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Biométrie et Intelligence Artificielle

Model-based adaptive spatial sampling for occurrence map construction

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ALIMENTATION
AGRICULTURE
ENVIRONNEMENT

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Département de mathématiques et informatique appliquées

INRA

Mapping spatial processes in environmental management

Different problems depending on observations nature

- Data visualization
  - Complete observations (everywhere)
  - Perfect observations (No errors/missing data)

  ⇒ How to visualize data?

- Map reconstruction
  - Complete observations
  - Noisy observations

  ⇒ How to reconstruct the “true” map?

- **Sampling and map construction**
  - Incomplete observations (not everywhere)
  - Noisy observations

  ⇒ Where to observe? / How to reconstruct?

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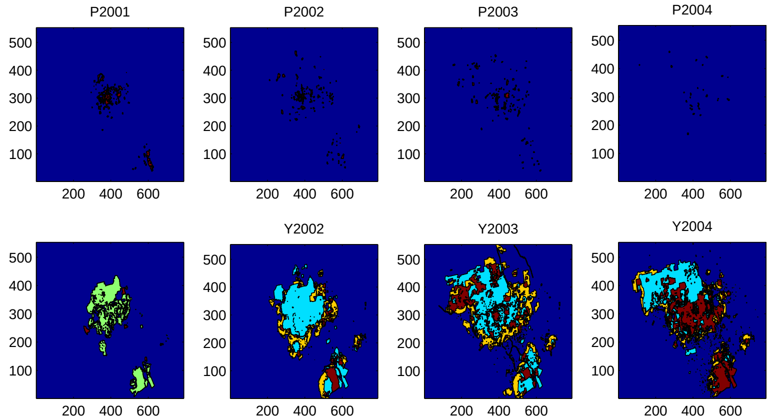
Mapping spatial processes in environmental management

Mapping pest occurrence

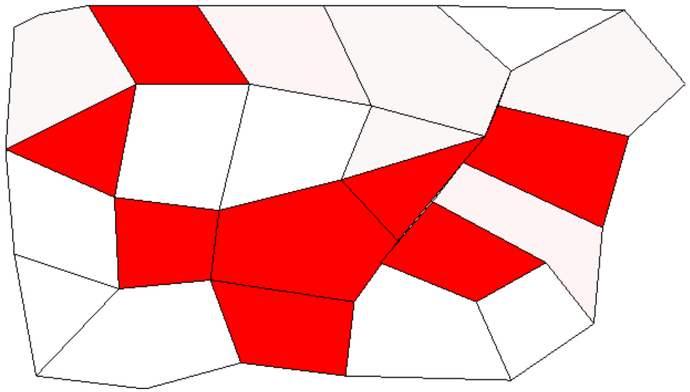
- Building pest occurrence map in order to eradicate
- Observations costly
- Errors in mapping also costly



Mapping spatial processes in environmental management

How to design an efficient spatial sampling method to estimate an occurrence (0/1) map when

- ✓ process to map has spatial structure
- ✓ observations are imperfect/incomplete
- ✓ sampling is costly
- ✓ process does not evolve during the sampling period



Overview of the proposed approach

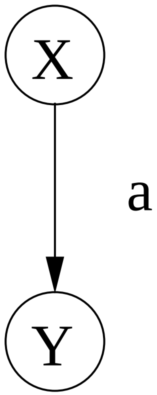
Optimization approach for designing spatial sampling policies

The Hidden Markov Random Field model is used for:

- Representing current uncertain knowledge about map to reconstruct
- Updating knowledge after observations
- Defining a unique criterion for
  - map reconstruction from observed data
  - spatial sampling actions selection
- Tradeoff between sampling actions cost and expected quality of the reconstructed map

Contents

Optimal sampling problem



Hidden variable X

Sampling action a

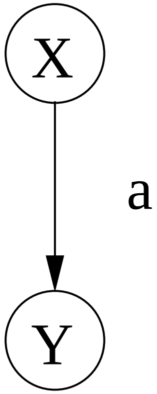
Observation model
$p(Y = o|x, a)$

Question: How to reconstruct hidden variable X using sampling actions?

1. Hidden variable model
2. Updated model after sampling result
3. Hidden variable reconstruction
4. Sampling action optimization

Optimal sampling problem

Hidden variable model



Prior model $P(x)$

Question: How to reconstruct hidden variable X using sampling actions?

1. Hidden variable model
2. Updated model after sampling result
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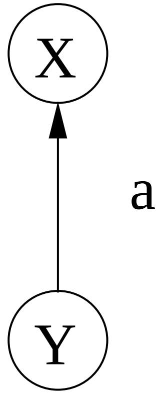
Optimal sampling problem

Updated model



Posterior: $P(x|o,a) = \frac{P(o|x,a)P(x)}{P(o|a)}$

Question: How to reconstruct hidden variable X using sampling actions?

1. Hidden variable model
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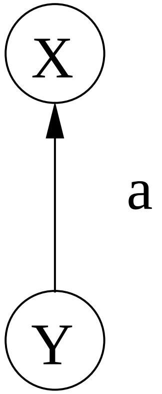
4. Sampling action optimization

Optimal sampling problem

Hidden variable reconstruction



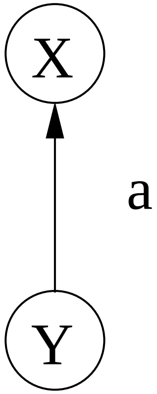
$$
\begin{aligned}
x^*(o,a) &= \arg\max_x P(x|o,a) \\
V(o,a) &= f(P(x^*|o,a))
\end{aligned}
$$

Question: How to reconstruct hidden variable X using sampling actions?

1. Hidden variable model
2. Updated model after sampling result
3. Hidden variable reconstruction
4. Sampling action optimization

Optimal sampling problem

Hidden variable reconstruction



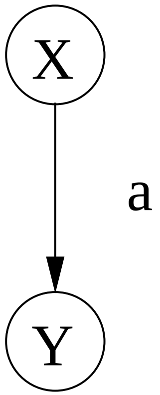
$$x^*(o,a) = \arg\max_x P(x|o,a)$$
$$V(o,a) = f(P(x^*|o,a))$$

Question: How to reconstruct hidden variable X using sampling actions?

- $x^*(o,a)$ is the best reconstruction given sampling result (o,a)
- $V(o,a)$ is the value of reconstructed variable after sampling result (o,a)

Optimal sampling problem

Sampling action optimization



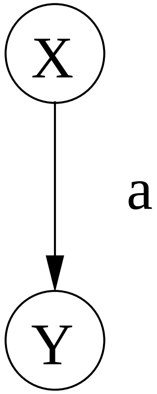
$$
\begin{aligned}
U(a) &= -c(a) + \sum_o P(o|a) V(o,a) \\
a^* &= \arg\max_a U(a)
\end{aligned}
$$

Question: How to reconstruct hidden variable X using sampling actions?

1. Hidden variable model
2. Updated model after sampling result
3. Hidden variable reconstruction
4. Sampling action optimization

Optimal sampling problem

Sampling action optimization



$$U(a) = -c(a) + \sum_o P(o|a)V(o,a)$$
$$a^* = \arg\max_a U(a)$$

Question: How to reconstruct hidden variable X using sampling actions?
The value of an action is a tradeoff between

- The cost $c(a)$ of the action and
- The expected quality of the reconstructed variable (over all possible sample results)

Contents

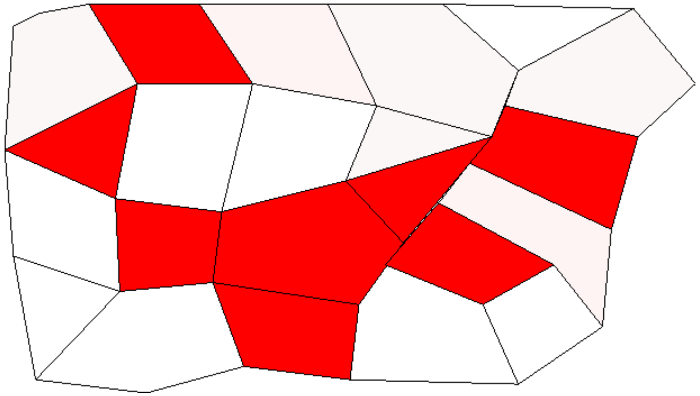
1- Optimal sampling of a hidden random variable

2- Defining optimal spatial sampling problems

3- Approximate computation of an optimal strategy

4- Evaluation of proposed method on simulated data

Spatial sampling optimization



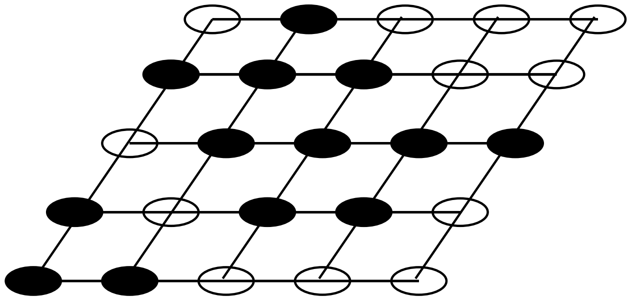
The hidden variable x is a map

$\Rightarrow$ The sampling optimization problem has to be revisited

Question: How to reconstruct hidden map x using sampling actions?

1. Hidden map model
2. Updated model after sampling result
3. Hidden map reconstruction
4. Sampling action optimization

Pairwise Markov random field (1)



- Multiple interacting variables
- Independence given neighborhood

$\Rightarrow$ Pairwise Markov random field

Question: How to reconstruct hidden map x using sampling actions?

1. Hidden map model
2. Updated model after sampling result
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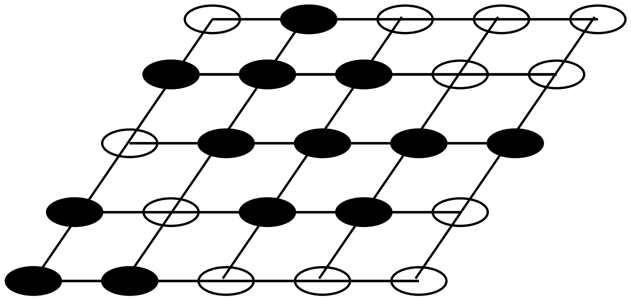
4. Sampling action optimization

Pairwise Markov random field (2)



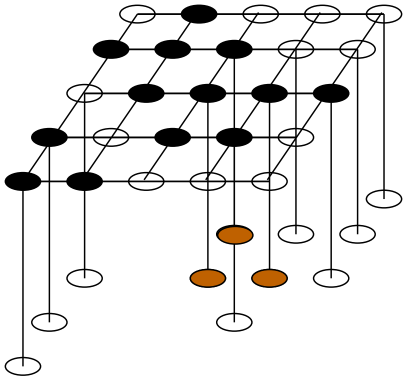
- Multiple interacting variables
- Independence given neighborhood

$\Rightarrow$ Pairwise Markov random field

- Interaction graph $G = (V, E)$
- ψ_i: "weights" on states of vertex i
- ψ_{ij}: correlations "strength" between neighbor vertices
- Z: normalizing constant / partition function

$$P(x) = \frac{1}{Z}\Big(\prod_{i \in V} \psi_i(x_i)\Big)\Big(\prod_{(i,j) \in E} \psi_{ij}(x_i, x_j)\Big)$$

Hidden Markov random field (1)



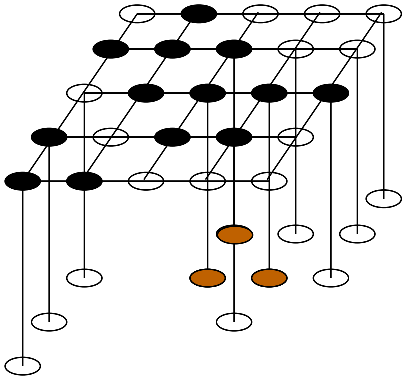
- $a \in \{0,1\}^{|V|}$: subset of V selected for sampling
- Independent observations:

$$P(o|x,a) = \prod_{i \in V} P_i(o_i|x_i,a_i)$$

Question: How to reconstruct hidden map x using sampling actions?

1. Hidden map model
2. Updated model after sampling result
3. Hidden map reconstruction
4. Sampling action optimization

Hidden Markov random field (2)



- $a \in \{0,1\}^{|V|}$: subset of V selected for sampling
- Independent observations:

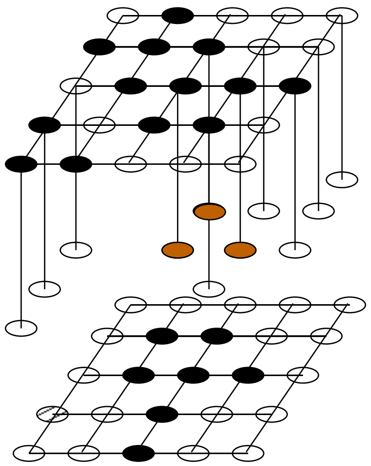
$$P(o|x,a) = \prod_{i \in V} P_i(o_i|x_i,a_i)$$

Updated Markov random field

$$P(x|o,a) = \frac{1}{Z}\Big(\prod_{i \in V} \psi'_i(x_i,o_i,a_i)\Big)\Big(\prod_{(i,j)\in E} \psi_{ij}(x_i,x_j)\Big) \text{ where}$$

$$\psi'_i(x_i,o_i,a_i) = \psi_i(x_i)P_i(o_i|x_i,a_i)$$

Hidden map reconstruction (1)



Global (MAP):

$$x^* = \arg\max_x P(x|o,a)$$

Local (MPM):

$$x_i^* = \arg\max_{x_i} P_i(x_i|o,a), \forall i \in V$$

Question: How to reconstruct hidden map x using sampling actions?

1. Hidden map model
2. Updated model after sampling result
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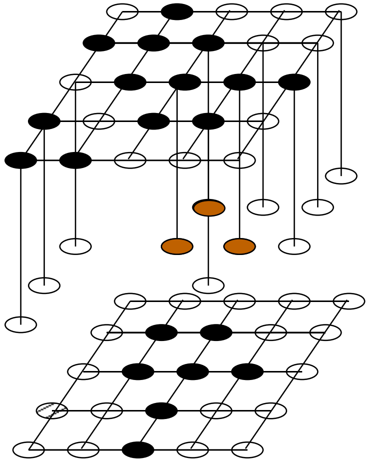
Hidden map reconstruction (2)



Global (MAP):

$$x^* = \arg\max_x P(x|o,a)$$

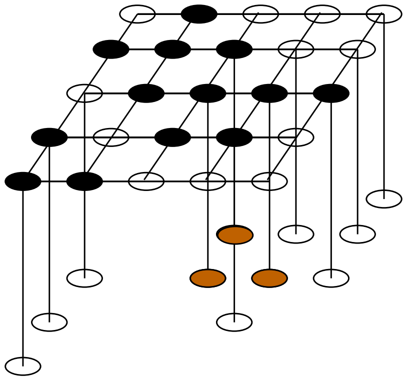
Local (MPM):

$$x_i^* = \arg\max_{x_i} P_i(x_i|o,a)$$

Value of reconstructed map

$$\begin{aligned} V^{MAP}(o,a) &= f\left(\max_x P(x|o,a)\right) \\ V^{MPM}(o,a) &= f\left(\sum_{i\in V} \max_{x_i} P_i(x_i|o,a)\right) \end{aligned}$$

Sampling action optimization (1)



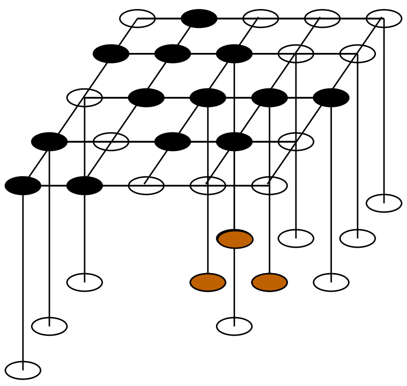
- $a \in \{0,1\}^{|V|}$ selected for sampling
- Independent observations $o \in \{0,1\}^{|V|}$

$\Rightarrow$ How to optimize the choice of a?

Question: How to reconstruct hidden map x using sampling actions?

1. Hidden map model
2. Updated model after sampling result
3. Hidden map reconstruction
4. Sampling action optimization

Sampling action optimization (2)



- $a \subseteq V$ selected for sampling
- Independent observations o result

$\Rightarrow$ How to optimize the choice of a?

$$
\begin{aligned}
U(a) &= -c(a) + \sum_y P(y|a)V(y,a) \\
a^* &= \arg\max_a U(a)
\end{aligned}
$$

- The computation of a^* is hard! (NP-hard)
- Only feasible for small problems or needs approximation!

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Approximate spatial sampling (1)

Approximate the computation of

$$a^* = \arg\max_a -c(a) + \sum_o P(o|a)V^{MPM}(o,a)$$

- Explore cells where initial knowledge is the most uncertain: marginal $P_i(x_i|o,a)$ closest to $\frac{1}{2}$

$$\tilde{a} = \arg\max_a -c(a) + f\left(\sum_{i,a_i=1} \min\left\{P_i(X_i=1|o,a), P_i(X_i=0|o,a)\right\}\right)$$

- Marginals computation is itself *NP-hard*

$\Rightarrow$ approximation using belief propagation (sum prod) algorithm

Approximate spatial sampling (2)

The approximation corresponds to two simplifying assumptions

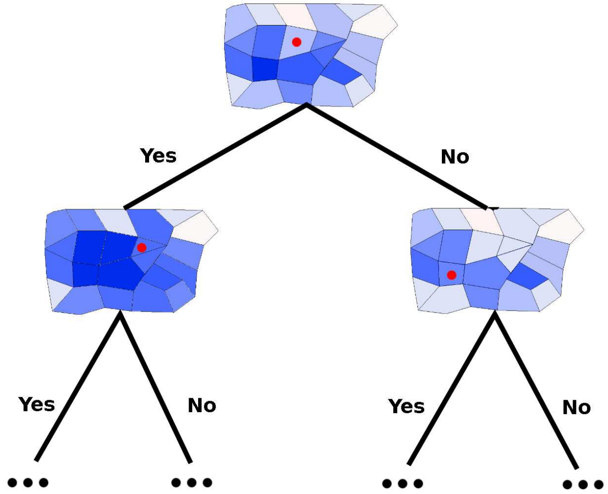
- Additional bservations are reliable and there are no passive observations: $\theta_1 = 1$ and $\theta_0 = 0$
- Joint probability approximated as a product of “approximate conditional marginals”: $P(x|o,a) \sim \prod_{i \in V} \tilde{P}_i(x_i|o,a)$

Adaptive spatial sampling (1)

- Idea:
  - Sampling locations not settled once for all before the sampling campaign
  - Intermediate observations are taken into account to design next sampling step
  - Possibility to visit a cell more than once

Adaptive spatial sampling (2)

- a sampling strategy δ is a tree
- a trajectory in δ: $\tau = (a^1, o^1, \ldots, a^K, o^K)$



Value of a leaf

$$U(\tau) = -\sum_{k=1}^{K} c(a^k) + V^{MPM}(o^0, o^1, \ldots, o^K, a^0, a^1, \ldots, a^K)$$

Value of a strategy $V(\delta) = \sum_\tau U(\tau) P(\tau \mid \delta)$

Heuristic adaptive spatial sampling

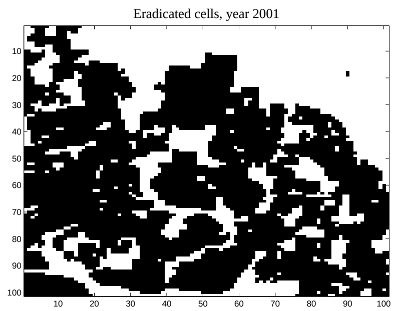
- Exact computation is *PSPACE-hard* !

⇒ Heuristic algorithm

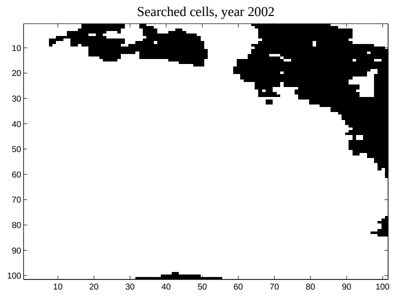
- on line computation
- approximate method for static sampling at each step

Contents

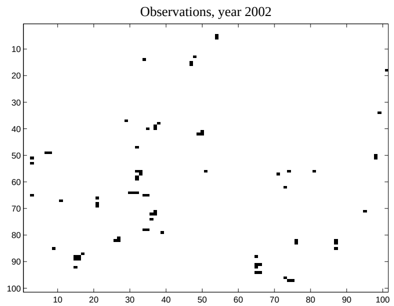
HMRF model for fire ants problem (1)



Eradication
(e)



Search actions
(a)



Observations
(o)

- eradication (at previous year): $e_i \in \{0, 1\},\ i = 1, \ldots n$
- search actions: passive search or active search, $a_i \in \{0, 1\},\ i = 1, \ldots n$
- observations: no nest detected / at least one nest detected, $o_i \in \{0, 1\},\ i = 1, \ldots n$

HMRF model for fire ants problem (2)

- Distribution on maps = Potts model

$$P_e(x \mid \alpha, \beta) = \frac{1}{Z} \exp\left(\sum_{i \in V} \alpha_{e_i} \operatorname{eq}(x_i, 1) + \beta \sum_{(i,j) \in E} \operatorname{eq}(x_i, x_j) \right)$$

- Distribution of observation given map, $P_{a_i}(o_i \mid x_i, \theta)$

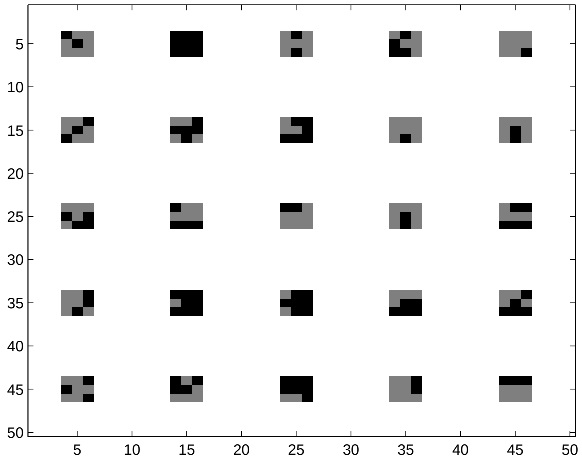
$o_i \setminus x_i$	0	1
0	1	$1 - \theta_{a_i}$
1	0	θ_{a_i}

with $\theta_0 < \theta_1$

HMRF model for fire ants problem (3)

An initial arbitrary sampling (a^0, o^0) is used for:

- Parameters estimation: $\lambda = (\alpha, \beta, \theta)$
  approximate version of EM for HMRF (Simul field EM)
  - identification problem between α and θ
  - OK if θ known: use of expert values
- Marginals computation: $P_i(x_i | o_i^0, a_i^0)$



Heuristic sampling methods evaluation (1)

- Evaluation on simulated data
- Comparison of behavior of
  - random sampling (RS)
  - adaptive cluster sampling (ACS)
  - static heuristic sampling (SHS)
  - adaptive heuristic sampling (AHS)

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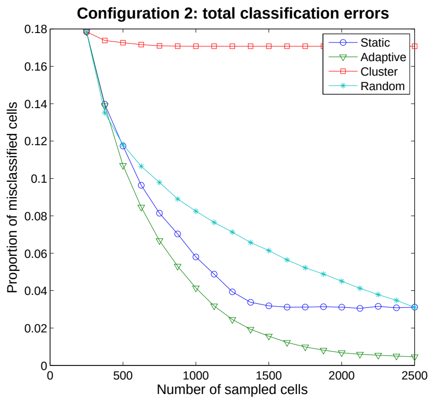
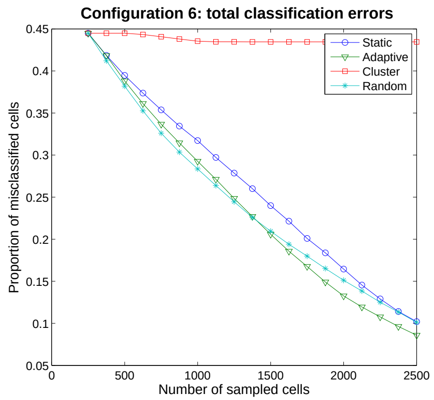
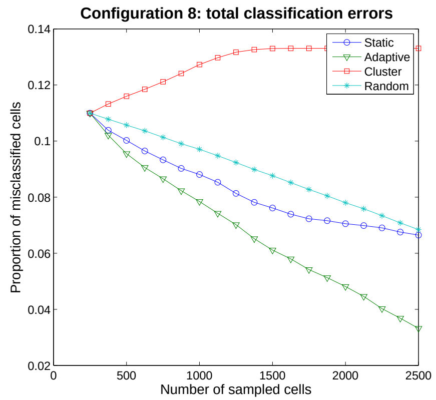
Heuristic sampling methods evaluation (2)

- Procedure: repeat 10 times
  - simulate hidden map x from $P(x \mid \alpha, \beta)$ (50×50 cells)
  - apply regular sampling (about 10% of area): a^0
  - simulate o^0 from $P_{a_i}(o_i \mid x_i, \theta)$ (regular sampling plus passive search)
  - estimate initial knowledge
  - apply RS, ACS, SHS, AHS, 10 times

Rate of misclassified cells







$\alpha = (0, -2), \beta = 0.8$ $\alpha = (0, 0), \beta = 0.5$ $\alpha = (1 - 1), \beta = 0.4$

$$\theta = (0, 0.8)$$

legend: SHS AHA ACS RS

Per color error rate

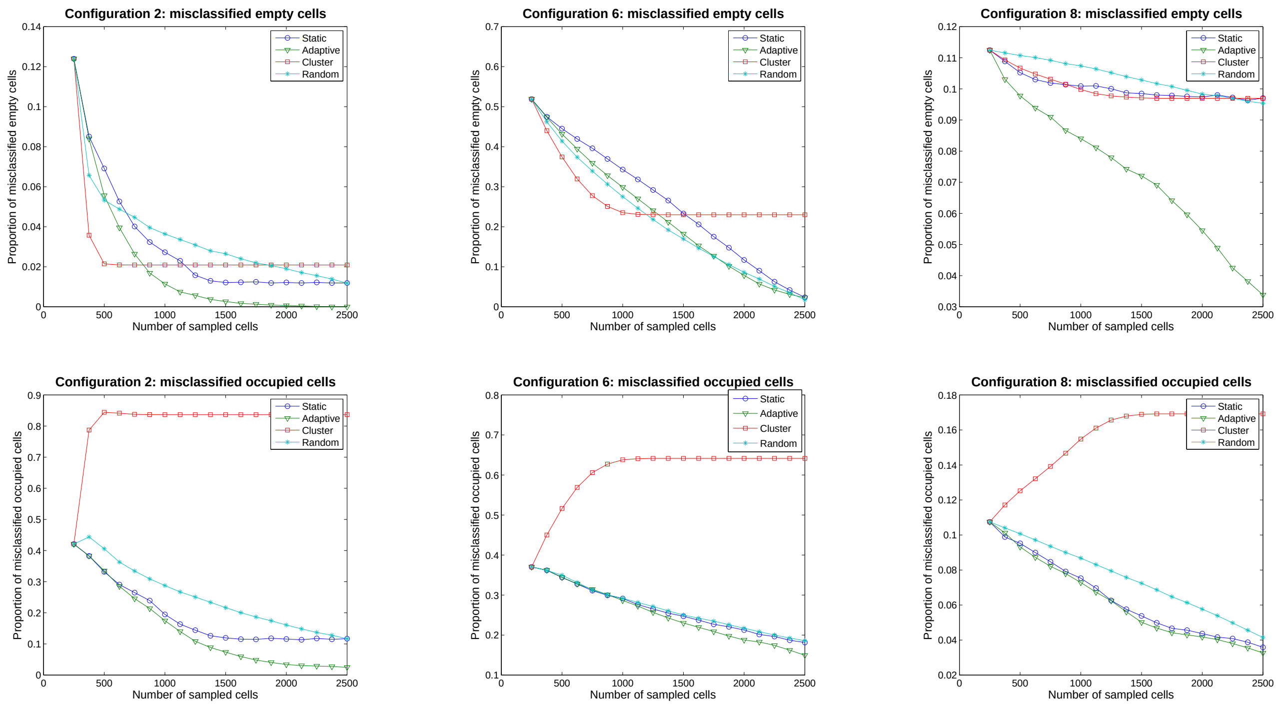
misclassified
empty cells

misclassified
occupied
cells

egend:

SHS AHA ACS

RS



$$\alpha = (0, -2) \quad \beta = 0.8$$

$$\alpha = (0, 0) \quad \beta = 0.5$$

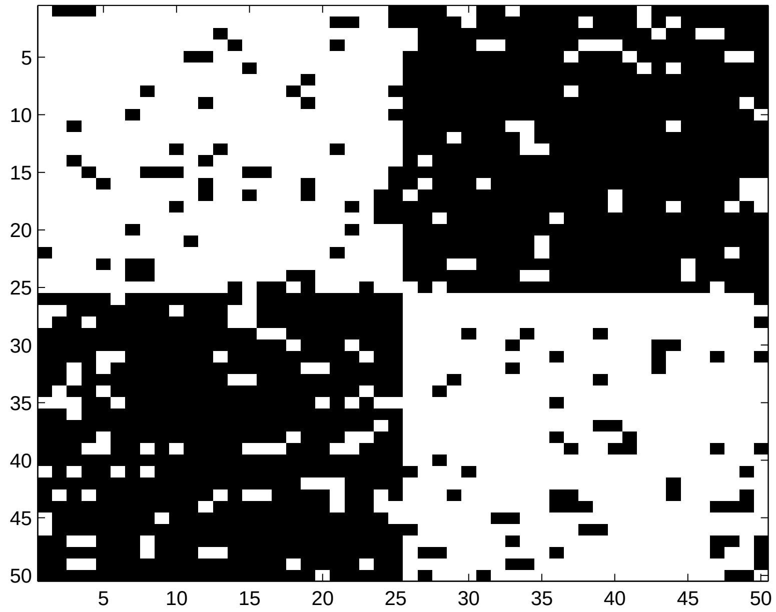
$$\alpha = (1 - 1) \quad \beta = 0.4$$

General behavior

- ACS is not adapted (as expected): poor results
- Adaptive HS $\geq$ Static HS $\geq$ Random S
- Discrepancy between Adaptive HS and Static HS increases with
  - sampling ressources
  - hidden map structure

Where do we sample?

Hidden map



$$\alpha = (1, -1), \beta = 0.4, \theta = (0, 0.8)$$

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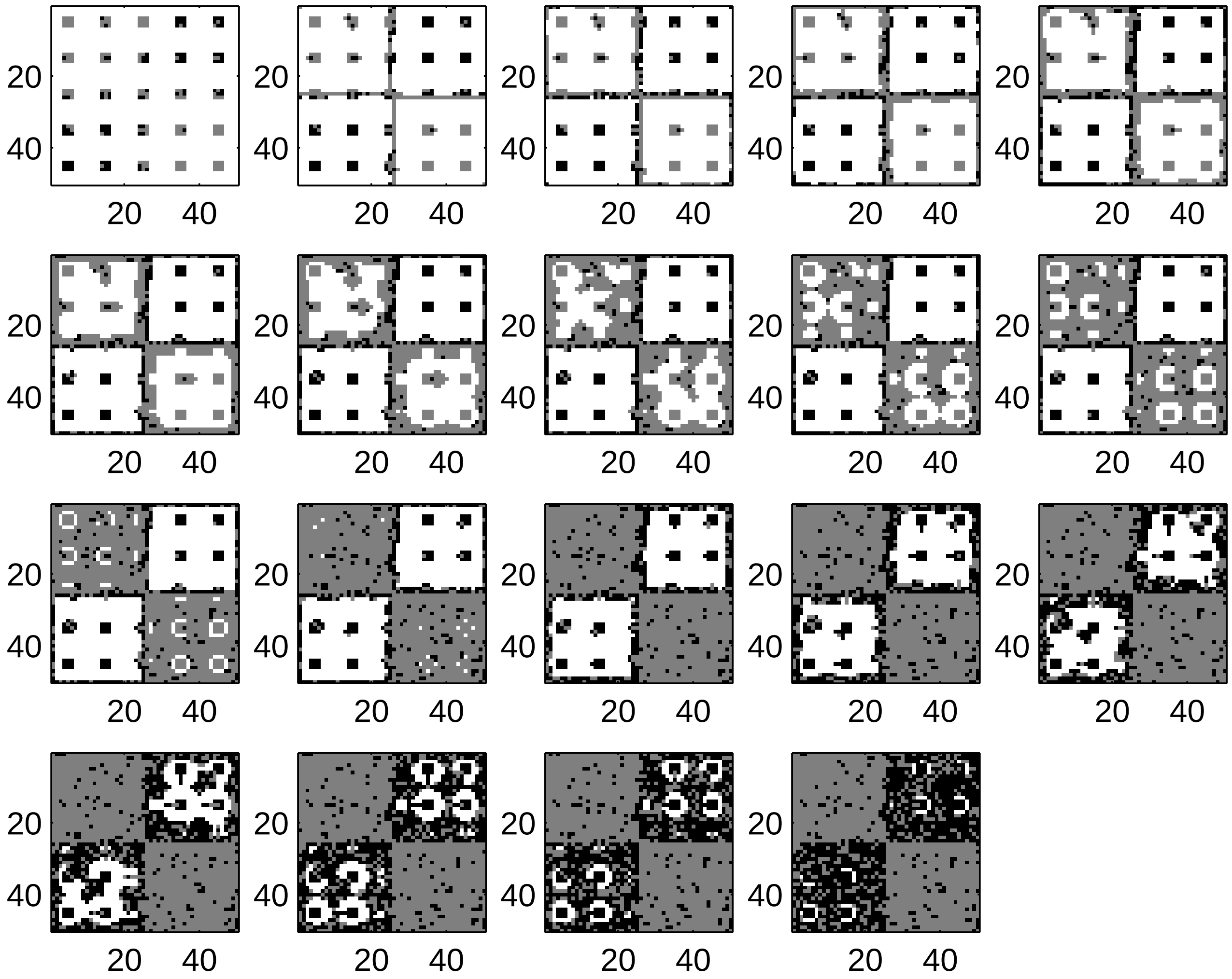
Where do we sample?

Static sampling: A and O



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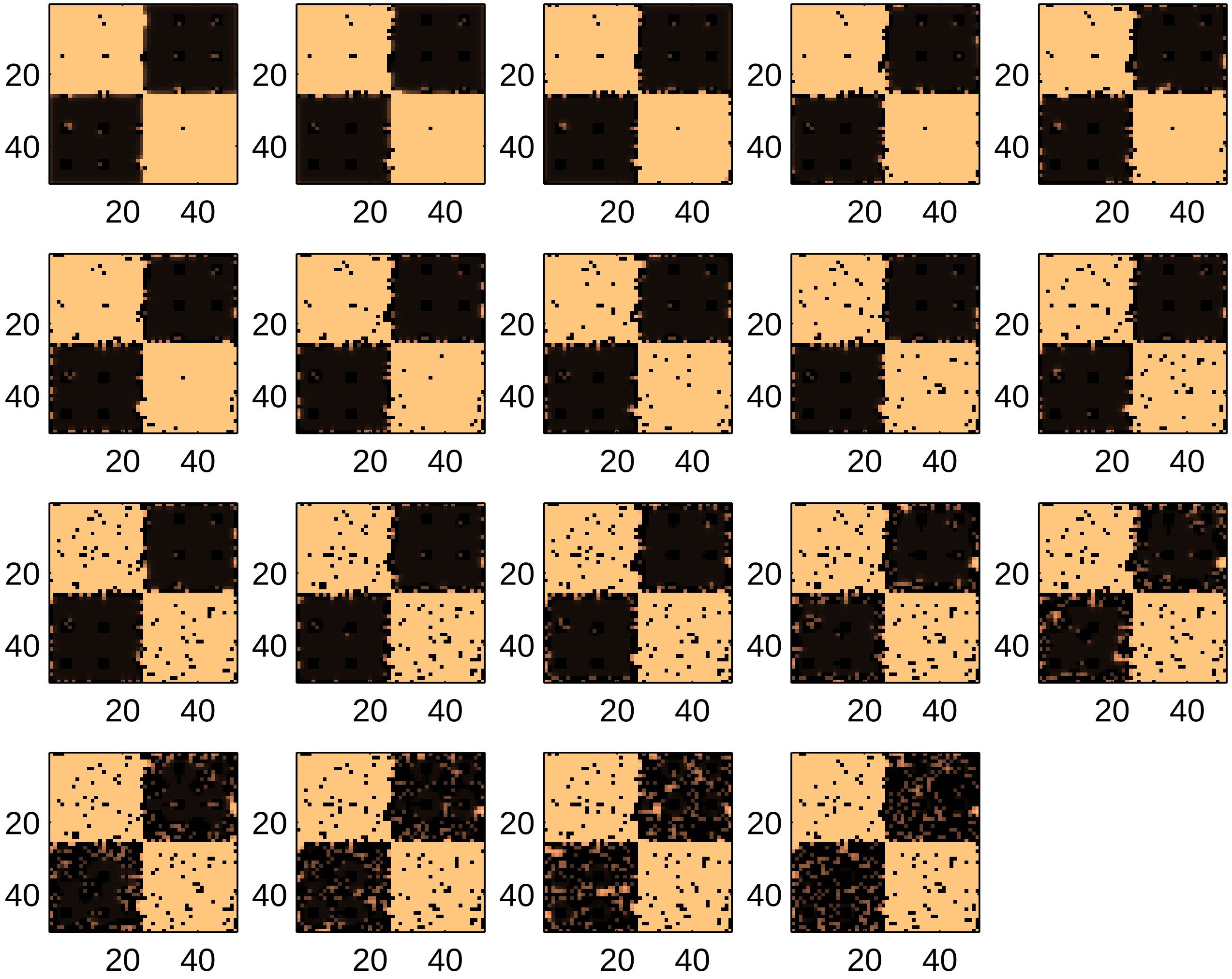
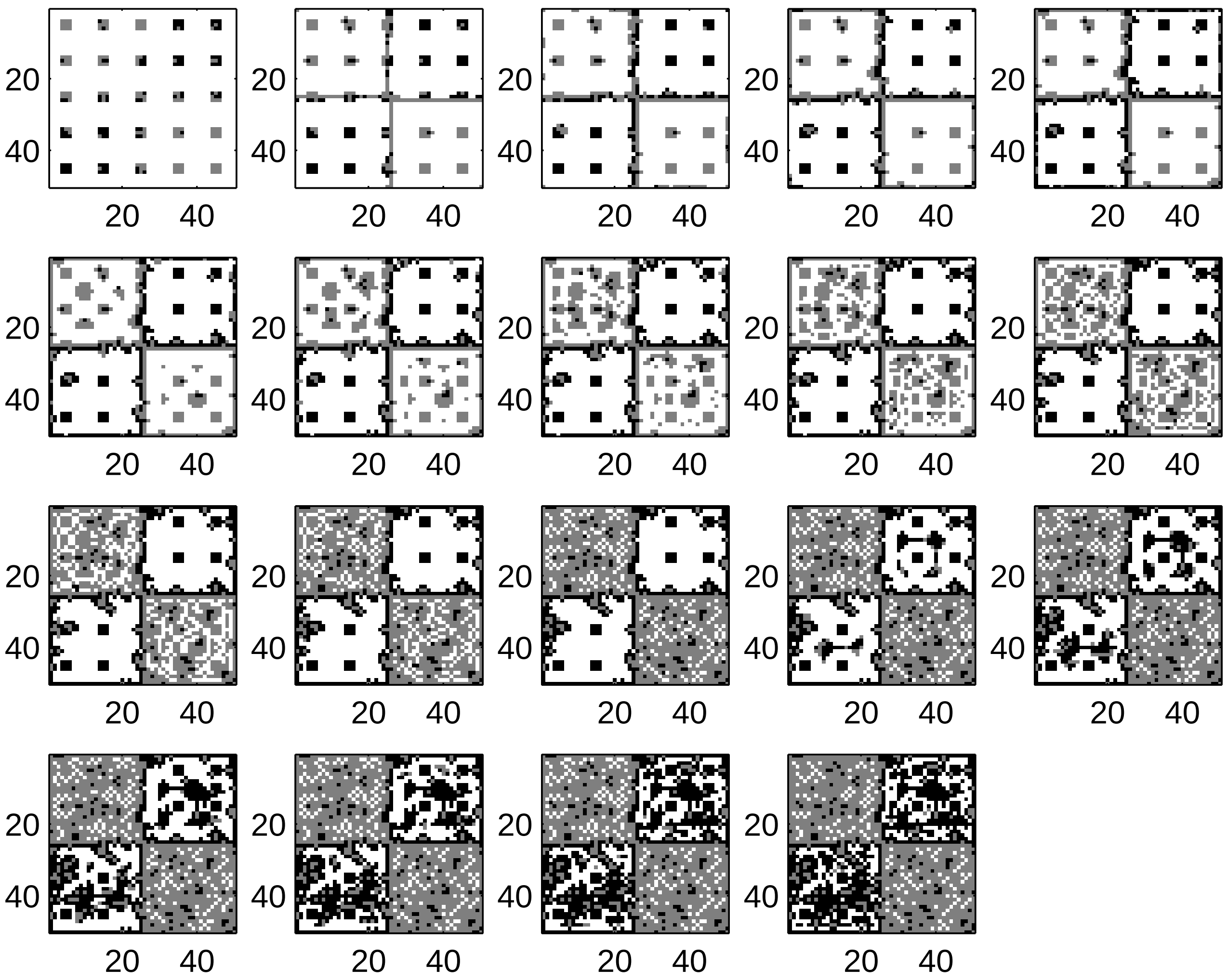
Where do we sample?

Static sampling:marginals



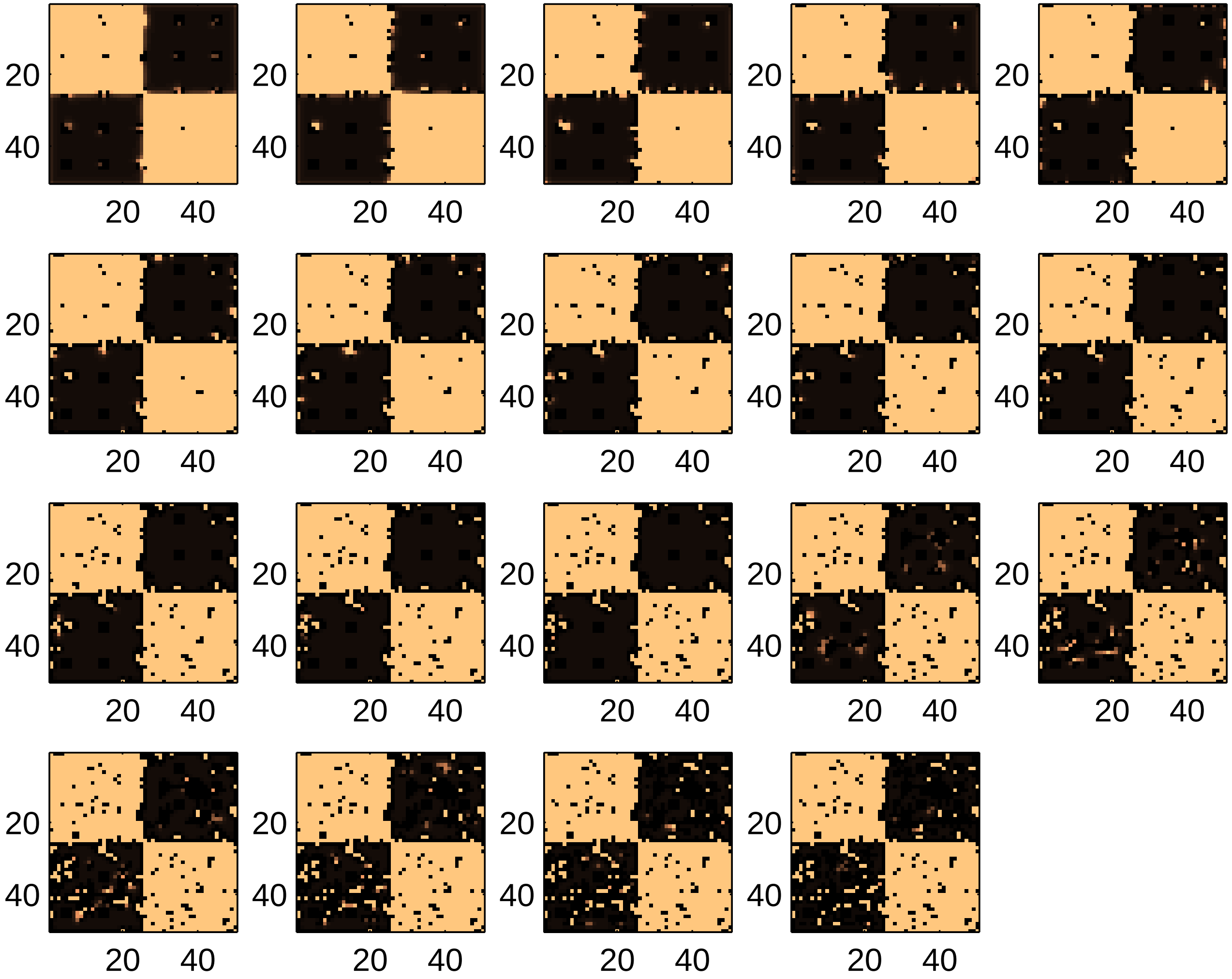
Where do we sample?

Adaptive sampling: A and O (cumul)



Where do we sample?

Adaptive sampling: marginals



Where do we sample?

- No sampling in large empty areas
- Sampling preferably near detected occupied sites within low density areas
- If sampling ressources increase
  - SHS complete exploration until the whole area is covered
  - AHA can visit several times a site before extending exploration to another area

Concluding remarks

- A framework for spatial sampling optimization:
  - based on Hidden Markov random fields
  - different map quality criteria
  - extended to “adaptive” sampling
- Problems too complex for exact resolution

  ⇒ Heuristic solution based on approximate marginals computation

Ongoing work

- Exact algorithms for small problems (Usman Farrokh): combining variable elimination and tree search
- “Random sets + kriging” approach (Mathieu Bonneau): development of a dedicated approximate method and comparison to the HMRF approach
- PhD thesis on *adaptive spatial sampling for weeds mapping at the scale of the agricultural area* (Sabrina Gaba, INRA-Dijon).

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Future work

- Smarter/finer approximation methods:
  - relaxing the assumptions of independence and reliable observation
  - simulation-based optimization (reinforcement learning)
- Using space-time data for estimating current prior
- Interleaving estimation and optimization (adaptive case)
- Considering controlled “dynamic” spatial processes (invasion/eradication):

  $\Rightarrow$ Spatial partially observed Markov decision processes

Questions?

Thanks for listening