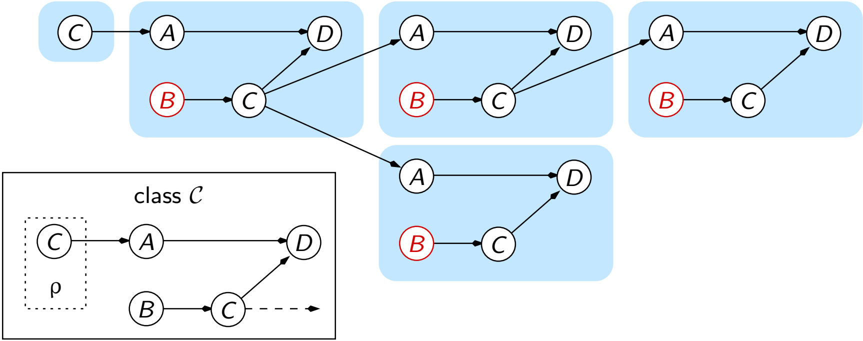

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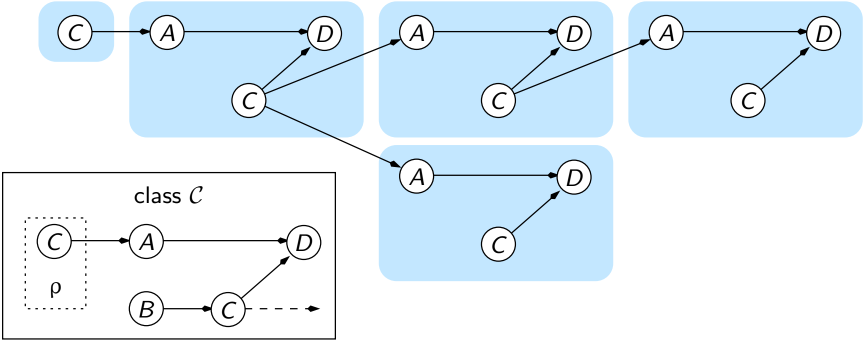

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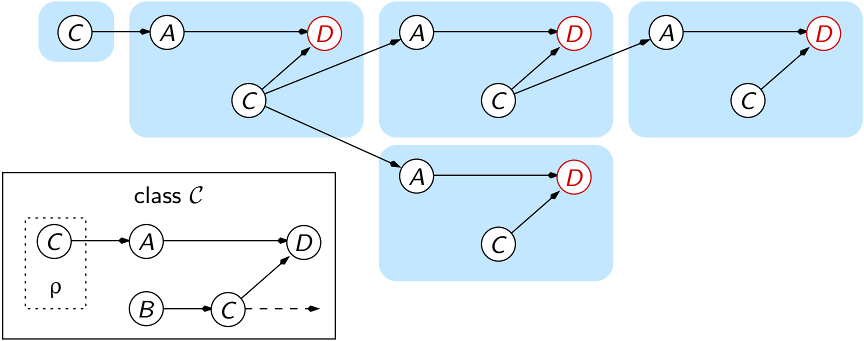

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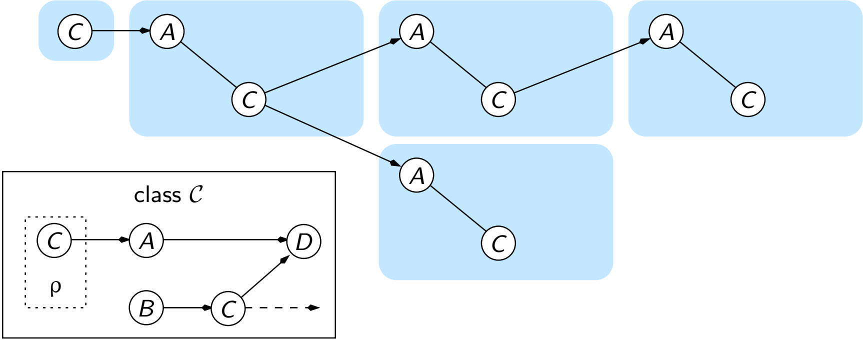

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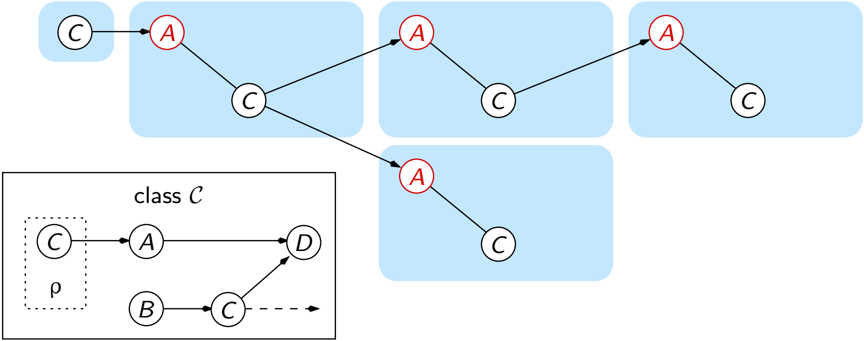

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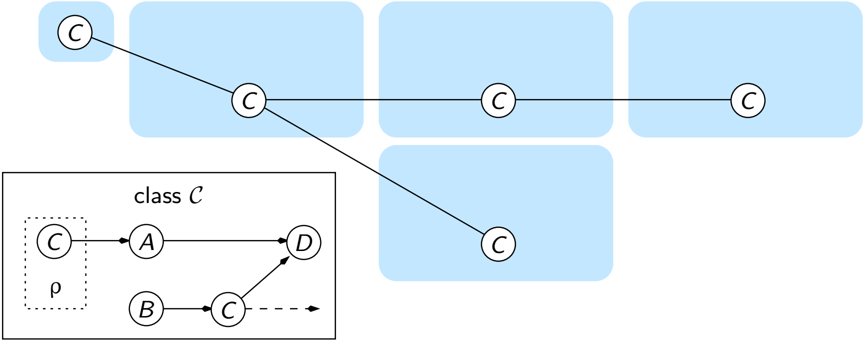

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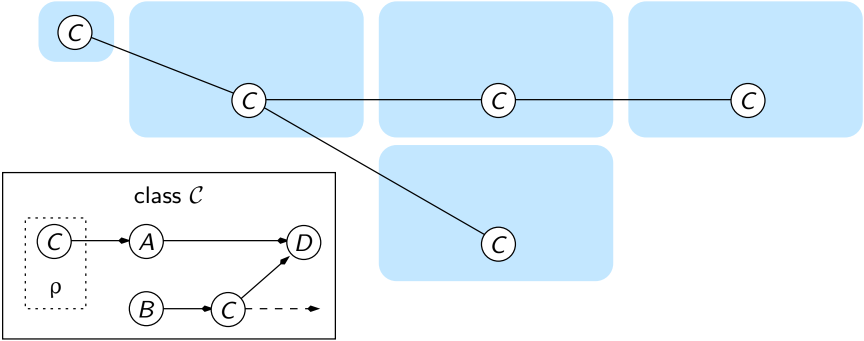

Elimination at class level : performed only once !

It works only if the inner attributes are not observed.
Only internal variables can be eliminated at class level !

D-Separation in PRMs ([Torti and Wuillemin, 2010])

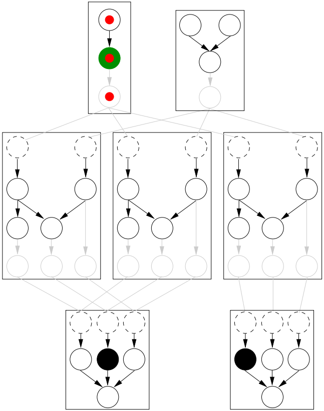

FIGURE: Different configurations due to evidence.

Integrating D-Separation in SVE

Following SVE bottom-up elimination order :

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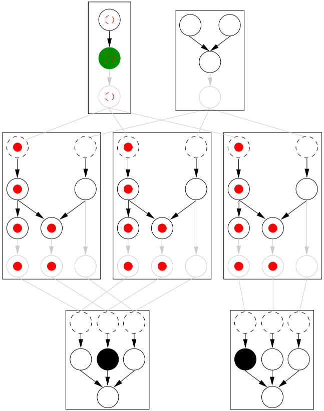

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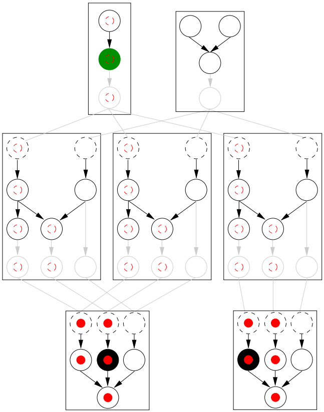

FIGURE: Different configurations due to evidence.

Integrating D-Separation in SVE

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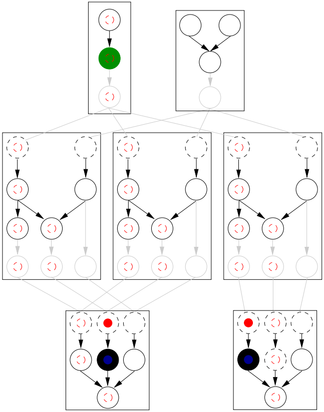

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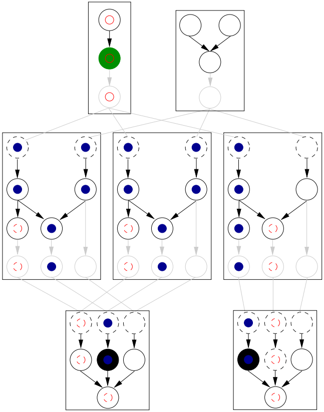

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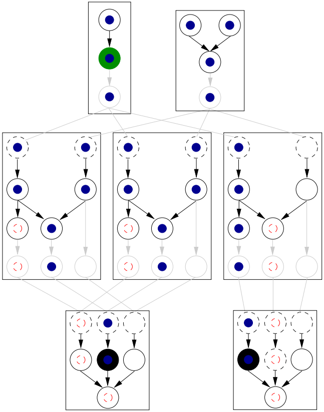

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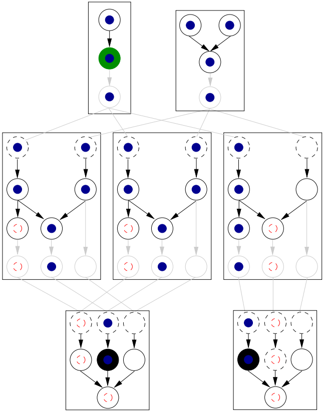

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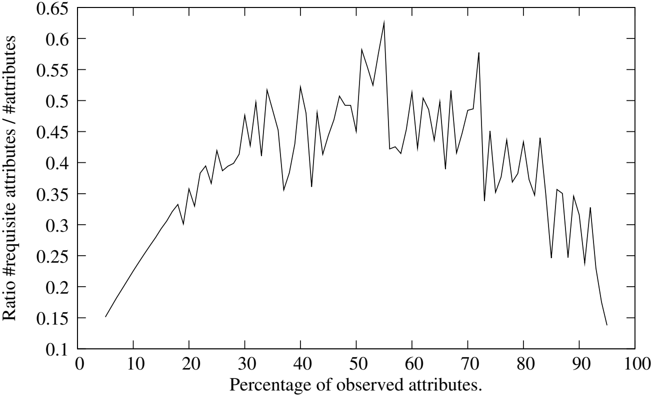
Generalized BayesBall rule

Experimental results

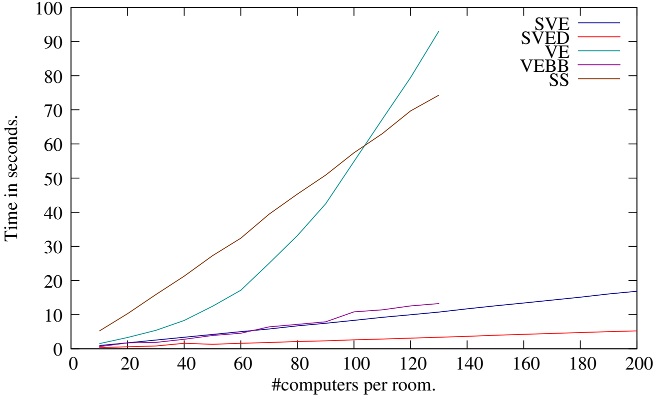
The example used for our experimentations

- We used the printer example with the following parameters :
  - different number of computers per room ;
  - different number of printers per room ;
  - different number of evidences.
- The examples we used have a 1400 to 24000 attributes.

Efficiency of D-Separation



Increasing the number of computers



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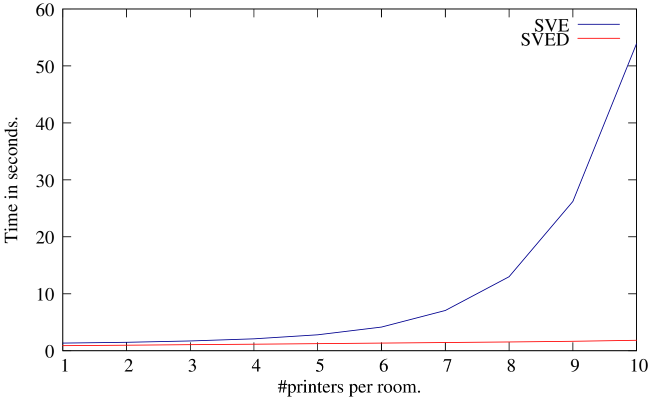

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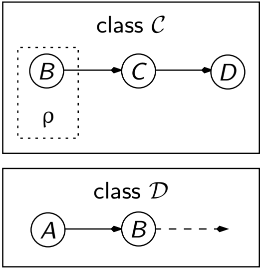

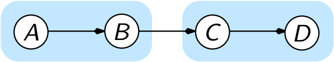

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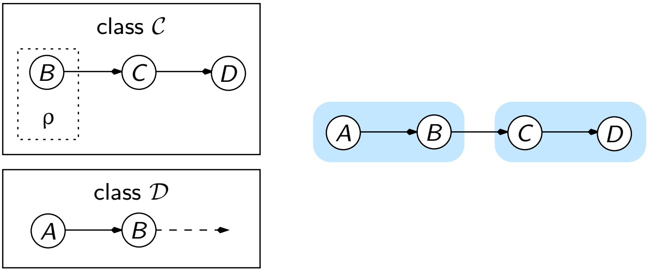

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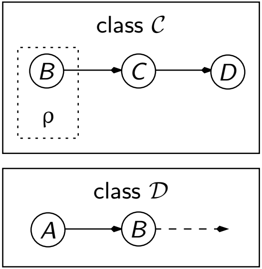

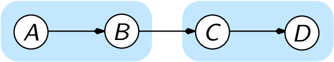

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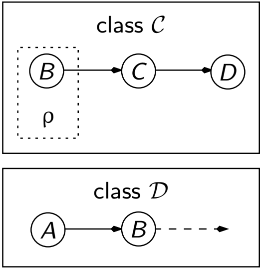

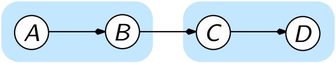

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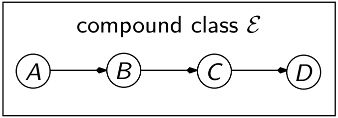
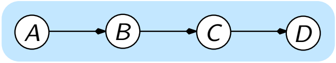
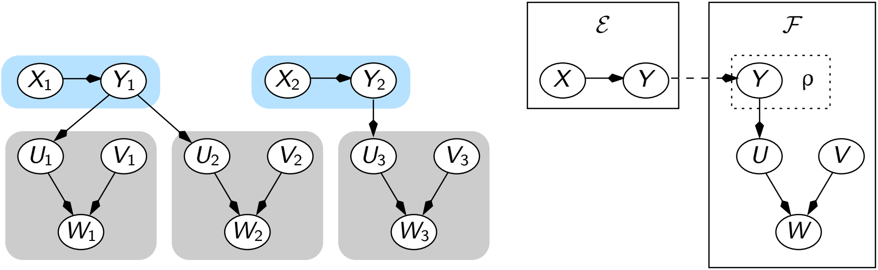




Substitution rule



Rule

One random variable can only belong to one instance.

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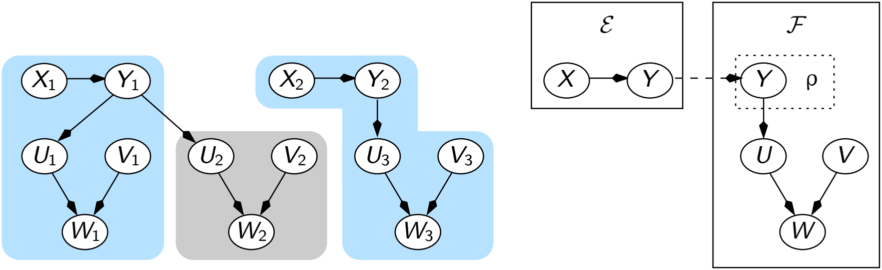

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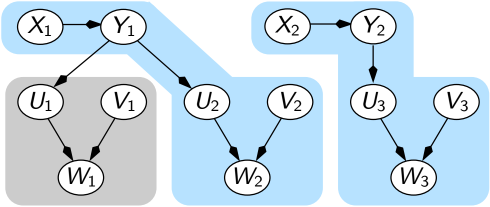

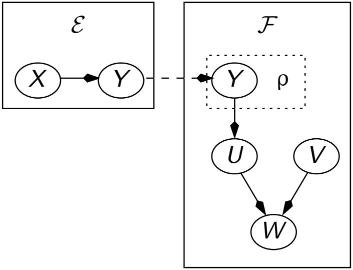

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The following problem is NP-hard :

Instance : a PRM, an integer $K \geq 0$

Question : does there exist a set of dynamic classes / substitutions s.t. the number of operations performed by structured inference is $\leq K$?

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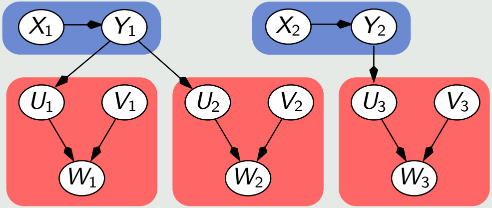
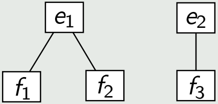
remains NP-hard even when classical inference is polynomial

Approximate algorithm

Boundary graph

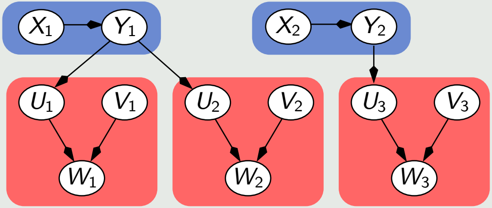


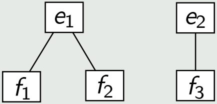


Relational skeleton

Approximate algorithm

Boundary graph





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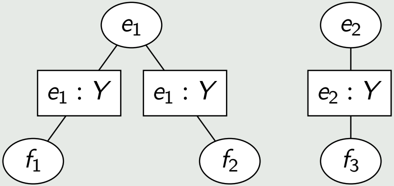

Boundary graph

$\Longrightarrow$ find frequent patterns in the boundary graph

Search tree

- Mining frequent subgraphs

  [Inokuchi et al, 05], [Kuramochi and Karypis, 01], [Yan and Han, 02]

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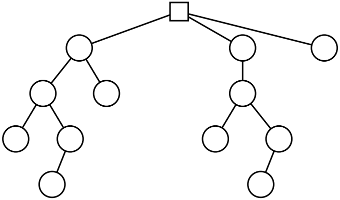
- Variant of the gSpan algorithm :

1 edge
2 edges
3 edges
4 edges



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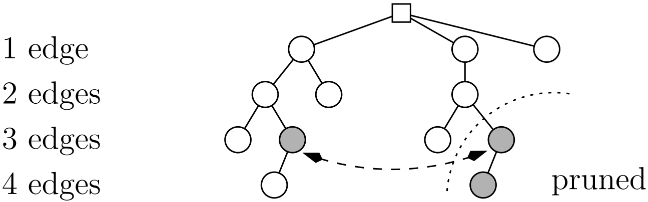

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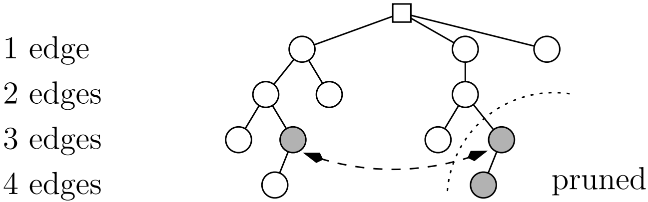

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- Possible substitutions (Rule 1) : Max Independence Set

Another pruning rule

Efficiency Pruning Rule

Prune nodes/classes whose substitutions do not speed-up inference.

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Prune nodes/classes whose substitutions do not speed-up inference.

- Gain estimation by dynamic programming

  $\Longrightarrow$ α-value : $\alpha > 0 \Longleftrightarrow$ class unattractive

search tree not monotonically α-decreasing !

- Rule applied : prune subtree whenever $\alpha > 0$

Experiments